Functional Programming

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K.U.Leuven

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Goal of the Lecture

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- remind/illustrate FP style, and related concepts;
- remind why FP is important and its strong points.[1]

- Introduce you to the syntax of Racket.

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Style of the Lecture

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INTERACTIVE

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With explanation on the side.

Organisation of the Lecture

- 1/2 (or more): FP

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- 1/2 (or more): FP
- 1/2 (or less): Racket syntax

Functional Programming: Motivation

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- modularisation;
- “no” side effects.

Outline

Motivation 1: Modular Code

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- functions as first-class citizens (higher-order functions);

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HOW? (what is the glue?)

- algebraic data types (seen yesterday);
- functions;
- functions as first-class citizens (higher-order functions);
- laziness.

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- code can be reasoned about ***in isolation***; (also, verified)
- code can be reused. (similar motivation for OO programming! what is the difference?)

Homework

Think about this!

Motivation 2: “No” Side Effects

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Maybe we need them.

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Achievable via Monads (more on Dave’s lectures).

Outline

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you tell me! you were told yesterday!

Algebraic Data Types and Pattern Matching

Coding time #1

Natural numbers, double, addition, multiplication, equality, maximum.

Outline

Functions as First-class Citizens

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This is lame...

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AND

The type checker can infer it *automatically* (more or less, in certain languages).

Functions as First-class Citizens

Coding time #2

Lists, sum of elements of a list, tail recursive sum, length, append.

Outline

Functions with *Better* Types

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Higher order!

Higher-Order Functions

Coding time #3

Map, filter, foldr.

Outline

foldr

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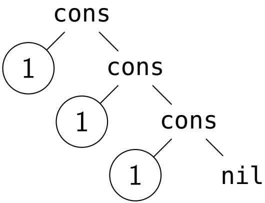

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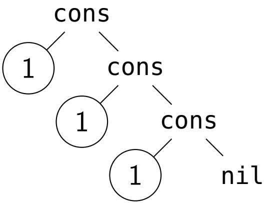

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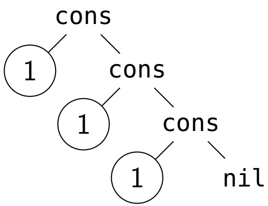

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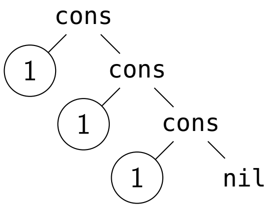

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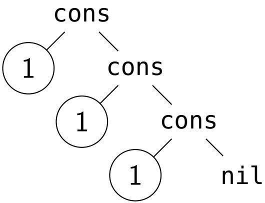

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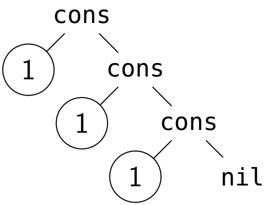

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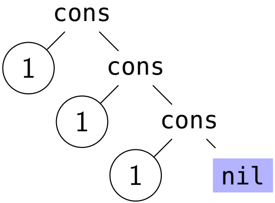

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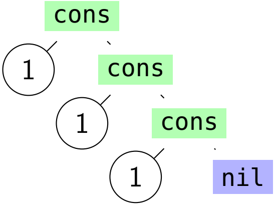

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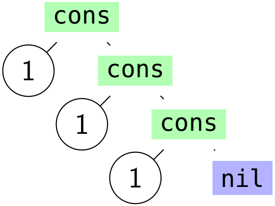

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More Modularisation

Coding time #4

sum via foldr, append via foldr, length via foldr, map via foldr.

The Type of Function Composition

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Question:

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Coding time #4 bis

map via foldr.

Outline

What About ... ?

more complex data structures?

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Does the `foldr` scale on them?

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more complex data structures?

Does the `foldr` scale on them?

YES!

All inductively-defined data structures implicitly have a `foldr`.
It is the concept of *catamorphism*.

Binary Trees

Coding time #5

BTrees, foldr on trees, map on trees, map on trees via foldr, depth via foldr.

Outline

Laziness

- What is laziness?

Laziness

- What is laziness?
- Why do we want/need laziness?

Laziness: What

Example: how do students study?

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Example: how do students study?

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- “call by need” in the λ-calculus;

Laziness: Why

- to avoid large and possibly diverging computation;

```
(2 == 2) || (isPrime 997)
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data Stream a = Elem a (Stream a)
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Once upon a time ...

Laziness

Coding time #6

All numbers, all the even ones, all the prime ones.

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- write something in the definition area and call it;
- Racket is: *functional* and *untyped*, so you can write functions that expect functions!

Racket

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Play with it before the next lectures.