Shape-Aware Analysis of End-to-End Latency Under LET

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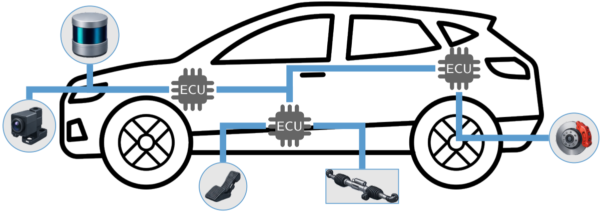
May 12, 2026 @RTAS 26

This result is part of a project (PropRT) that has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 865170). This work has been funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Project No. 569077889 (PEACH). This work was partially funded by the Swedish Research Council (VR) under the project nr. 2023-04773 and by Sweden's Innovation Agency via the NFFP8 project 2024-01267: PARTI. This work has been partially supported by the project SERICS (PE00000014) under the MUR National Recovery and Resilience Plan funded by the European Union – NextGenerationEU.

Motivation: Cause-Effect Chains in Automotive Systems

- Automotive Systems: Tasks work together to perform functionality



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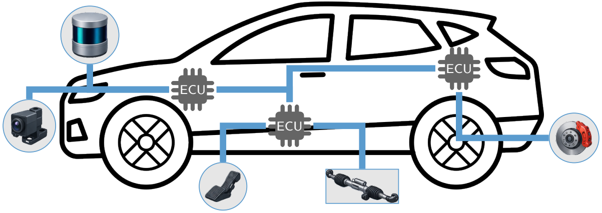

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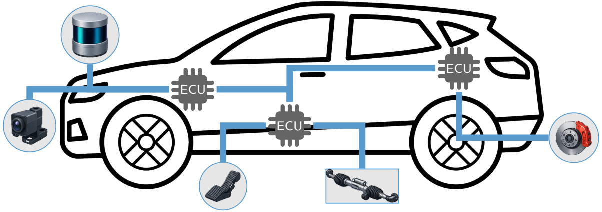

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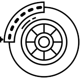
$\Longrightarrow$ Get Camera Image $\rightarrow$ Image Recognition $\rightarrow$ Danger Analysis $\Longrightarrow$



(= Cause-effect chain $E = (\tau_1 \rightarrow \tau_2 \rightarrow \tau_3)$)

- Want: Analyze timing behavior of full cause-effect chain

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Reaction Time: "How fast does the system *react* to a change?"

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This paper:

- No! We can analyze the *full shape once* and derive metrics from that.
- For many metrics, we provide *first* analysis

Outline

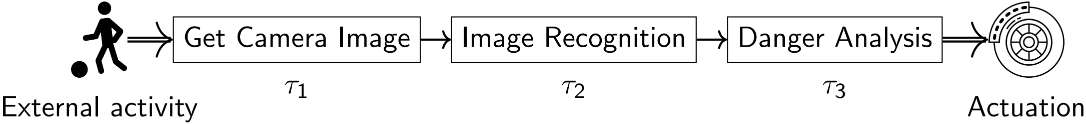
End-to-End Latency of Cause-Effect Chains

Shape-Aware Analysis of Reaction Time

Evaluation

Cause-Effect Chains

- Functionality is described by a sequence of tasks



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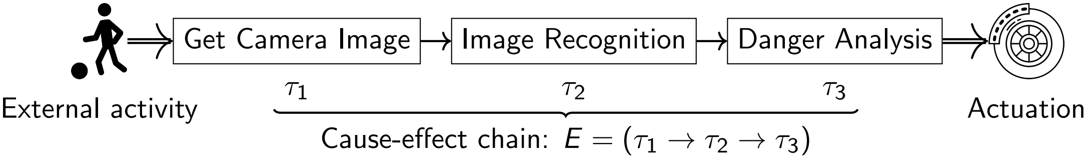

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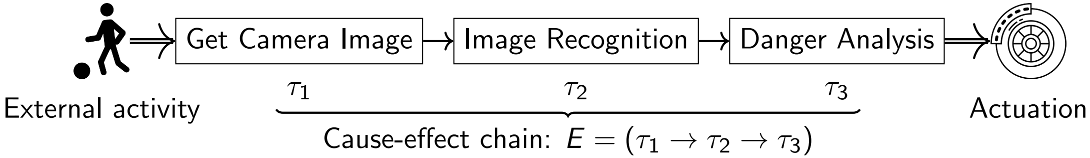
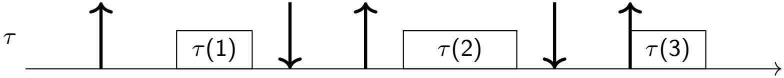


Cause-Effect Chain

Sequence $E = (\tau_1 \to \tau_2 \to \cdots \to \tau_n)$ of tasks τ_i

(Real-Time) Task Model

$\mathbb{T}$ task set, $\tau \in \mathbb{T}$ task, releases jobs $\tau(i)$



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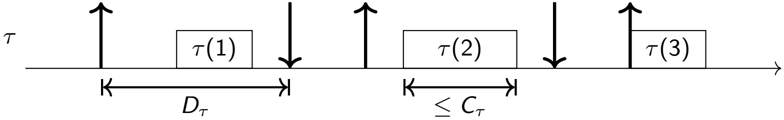

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- Relative deadline D_τ

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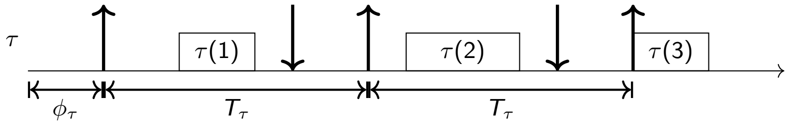
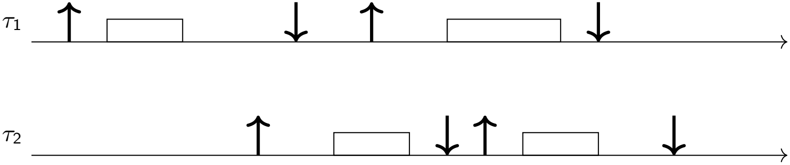


- Worst-case execution time (WCET) C_τ
- Relative deadline D_τ
- Period T_τ
- Phase ϕ_τ

Communication Under Logical Execution Time (LET)

- Communication modeled through shared resource



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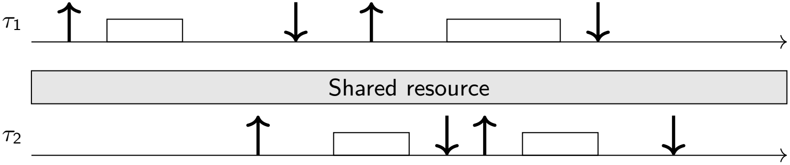

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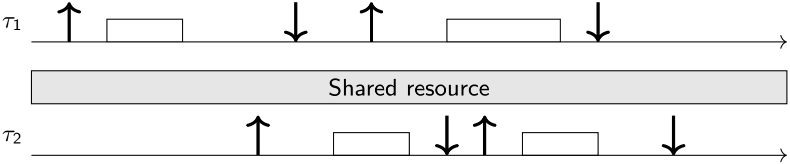

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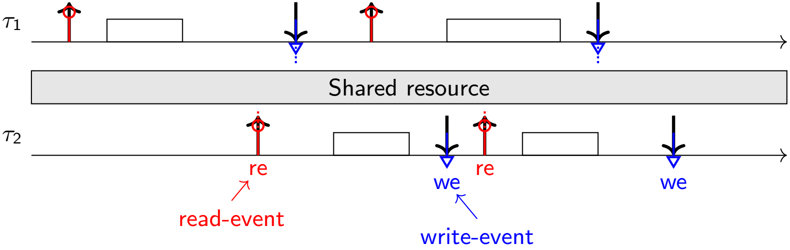

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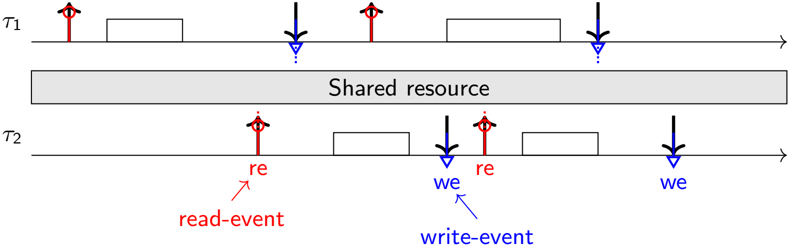

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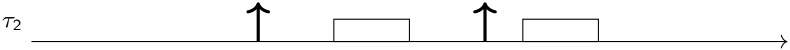

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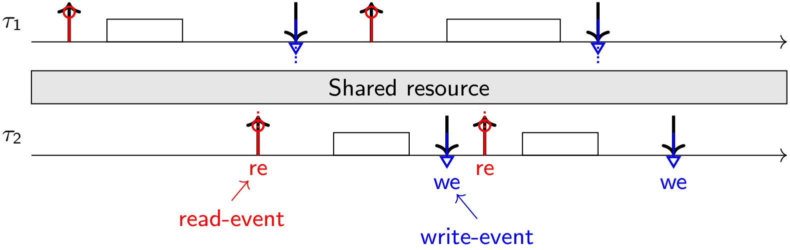

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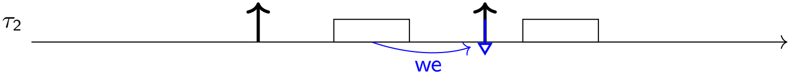

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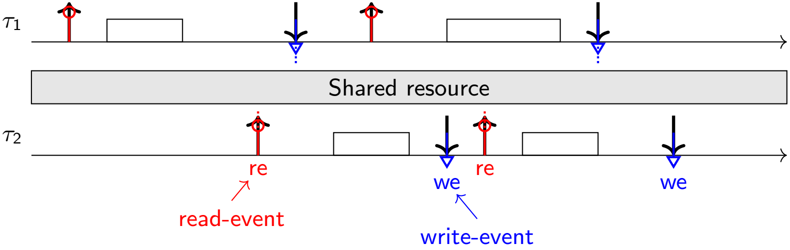

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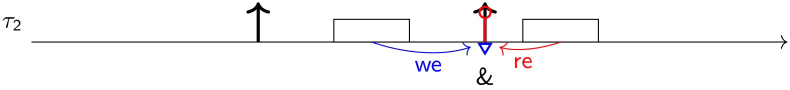

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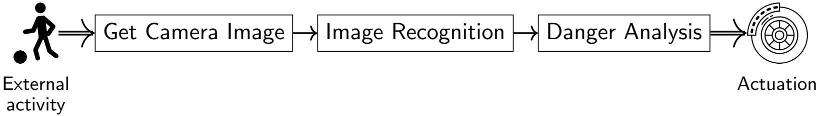

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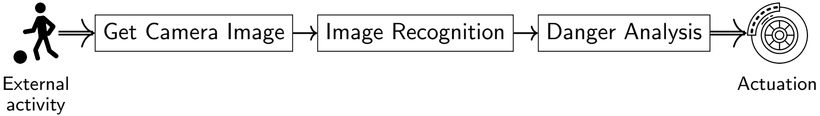

Reaction Time $RT(z)$

Given an external activity at time z, how long does it take until this external activity is fully processed?

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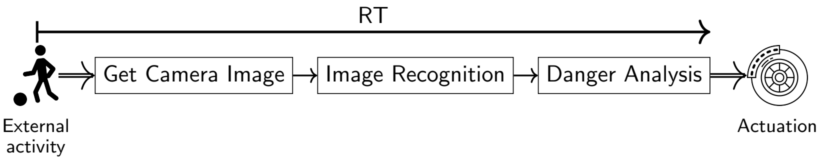

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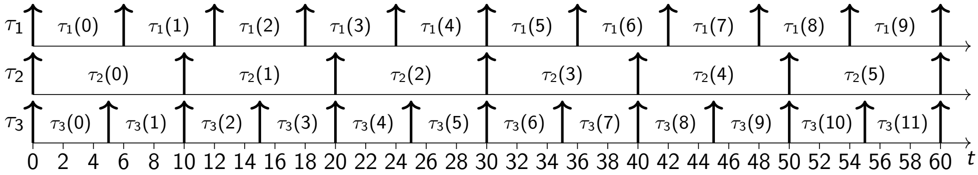

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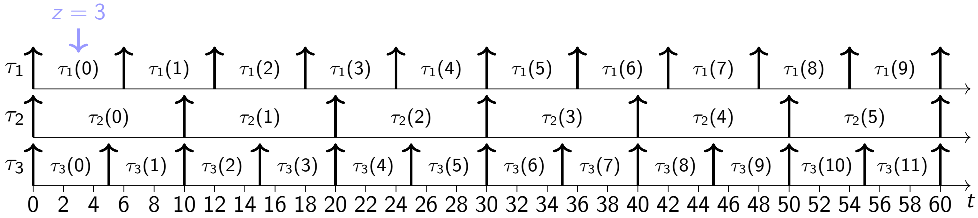

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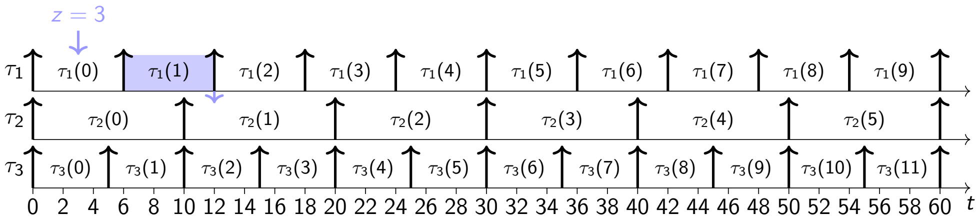

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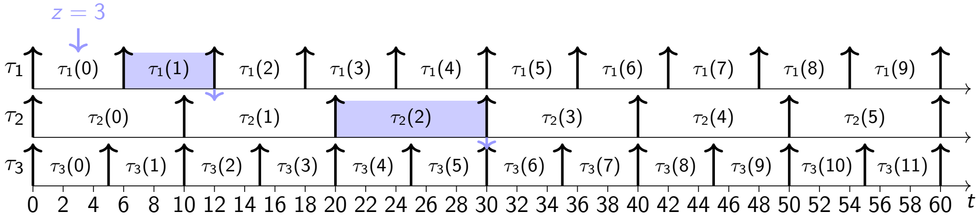

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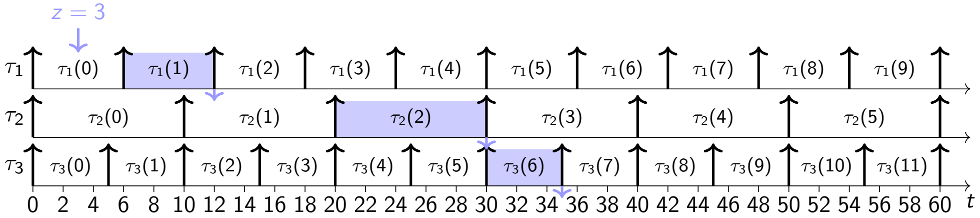

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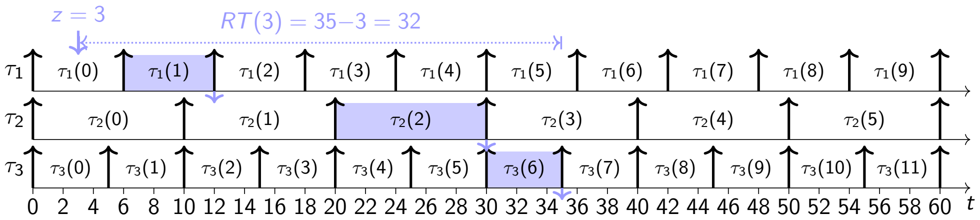

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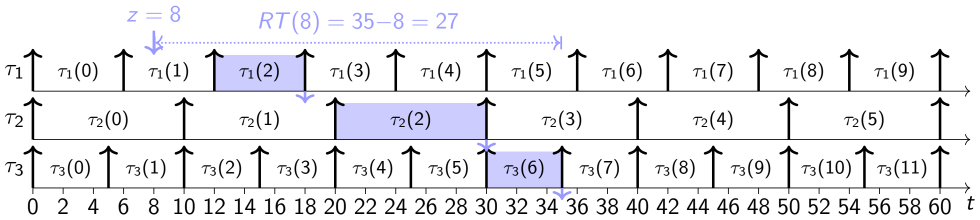

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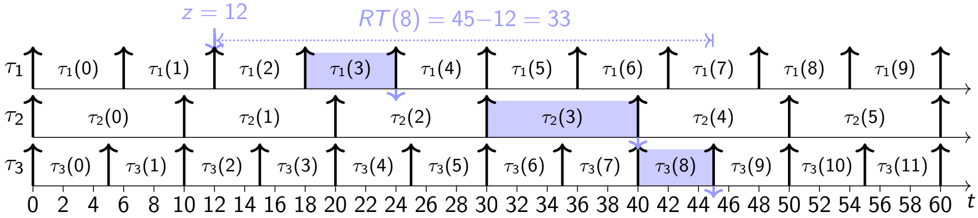

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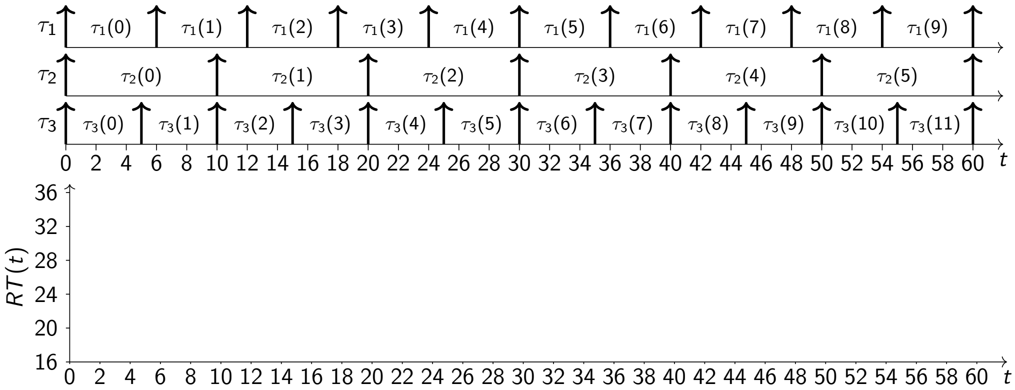

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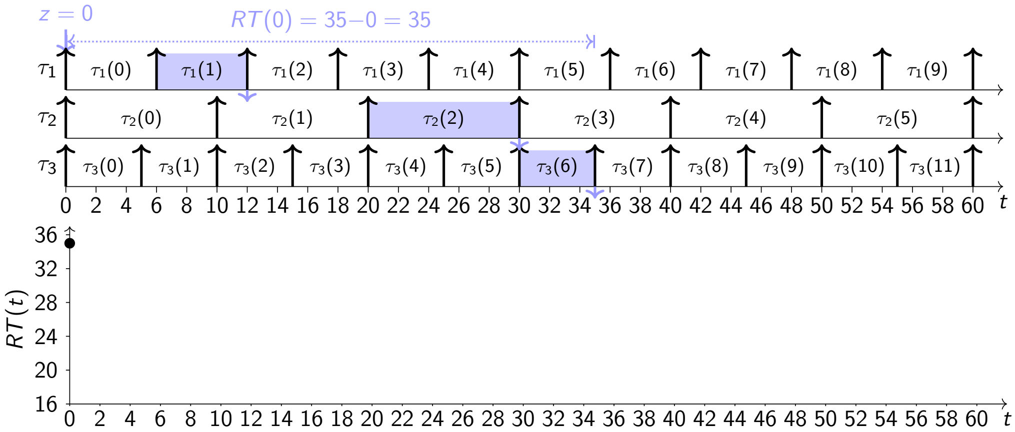

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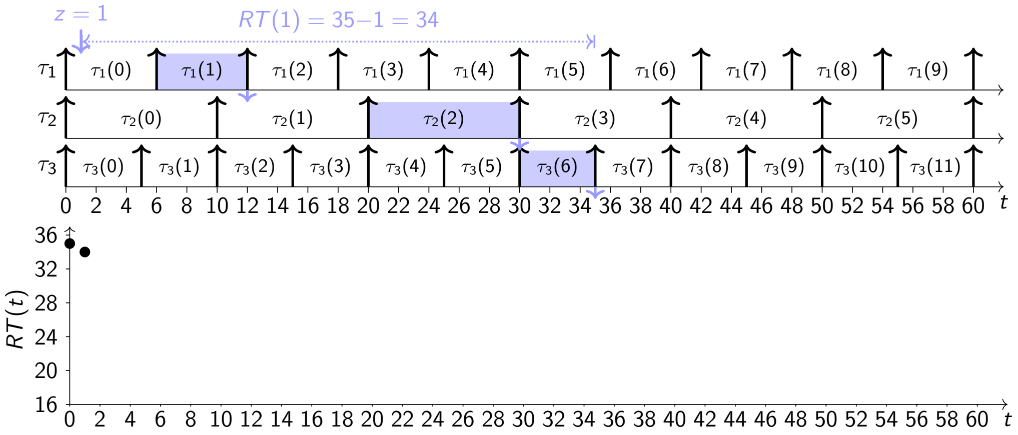

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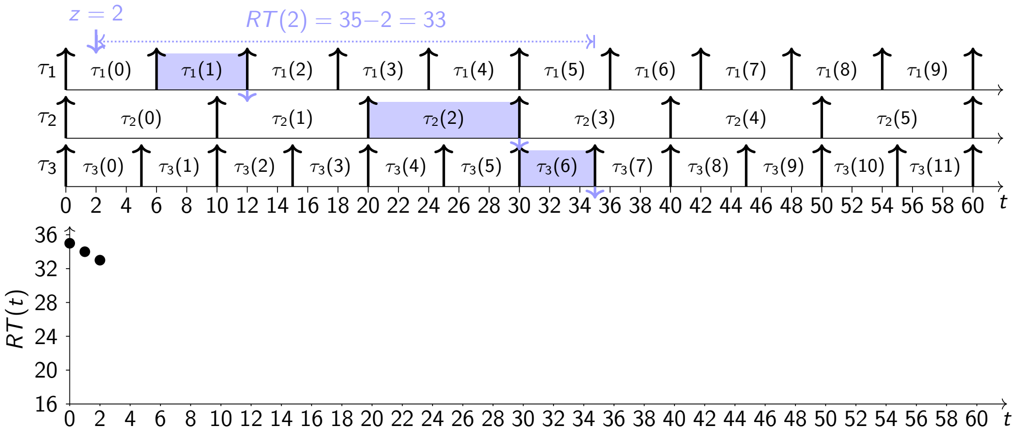

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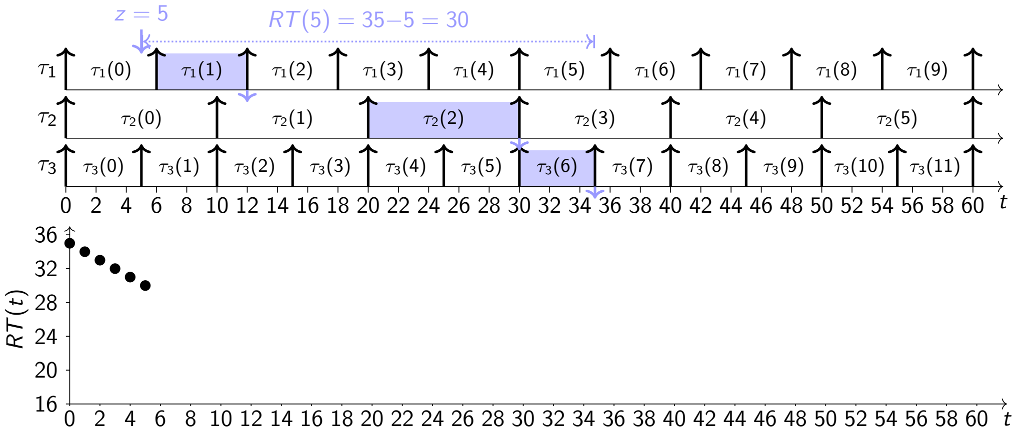

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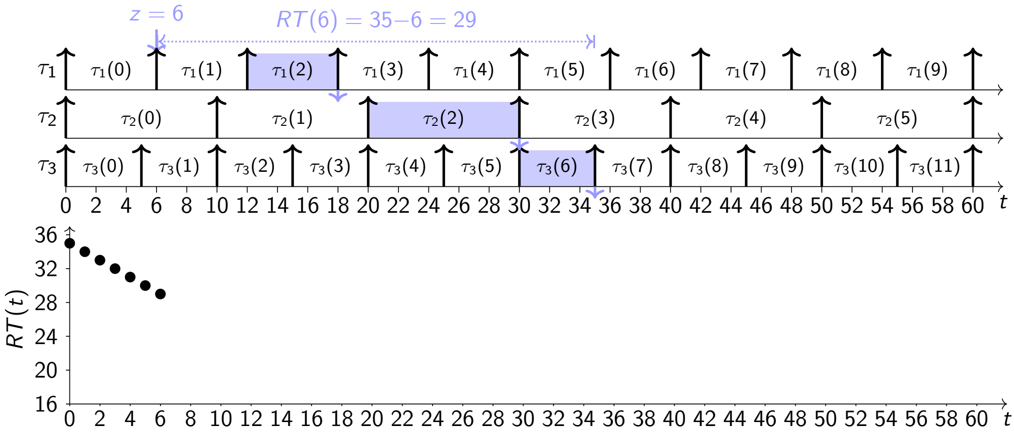

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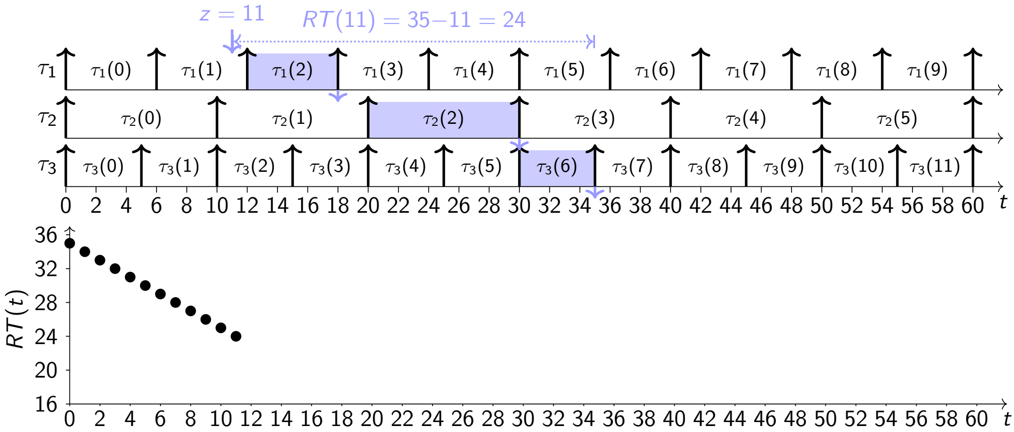

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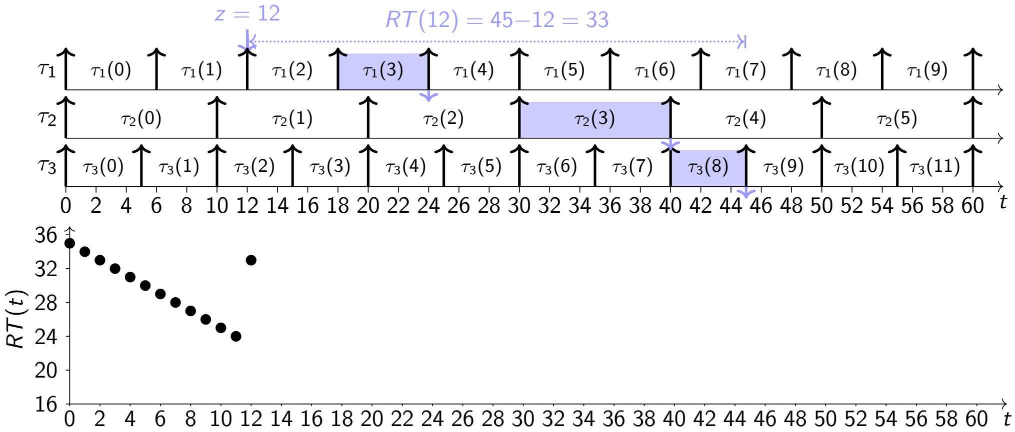

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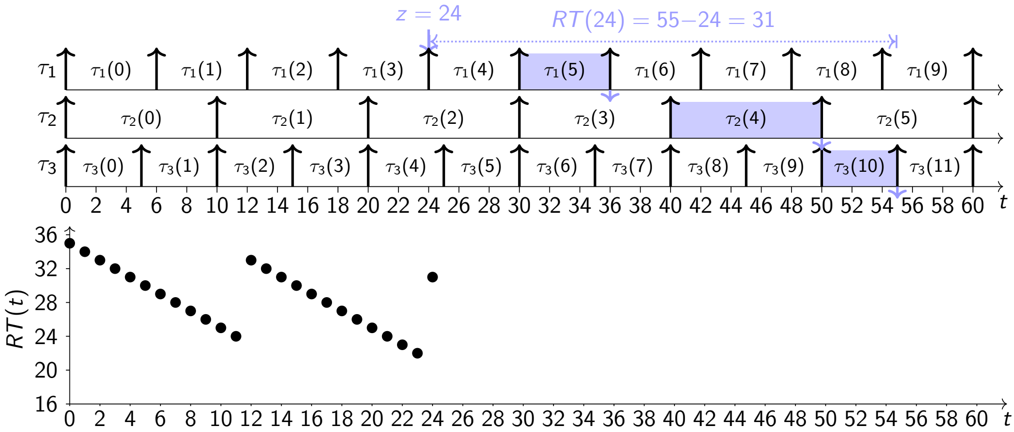

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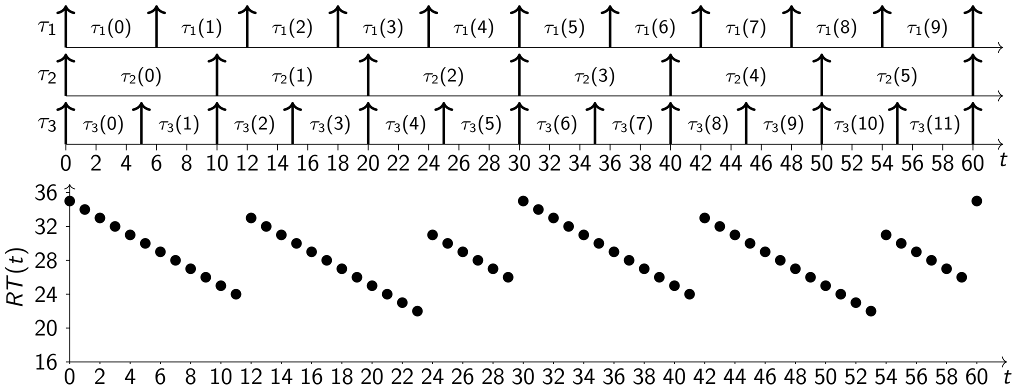
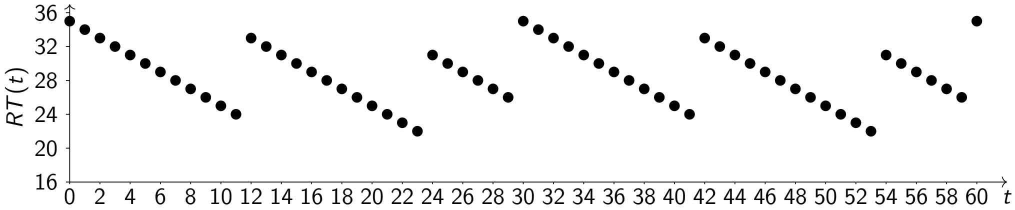


Observations



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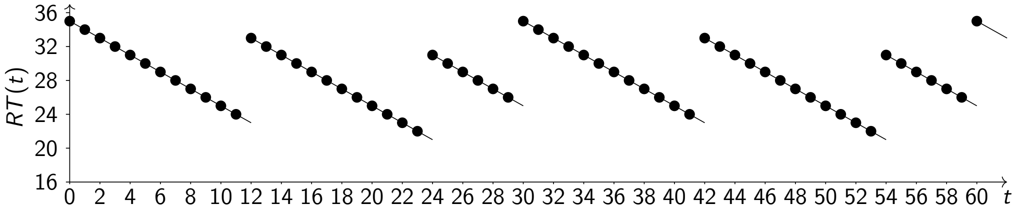

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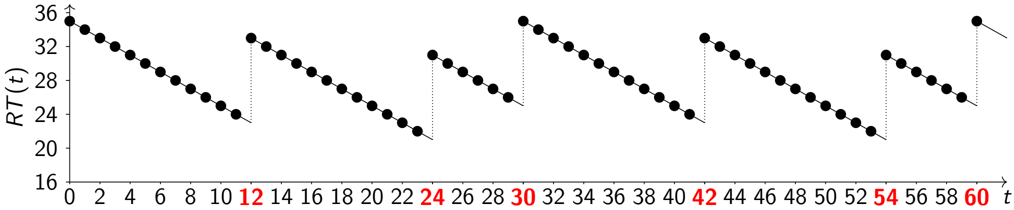

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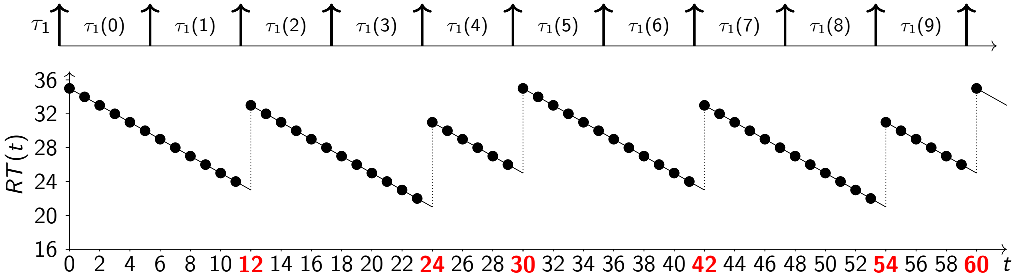

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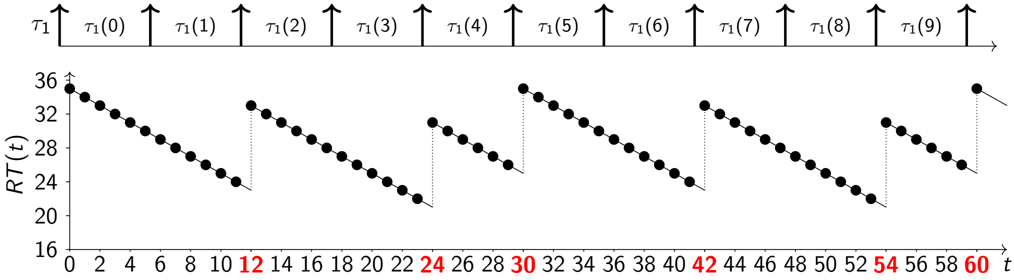

1. RT Linearly decreasing
2. Jumps up can only occur at multiples of 6 (= read-events of task τ_1)

Formalization of Observations

Lemma 8

RT restricted to the interval $[\mathrm{re}(\tau_1(m)), \mathrm{re}(\tau_1(m+1)))$, $m \in \mathbb{N}$ is linearly decreasing:

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RT restricted to the interval $[\mathrm{re}(\tau_1(m)), \mathrm{re}(\tau_1(m+1)))$, $m \in \mathbb{N}$ is linearly decreasing:

$$RT(t) = RT(\mathrm{re}(\tau_1(m))) - (t - \mathrm{re}(\tau_1(m)))$$

$\forall\ t \in [\mathrm{re}(\tau_1(m)), \mathrm{re}(\tau_1(m+1)))$

Formalization of Observations

Lemma 8

RT restricted to the interval $[\mathrm{re}(\tau_1(m)), \mathrm{re}(\tau_1(m+1)))$, $m \in \mathbb{N}$ is linearly decreasing:

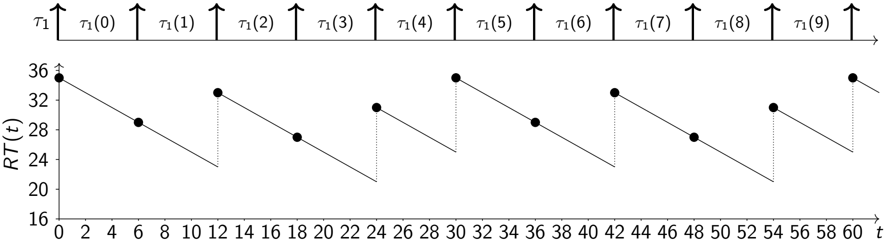
$$RT(t) = RT(\mathrm{re}(\tau_1(m))) - (t - \mathrm{re}(\tau_1(m)))$$

$$\forall\ t \in [\mathrm{re}(\tau_1(m)), \mathrm{re}(\tau_1(m+1)))$$

$\Rightarrow$ Full shape of RT can be determined by evaluating at read-events of τ_1

Shape of RT: Specification by Anchor Points

This gives us the following shape:



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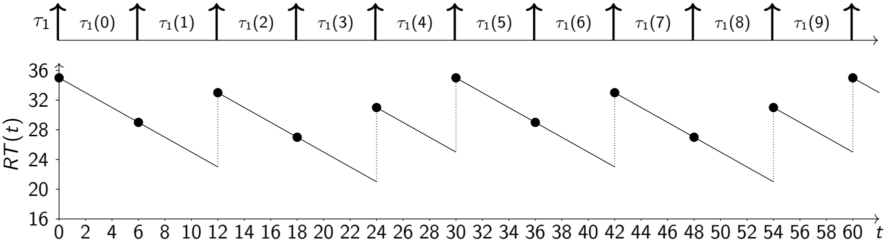

Definition: Anchor Points

$$\mathcal{A}^1 := \left\{ \left(x, RT(x)\right) \,\middle|\, x = \mathrm{re}(\tau_1(m)), m \in \mathbb{N} \right\}$$

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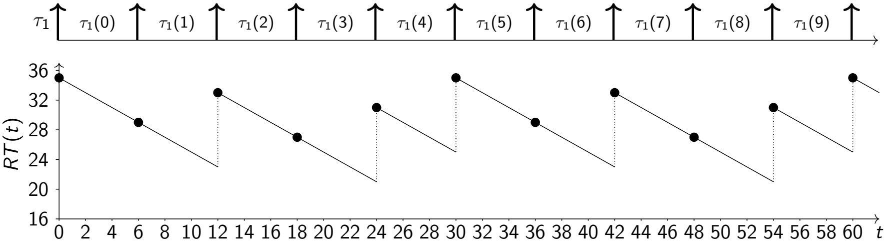

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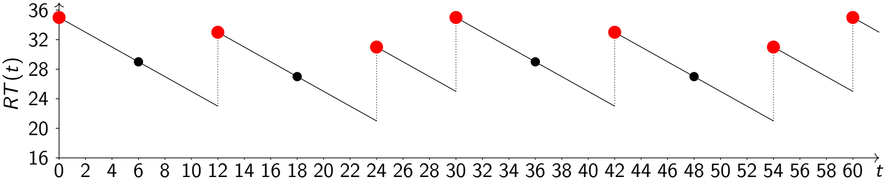
$$\mathcal{A}^1 = \left\{ \begin{array}{l} (0,35), (6,29), (12,33), \\ (18,27), (24,31), (30,35), \\ (36,29), (42,33), (48,27), \\ (54,31), (60,35), \ldots \end{array} \right\}$$

Minimal Set of Anchor Points



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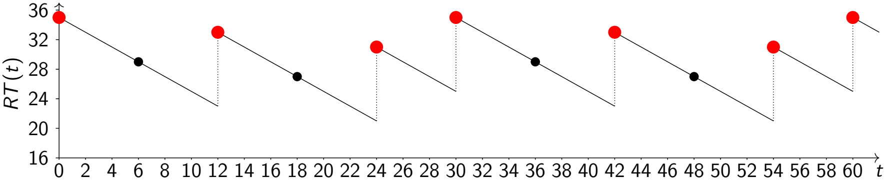

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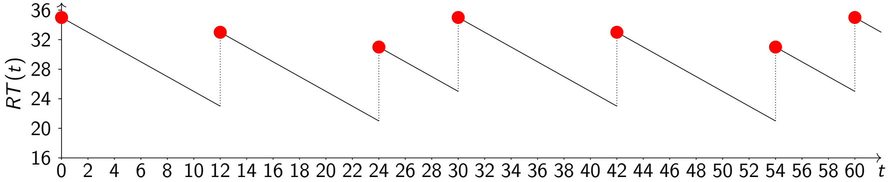

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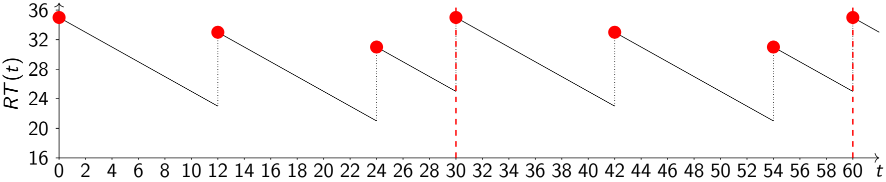

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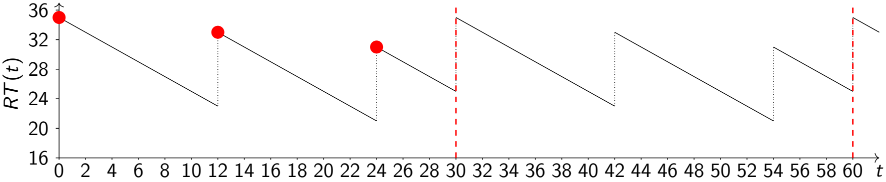

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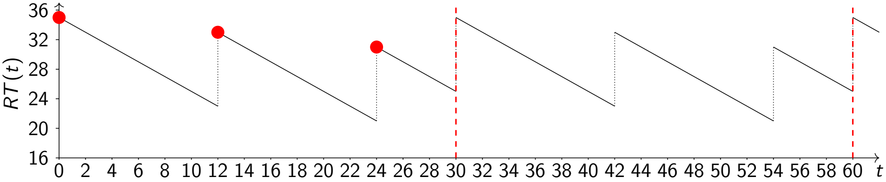

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$\Rightarrow \mathcal{A}^*|_H$ is sufficient to describe the whole shape of RT

Analysis of Metrics Becomes Simple

Given: Anchor points $\mathcal{A}^*|_H = \{(x_0, y_0), \dots, (x_g, y_g)\}$ over one hyperperiod $[0, H)$

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Running example: $\mathcal{A}^*|_H = \{(0, 35), (12, 33), (24, 31)\}$

- $MaxRT = \max(35, 33, 31) = 35$

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- $Thr = \frac{1}{30} \cdot 3 = 0.1$

More Metrics and Insights in the Paper

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- Determine $\mathcal{A}^*|_H$ efficiently using *partitioned job chains*
  $\Rightarrow$ Speedup of $\approx$1000x for automotive tasks

Implementation in Python

https://github.com/marioguenzel/Shape-Aware-E2E.git

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- Simple specification of cause-effect chains:

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Evaluation of Case Studies

Case Study	Max	Min	Av	Thr	(m,k)	LE
Wat17-C1	50	40	45.0	0.100	(0,10)	2.50
Wat17-C2	212	112	162.0	0.010	(0,10)	10.60
Wat19-C1	908	470	689.0	0.003	(1,10)	45.40
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Fre10-C4	45	35	40.0	0.100	(0,10)	2.25
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- **Low Runtime**:
  Total analysis time: 0.062 seconds (single core with 2.25 GHz)

Runtime Comparison with SOTA

Two benchmarks:

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Approaches for Comparison:

Paper	**Approach**	**Metrics**
RTNS 2023 (FW)	Forward	MaxRT
RTAS 2023 (BW)	Backward	Reac
ECRTS 2023 (P)	Partitioned	MaxRT, MaxRedRT
This paper (Our)	Partitioned	MaxRT, MinRT, MaxRedRT, Reac, AvRT, Thr, (m,k), LE

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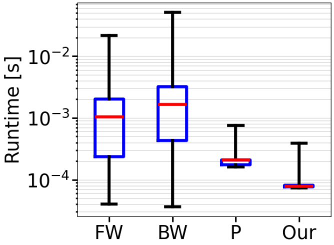
Runtime Comparison with SOTA

Plots with $N = 50$ tasks per chain (more in the paper):



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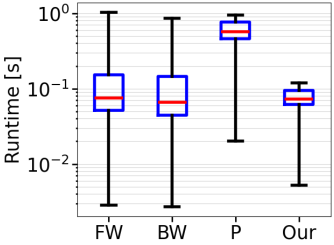

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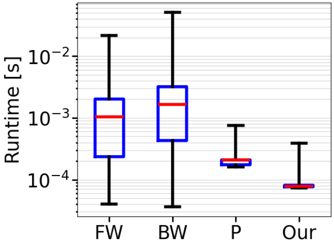
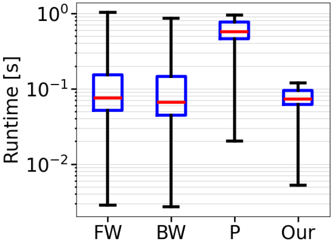


(WATERS)



(UNIFORM)

- Comparable runtime for UNIFORM and improved runtime for WATERS

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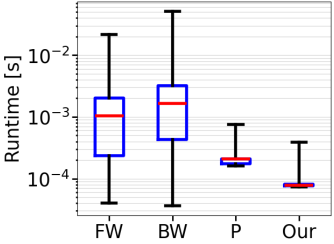

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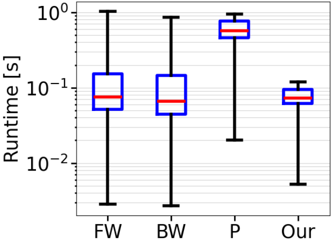

(UNIFORM)

- Comparable runtime for UNIFORM and improved runtime for WATERS
- **Note:** Major effort (= determining anchor points) only *once*!
  $\Rightarrow$ Our analyzes *multiple* metrics in similar time

Conclusion and Future Work

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Future work:

- Further metrics and exploitation of shape for design choices
- Data-age related metrics
- Other communication paradigms such as implicit communication

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