$\mathit{prog} \quad ::=$

$\mathit{dec}_1\langle\texttt{,}\rangle\ldots\langle\texttt{,}\rangle\mathit{dec}_n$
$\mathit{mod} \quad \rightsquigarrow \texttt{unit it = }\mathit{mod}$

$\mathit{mod} \quad ::=$

x
$\texttt{[]}$
$\texttt{[val }\mathit{exp}\texttt{]}$
$\texttt{[val :}\mathit{typ}\texttt{]}$
$\texttt{[type }\mathit{typ}\texttt{]}$
$\texttt{[type :}\mathit{kind}\texttt{]}$
$\texttt{[data }\mathit{typ}\texttt{]}$
$\texttt{[data :}\mathit{typ}\texttt{]}$
$\texttt{[unit }\mathit{mod}\texttt{]}$
$\texttt{[unit :}\mathit{sig}\texttt{]}$
$\texttt{new }\mathit{mod}$
$\{\texttt{module }l\texttt{ = }\mathit{mod}\}$
$\mathit{mod}\texttt{.}l$
$\texttt{link }x\texttt{ = }\mathit{mod}_1\texttt{ with }\mathit{mod}_2$
$\texttt{link }x\texttt{ = }\mathit{mod}_1\texttt{ seals }\mathit{mod}_2$
$\mathit{mod}\texttt{ :> }\mathit{sig}$
$\texttt{(}\mathit{mod}\texttt{)}$
$\texttt{[val }\mathit{exp}\texttt{:}\mathit{typ}\texttt{]} \quad \rightsquigarrow \texttt{[val }\mathit{exp}\texttt{] with [val :}\mathit{typ}\texttt{]}$
$\texttt{[type]} \quad \rightsquigarrow \texttt{[type : \#]}$
$\texttt{[type }\mathit{typ}\texttt{:}\mathit{kind}\texttt{]} \quad \rightsquigarrow \texttt{[type }\mathit{typ}\texttt{] with [type :}\mathit{kind}\texttt{]}$
$\texttt{[data }\mathit{typ}\texttt{:}\mathit{kind}\texttt{]} \quad \rightsquigarrow \texttt{[data }\mathit{typ}\texttt{] with [type :}\mathit{kind}\texttt{]}$
$\texttt{[data :}\mathit{typ}\texttt{:}\mathit{kind}\texttt{]} \quad \rightsquigarrow \texttt{[data :}\mathit{typ}\texttt{] with [type :}\mathit{kind}\texttt{]}$
$\texttt{[unit }\mathit{mod}\texttt{:}\mathit{sig}\texttt{]} \quad \rightsquigarrow \texttt{[unit }\mathit{mod}\texttt{] with [unit :}\mathit{sig}\texttt{]}$
$\texttt{[module }\mathit{mod}\texttt{]} \quad \rightsquigarrow \mathit{mod}$
$\texttt{[module :}\mathit{mod}\texttt{]} \quad \rightsquigarrow \texttt{!}\mathit{mod}$
$\texttt{[module }\mathit{mod}_1\texttt{:}\mathit{mod}_2\texttt{]} \quad \rightsquigarrow \mathit{mod}_1\texttt{ with !}\mathit{mod}_2$
$\{\mathit{dec}_1\langle\texttt{,}\rangle\cdots\langle\texttt{,}\rangle\mathit{dec}_n\} \quad \rightsquigarrow \texttt{link }x\texttt{ = }\{\mathit{dec}_1\}\texttt{ with }\cdots\texttt{ link }x\texttt{ = }\{\mathit{dec}_n[x\texttt{.}x_i/x_i]\}\texttt{ with }\{\}$
$\texttt{!}\mathit{mod} \quad \rightsquigarrow \texttt{new }\mathit{mod}$
$\mathit{mod}_1\texttt{ with }\mathit{mod}_2 \quad \rightsquigarrow \texttt{link }x\texttt{ = }\mathit{mod}_1\texttt{ with }\mathit{mod}$
$\mathit{mod}_1\texttt{ seals }\mathit{mod}_2 \quad \rightsquigarrow \texttt{link }x\texttt{ = }\mathit{mod}_1\texttt{ seals }\mathit{mod}$
$\texttt{link }x\texttt{ : }\mathit{mod}_1\texttt{ with }\mathit{mod}_2 \quad \rightsquigarrow \texttt{link }x\texttt{ = !}\mathit{mod}_1\texttt{ with }\mathit{mod}$
$\texttt{link }x\texttt{ : }\mathit{mod}_1\texttt{ seals }\mathit{mod}_2 \quad \rightsquigarrow \texttt{link }x\texttt{ = !}\mathit{mod}_1\texttt{ seals }\mathit{mod}$
$\texttt{let }\mathit{dec}_1\langle\texttt{,}\rangle\ldots\langle\texttt{,}\rangle\mathit{dec}_n\texttt{ in }\mathit{mod} \quad \rightsquigarrow \{\mathit{dec}_1\langle\texttt{,}\rangle\ldots\langle\texttt{,}\rangle\mathit{dec}_n\texttt{, module }l\texttt{ = }\mathit{mod}\}\texttt{.}l$
$\texttt{let }x\texttt{ = }\mathit{mod}_1\texttt{ in }\mathit{mod}_2 \quad \rightsquigarrow \texttt{let module }x\texttt{ = }\mathit{mod}_1\texttt{ in }\mathit{mod}_2$

$\mathit{dec} \quad ::=$

$\texttt{val }p\ \langle\texttt{[}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{]}\rangle\ \texttt{(}x_1\texttt{:}\mathit{typ}_1\texttt{)}\cdots\texttt{(}x_m\texttt{:}\mathit{typ}_m\texttt{) : }\mathit{typ}$
$\quad \rightsquigarrow p\texttt{ = [val : }\langle\texttt{forall [}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{] ->}\rangle\ \mathit{typ}_1\texttt{ ->}\cdots\texttt{-> }\mathit{typ}_m\texttt{ -> }\mathit{typ}\texttt{]}$
$\texttt{val }p\ \langle\texttt{[}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{]}\rangle\ \texttt{(}x_1\texttt{:}\mathit{typ}_1\texttt{)}\cdots\texttt{(}x_m\texttt{:}\mathit{typ}_m\texttt{)}\ \langle\texttt{: }\mathit{typ}\rangle\texttt{ = }\mathit{exp}$
$\quad \rightsquigarrow p\texttt{ = [val }\langle\texttt{fn [}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{] ->}\rangle\texttt{ fn }x_1\texttt{:}\mathit{typ}_1\texttt{ ->}\cdots\texttt{fn }x_m\texttt{:}\mathit{typ}_m\texttt{ -> }\mathit{exp}$
$\qquad \langle\texttt{: }\langle\texttt{forall [}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{] ->}\rangle\ \mathit{typ}_1\texttt{ ->}\cdots\texttt{-> }\mathit{typ}_m\texttt{ -> }\mathit{typ}\rangle\texttt{]}$
$\texttt{type }p\ \langle\texttt{[}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{]}\rangle \quad \rightsquigarrow p\texttt{ = [type }\langle\texttt{: \#}n\texttt{ -> \#}\rangle\texttt{]}$
$\texttt{type }p\ \langle\texttt{[}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{]}\rangle\texttt{ : }\mathit{kind} \quad \rightsquigarrow p\texttt{ = [type : }\langle\texttt{\#}n\texttt{ ->}\rangle\ \mathit{kind}\texttt{]}$
$\texttt{type }p\ \langle\texttt{[}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{]}\rangle\ \langle\texttt{: }\mathit{kind}\rangle\texttt{ = }\mathit{typ} \quad \rightsquigarrow p\texttt{ = [type }\langle\texttt{fn [}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{] ->}\rangle\ \mathit{typ}\ \langle\texttt{: }\langle\texttt{\#}n\texttt{ ->}\rangle\ \mathit{kind}\rangle\texttt{]}$
$\texttt{data }p\ \langle\texttt{[}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{]}\rangle\ \langle\texttt{: }\mathit{kind}\rangle\texttt{ = }\mathit{typ} \quad \rightsquigarrow p\texttt{ = [data }\langle\texttt{fn [}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{] ->}\rangle\ \mathit{typ}\ \langle\texttt{: }\langle\texttt{\#}n\texttt{ ->}\rangle\ \mathit{kind}\rangle\texttt{]}$
$\texttt{data }p\ \langle\texttt{[}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{]}\rangle\ \langle\texttt{: }\mathit{kind}\rangle\texttt{ : }\mathit{typ} \quad \rightsquigarrow p\texttt{ = [data : }\langle\texttt{fn [}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{] ->}\rangle\ \mathit{typ}\ \langle\texttt{: }\langle\texttt{\#}n\texttt{ ->}\rangle\ \mathit{kind}\rangle\texttt{]}$
$\texttt{unit }p\texttt{ : }\mathit{sig} \quad \rightsquigarrow p\texttt{ = [unit :}\mathit{sig}\texttt{]}$
$\texttt{unit }p\ \langle\texttt{: }\mathit{sig}\rangle\texttt{ = }\mathit{mod} \quad \rightsquigarrow p\texttt{ = [unit }\mathit{mod}\langle\texttt{:}\mathit{sig}\rangle\texttt{]}$
$\texttt{module }p\texttt{ : }\mathit{mod} \quad \rightsquigarrow p\texttt{ = [module :}\mathit{mod}\texttt{]}$
$\texttt{module }p\ \langle\texttt{: }\mathit{mod}_1\rangle\texttt{ = }\mathit{mod}_2 \quad \rightsquigarrow p\texttt{ = [module }\mathit{mod}_2\langle\texttt{:}\mathit{mod}_1\rangle\texttt{]}$
$\texttt{module }x\texttt{.}\mathit{ls}\texttt{ = }\mathit{mod} \quad \rightsquigarrow x\texttt{ = }\{\texttt{module }l_1\texttt{ =}\cdots\{\texttt{module }l_n\texttt{ = }\mathit{mod}\}\cdots\}$
$\texttt{do }\mathit{exp} \quad \rightsquigarrow \texttt{val }x\texttt{ = }\mathit{exp}$

sig	$::=$		
		$mod\ \texttt{import (}ls_1\texttt{,}\ldots\texttt{,}ls_n\texttt{)}$	
		$mod\ \texttt{export (}ls_1\texttt{,}\ldots\texttt{,}ls_n\texttt{)}$	
		mod	$\leadsto mod\ \texttt{import ()}$
ls	$::=$	$\epsilon \mid ls.l$	
p	$::=$	$x \mid x\,.\,ls$	
$kind$	$::=$		
		$\texttt{\#}$	
		$\texttt{\#}n\ \texttt{-> \#}$	
		$\texttt{\# -> \#}$	$\leadsto \texttt{\#1 -> \#}$
typ	$::=$		
		$\texttt{!}mod$	
		$\texttt{bool}$	
		$\texttt{int}$	
		$\texttt{string}$	
		$\texttt{(}typ_1\texttt{,}\ldots\texttt{,}typ_n\texttt{)}$	
		$typ_1\ \texttt{->}\ typ_2$	
		$\texttt{forall [}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{] ->}\ typ$	
		$\texttt{fn [}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{] ->}\ typ$	
		$typ\ \texttt{[}typ_1\texttt{,}\ldots\texttt{,}typ_n\texttt{]}$	
		$\texttt{(}typ\texttt{)}$	
		p	$\leadsto \texttt{!}p$
exp	$::=$		
		$\texttt{!}mod$	
		$\texttt{false}$	
		$\texttt{true}$	
		n	
		s	
		$exp_1\ \texttt{+}\ exp_2$	
		$exp_1\ \texttt{-}\ exp_2$	
		$exp_1\ \texttt{==}\ exp_2$	
		$exp_1\ \texttt{<}\ exp_2$	
		$exp_1\ \texttt{++}\ exp_2$	
		$\texttt{if}\ exp_1\ \texttt{then}\ exp_2\ \texttt{else}\ exp_3$	
		$\texttt{(}exp_1\texttt{,}\ldots\texttt{,}exp_n\texttt{)}$	
		$exp\texttt{\#}n$	
		$\texttt{fn}\ x\texttt{:}typ\ \texttt{->}\ exp$	
		$exp_1\ exp_2$	
		$\texttt{fn [}\alpha_1\texttt{,}\ldots\texttt{,}\alpha_n\texttt{] ->}\ exp$	
		$exp\ \texttt{[}typ_1\texttt{,}\ldots\texttt{,}typ_n\texttt{]}$	
		$\texttt{in}\ mod\ \texttt{[}typ_1\texttt{,}\ldots\texttt{,}typ_n\texttt{]}\ exp$	
		$\texttt{out}\ mod\ \texttt{[}typ_1\texttt{,}\ldots\texttt{,}typ_n\texttt{]}\ exp$	
		$\texttt{let module}\ x\ \texttt{=}\ mod\ \texttt{in}\ exp$	
		$\texttt{print}\ exp$	
		$\texttt{(}exp\texttt{)}$	
		p	$\leadsto \texttt{!}p$
		$\texttt{let}\ dec_1\langle,\rangle\ldots\langle,\rangle dec_n\ \texttt{in}\ exp$	$\leadsto \texttt{let}\ dec_1\ \texttt{in}\ \ldots\ \texttt{let}\ dec_n\ \texttt{in}\ exp$
		$\texttt{let}\ x\ \texttt{=}\ exp_1\ \texttt{in}\ exp_2$	$\leadsto \texttt{let}\ x\ \texttt{= [val}\ exp_1\texttt{] in}\ exp_2$
		$\texttt{(}exp\ \texttt{:}\ typ\texttt{)}$	$\leadsto \texttt{let}\ x\ \texttt{= [val}\ exp\texttt{:}typ\texttt{] in}\ x$
		$exp_1\ \texttt{;}\ exp_2$	$\leadsto \texttt{let}\ x\ \texttt{=}\ exp_1\ \texttt{in}\ exp_2$