Exercise 1 (30 points).
Let \((D, \leq)\) be a lattice and \(f : D \to D\) be a function.

a) Prove that if \(f\) is monotonic, then \(f(d_1) \sqcup f(d_2) \leq f(d_1 \sqcup d_2)\) for all \(d_1, d_2 \in D\).

b) The function \(f\) is called distributive if \(f(d_1) \sqcup f(d_2) = f(d_1 \sqcup d_2)\) for all \(d_1, d_2 \in D\). Prove that if \(f\) is distributive, then \(f\) is also monotonic.

Exercise 2 (30 points).
Let \((D, \leq)\) be a complete lattice that satisfies the (ACC) condition. Show that if the function \(f : D \to D\) is distributive, then for all \(X \subseteq D\) where \(X \neq \emptyset\) it holds that \(f(\bigsqcup X) = \bigsqcup \{f(x) \mid x \in X\}\).

Exercise 3 (40 points).
Suppose that your goal is to define an analysis that can be used to check if a program contains division by 0. The analysis should be sound, meaning that if it reports that there is no division by 0, then this indeed holds.

You are given the following program:

\[
[y := 1]; \\
\textbf{while} \ [y < 42] \ \textbf{do} \{ \\
\quad [y := y \ast 2] \\
\}\] 
\[
\textbf{print}(1/y) 
\]
a) Give the control flow graph $G$ for this program.

b) What lattice $(D, \leq)$ will you use for the analysis?

c) Define functions $f_1, f_2, f_3, f_4$ associated with the blocks of the program, and the initial value $i \in D$ for the initial block in $G$.

d) Give the resulting system of equations for this program and solve it by fixpoint computation.