Assignment #1

Name: ____________________ ID: ____________________

This assignment has **5** questions, for a total of **25** marks.

Recall the following acronyms: SOS (structural operational semantics), COS (contextual operational semantics), SM (small step), BG (big step), CBV (call by value), CBN (call by name).

Question 1: **Reductions** . . . . 3 marks

Write what these terms reduce to, using the reduction strategy indicated before each of them. You can add the name of each rule applied on top of the arrow in order to identify what reduction steps have been taken (e.g., $(\lambda x.\, x)\ 3 \xrightarrow{beta} 3$).

1. SM-CBV $(\lambda x.\, \lambda y.\, \lambda z.\, ((x\ y)(x\ z)))\ (\lambda u.\, u + u)\ 4\ 5$

2. BG-CBV $(\lambda x.\, \lambda y.\, \lambda z.\, ((x\ y)(x\ z)))\ (\lambda u.\, u + u)\ 4\ 5$

3. SM-CBN $(\lambda x.\, \lambda y.\, y\ x\ (x + x))\ 7\ (\lambda z.\, \lambda u.\, u)$

Question 2: **Big step-call by name** . . . . 4 marks

Write the operational semantics rules for a big-step, call-by-name reduction. Write the semantically correct ones only, but write them all.

Question 3: **CBV and stuckness** . . . . 5 marks

Write a term t such that t in SM-CBV will get stuck (i.e., reduce to fail) but the same term t in SM-CBN will not. Show the reductions for each case.

- $t \stackrel{\text{def}}{=}$

1. SM-CBV

2. SM-CBN

Question 4: **Equivalence of SOS and COS** . . . . 8 marks

We saw some cases in class of the proof showing that small-step, call-by-value structured and contextual operational semantics (i.e., SOS-SM-CBV and COS-SM-CBV) are equivalent. Show the missing cases. Consider only the semantically correct rules for both semantics, i.e., BETA, OP, APP1, APP2, OP1, OP2 (TAPL page 72 plus in-class additions) for SOS-SM-CBV and rules CTX-BETA, CTX-OP, CTX for COS-SM-CBV.

1. If $t \to t'$ then $t \rightsquigarrow t'$

2. If $t \rightsquigarrow t'$ then $t \to t'$

Question 5: **Distinguish terms** .... 5 marks

Write out a term t that will reduce to two different values once applied to terms t_1 and t_2 below, i.e., such that $t\ t_1$ and $t\ t_2$ respectively reduce to v_1 and v_2 such that $v_1 \neq v_2$. The reduction strategy is SOS-SM-CBV, recall that if $n > m$ then $m - n = 0$. Write out the reductions too.

- $t_1 \stackrel{\text{def}}{=} \lambda x.\, \lambda y.\, (2 * x) - (3 * x) + ((\lambda z.\, y\ z\ x)\ 0)$
- $t_2 \stackrel{\text{def}}{=} \lambda x.\, \lambda y.\, (1 + x) - (3 + x) + ((\lambda z.\, y\ z\ x)\ 1)$

1. $t \stackrel{\text{def}}{=}$
2. $t\ t_1$ reductions.

3. $t\ t_2$ reductions.