Complexity Theory (Winter 2016/17)	Problem Set #2 (Due 30.11.2016)

Problem 1.1. In the time hierarchy theorem, where did we use time constructibility?

Problem 1.2. Prove the space hierarchy theorem.

Problem 1.3. Explain why the following argument does not show $P \neq NP$. Assume $P = NP$ and obtain a contradiction. If $P = NP$, then $SAT \in P$, and so for some k, $SAT \in DTIME(n^k)$. Since every language in NP is polynomial time reducible to SAT, we have $NP \subseteq DTIME(n^k)$. So $P \subseteq DTIME(n^k)$. But by the time hierarchy theorem, $DTIME(n^{k+1})$ contains a language that is not in $DTIME(n^k)$. This language is in P. This is a contradiction, so $P \neq NP$.

Problem 1.4. Prove that if $P = NP$ then $NP = coNP$.

Problem 1.5. Show that NL is closed under union, intersection, and star. For any language L, the *star* L^* is defined as the language $L^* = \{w \mid \exists k > 0. \exists u_1, \ldots, u_k : w = u_1 \ldots u_k \text{ and each } u_i \in L\}$.

Problem 1.5. A language is called *unary* if it is a subset of 1^*. Show that there is an undecidable unary language. (Somewhat hard.) Show that if there is an NP-complete unary language, then $P = NP$. [Hint: Try solving SAT by making queries to the unary NP-complete language, and caching all queries. Argue the cache has polynomially many entries.]

Problem 1.6. Show that SAT is NP-complete and QBF is $PSPACE$-complete under *logspace* reductions. [Sketch the argument briefly. You need not write down the logarithmic space reduction, but read the proof of Cook-Levin Theorem and the proof that QBF is $PSPACE$-complete from the book to convince yourself that the reduction can be performed in logarithimic space.]

Problem 1.7. Let

$$SC = \bigcup_{c>0} TIME-SPACE(n^c, \log^c n)$$

be the class of problems solvable by deterministic TMs running in polynomial time and using poly-logarithmic space. It is open if $PATH \in SC$. Why doesn't Savitch's theorem resolve this question?