Assignment #7

Name: _____ ID: _____

This assignment has **2** questions, for a total of **25** marks.

Question 1: **Fuel** ..... 15 marks

Consider the STLCN calculus from questions 1 and 2 in assignment 3. Add a cost to all reduction: function calls to private functions cost 10, beta reductions cost 5, projections and case cost 2, arithmetic operations cost 1. Change the program state and the reduction rules to correctly track and account for the cost of operations. Change the semantics of the language so that a function call to a private function can only run for 50 steps, otherwise a default value is returned. Change the type of functions accordingly so that it is possible to know whether the execution was correct and successful.

The intuition is that public functions are used to call private functions. The latter ones are only allowed to run until they have fuel, i.e., for 50 steps, then execution resumes in public functions, that can call each other freely.

Question 2: **Program equivalence** ..... 10 marks

For each of these programs, tell if they are equivalent or not. If they are equivalent, show what they reduce to, no matter the input. If they are not, argue why and if possible, show a context that tells them apart.

1. $z : \mathit{Ref}\ (N \to N)$.
   $t_1 = \lambda x : N.\, !z\ 0; 2 + x$
   $t_2 = \lambda x : N.\, \mathit{if}\ x > 0\ \mathit{then}\ x + 2\ \mathit{else}\ !z\ x; x + 2.$

2. $t_1 = \mathit{let}\ x : N \to N = \lambda y : \forall\alpha.\alpha \to \alpha.\, \lambda z : N.\, y\ [N]\ (z + 1)\ \mathit{in}\ x$
   $t_2 = \lambda y : \forall\alpha.\alpha \to \alpha.\, \lambda x : N.\, (y\ [N]\ x) + 1.$

3. $f : (\mathit{Ref}\ N) \to N$.
   $t_1 = \mathit{let}\ x = \mathit{new}\ 0\ \mathit{in}\ f\ x; !x$
   $t_2 = \mathit{let}\ x = \mathit{new}\ 1\ \mathit{in}\ f\ (\mathit{new}\ 0); x := (!x - 1).$

4. $r : \mathit{Ref}\ N$.
   $t_1 = \mathit{let}\ x = !r\ \mathit{in}\ \mathit{let}\ y = \mathit{new}\ x\ \mathit{in}\ r := !y; !y$
   $t_2 = \mathit{let}\ x = \mathit{new}\ 0\ \mathit{in}\ \mathit{let}\ y = !x; !r\ \mathit{in}\ y.$

5. $t_1 = \lambda x : N.\, \langle x, 1\rangle .1$
   $t_2 = \mathit{let}\ x = \Lambda\alpha.\lambda x : \alpha.\, x\ \mathit{in}\ x.$