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Finite-valued Semantics for Canonical Labelled Calculi
Joint work with: Matthias Baaz (Vienna University of Technology).

A useful semantics is an important property of formal calculi. In addition to providing real insights into their underlying logic, such semantics should also be *effective* in the sense of naturally inducing a decision procedure for its calculus. Another desirable property of such semantics is the possibility to apply it for characterizing important syntactic properties of the calculi, which are hard to establish by other means.

Recently some systematic methods for constructing such semantics for various calculi have been formulated. In particular, *labelled sequent calculi* with generalized forms of cuts and identity axioms and natural forms of logical rules were studied in this context. Such calculi, satisfying a certain coherence condition, have a semantic characterization using a natural generalization of the usual finite-valued matrix called *non-deterministic matrices*, which is effective in the above sense.

In this talk, we show that the class of labelled calculi that have a finite-valued effective semantics is substantially larger than all the families of calculi considered in the literature in this context. We start by defining a general class of fully-structural and propositional labelled calculi, called *canonical labelled calculi*, of which the previosuly considered labelled calculi are particular examples. In addition to the weakening rule, canonical labelled calculi have rules of two forms: primitive rules and introduction rules. The former operate on labels and do not mention any connectives, while the latter introduce exactly one logical connective of the language. To provide semantics for all of these calculi in a systematic and modular way, we generalize the notion of non-deterministic matrices to *partial non-deterministic matrices* (PNmatrices), in which empty sets of options are allowed in the truth tables of logical connectives. Although applicable to a much wider range of calculi, the semantic framework of finite PNmatrices shares the following attractive property with both usual and non-deterministic matrices: any calculus that has a characteristic PNmatrix is decidable. We then apply PNmatrices to provide simple *decidable* characterizations of the crucial syntactic properties of strong analyticity and strong cut-admissibility in canonical labelled calculi. Finally, we demonstrate how the theory of labelled canonical calculi developed here can be exploited to provide effective semantics also for a variety of logics induced by calculi which are not canonical. One such example is calculi for paraconsistent logics known as C-systems.

[1]Supported by The Israel Science Foundation (grant no. 280-10) and by FWF START Y544-N23.
[2]Supported by The European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 252314.