Assignment #4

Name: ______________________ ID: ______________________

This assignment has **6** questions, for a total of **25** marks.

Question 1: **Polymorphic behaviour** . . . . 5 marks

Given a closed term t of type $\forall\alpha.\, \alpha \to \alpha \to \alpha$, and two closed values v_1, v_2, prove that we have either $t\ N\ v_1\ v_2 \leadsto^* v1$ or $t\ N\ v_1\ v_2 \leadsto^* v2$.

Question 2: **Compatibility lemma for pair** .... 3 marks

Assuming:

- $\Gamma \vDash t_1 : \tau_1$
- $\Gamma \vDash t_2 : \tau_2$

Prove:

- $\Gamma \vDash \langle t_1, t_2 \rangle : \tau_1 \times \tau_2$

Question 3: **Compatibility lemma for projection** .... 3 marks

Assuming:

- $\Gamma \vDash t_1 : \tau_1 \times \tau_2$

Prove:

- $\Gamma \vDash t_1.1 : \tau_1$

Question 4: **Compatibility lemma for inl** .... 3 marks

Assuming:

- $\Gamma \vDash t_1 : \tau_1$

Prove:

- $\Gamma \vDash \mathit{inl}\ t_1 : \tau_1 \uplus \tau_2$

Question 5: **Compatibility lemma for case** .... 4 marks

Assuming:

- $\Gamma \vDash t_0 : \tau_1 \uplus \tau_2$
- $\Gamma, x_1 : \tau_1 \vDash t_1 : \tau$
- $\Gamma, x_2 : \tau_2 \vDash t_2 : \tau$

Prove:

- $\Gamma \vDash \mathit{case}\ t_0\ \mathit{of}\ \mathit{inl}\ x_1 \mapsto t_1 \mid \mathit{inr}\ x_2 \mapsto t_2 : \tau$

Question 6: **Compatibility lemma for unpack** .... 7 marks

Assuming:

- $\Delta \vdash \tau'$
- $\Delta, \Gamma \vDash t : \exists\alpha.\tau$
- $\Delta; \alpha, \Gamma; x : \tau \vDash t' : \tau'$

Prove:

- $\Delta, \Gamma \vDash \mathsf{unpack}\ t\ \mathsf{as}\ x\ \mathsf{in}\ t' : \tau'$