Lecture 4: Translating the Meaning of Safety

CS350

Marco Patrignani

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A FA compiler does not preserve simple safety
under an intuitive translation
because of responses to invalid input

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- Safety property in T: never output 0

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- $[\![\cdot]\!]^{\mathsf{S}}_{\mathbf{T}}$ is FA
- Property in S (and $\mathbf{T}$): Do not output the secret until the 10th input
- timing is key to failure here

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Q1: how do we bridge this gap?

- (otherwise no correct compilation is possible)

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Different Trace Alphabets

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Q1: how do we bridge this gap?

Q2: how do we preserve the meaning of properties across languages?

- A … ne

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Different Trace Alphabets

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Recommended reading:

Patrignani and Garg. 2017. Secure Compilation as Hyperproperty Preservation.

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- What does this mean?

- We only know for safety and hypersafety

Safety Preservation with TPC

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dually

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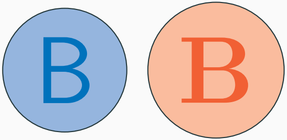
- safety = set of bad prefixes



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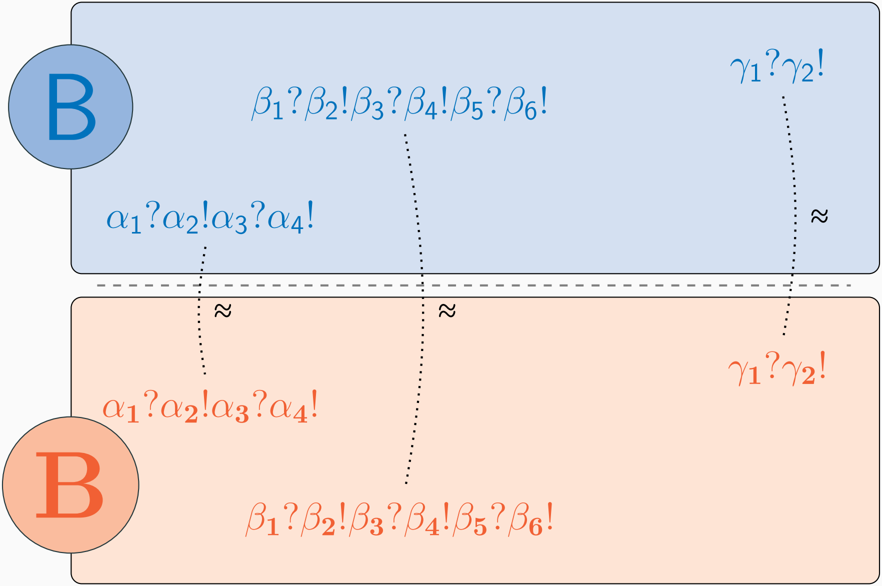

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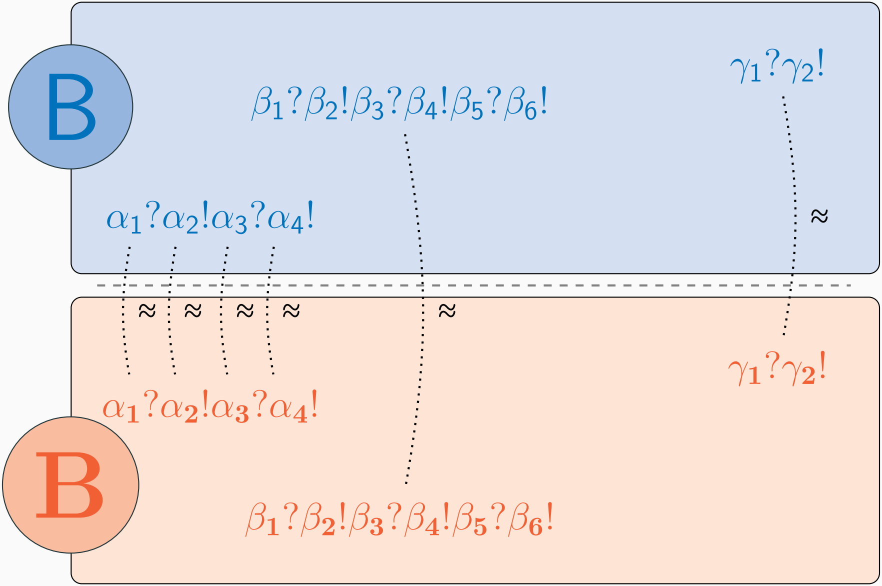

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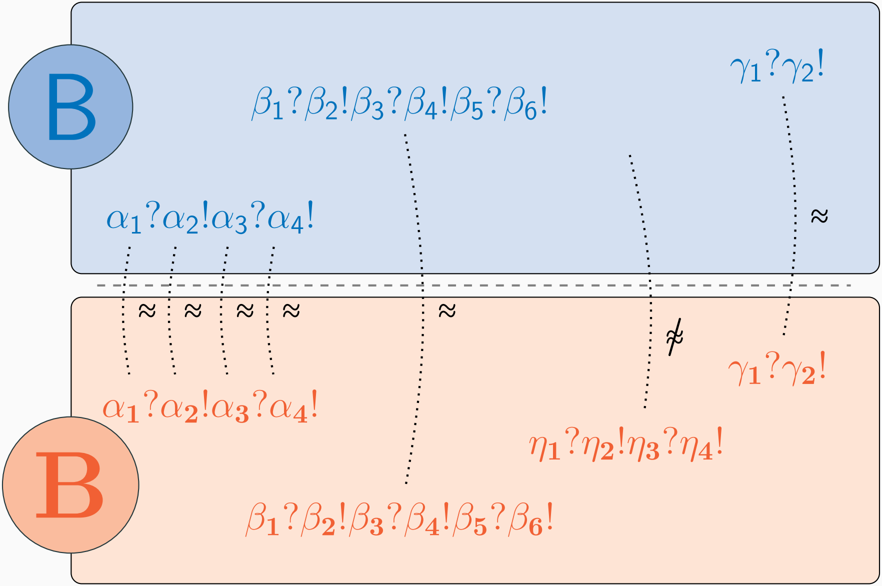

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outgoing action (!), violate safety

We have no control over η?

But if we only use $\checkmark$ for η! we can

preserve safety and hypersafety

Recall

- π is a property: $\{t\}$
- if π is safety, we can express it dually via the set of bad prefixes $\{m\}$ such that $\{m\} :: \pi$
- compact notation: $\{m\}$ = m

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- if π is safety, we can express it dually via the set of bad prefixes $\{m\}$ such that $\{m\} :: \pi$
- compact notation: $\{m\}$ = m
- H is a hyperproperty: $\{\pi\}$, $\{\{t\}\}$
- if H is hypersafety we can express it dually via the set of sets of bad prefixes $\{\{m\}\}$ such that $\{\{m\}\} :: H$
- compact notation: $\{\{m\}\}$ = $\mathbb{M}$

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Singletons: minimum addition s.t. any $\mathbf{P}$ not responding even once as $\surd$ are bad

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Theorem (Non-interference is preserved)

Let $\mathbb{M} :: \mathbb{NI}$. *Let* $\mathbb{M} \overset{\text{SHP}}{\approx} \mathbb{M}$ *and let* $\mathbb{S}$ *be a hyperproperty such that* $\mathbb{M} :: \mathbb{S}$. *Then,* $\forall \mathbf{t} \in \mathbb{S}, \mathbf{t} \in \mathbb{NI}$.

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- Can we devise a compiler that preserves hypersafety across languages?
- How will it differ from RSC / RHS ?

Trace-Preserving Compilation

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- reactive setting: only I/O traces (implicitly robust)
- traces capture all form of behaviour

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TPC: $\forall \mathsf{C} \in \mathsf{S}.\ \mathsf{TR}(\llbracket \mathsf{C} \rrbracket^{\mathsf{S}}_{\mathbf{T}}) = \mathsf{TR}(\mathsf{C}) \cup \mathbf{B}_{\mathsf{C}}$.

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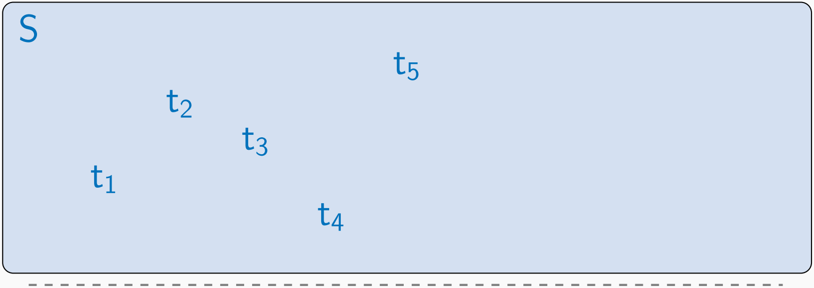

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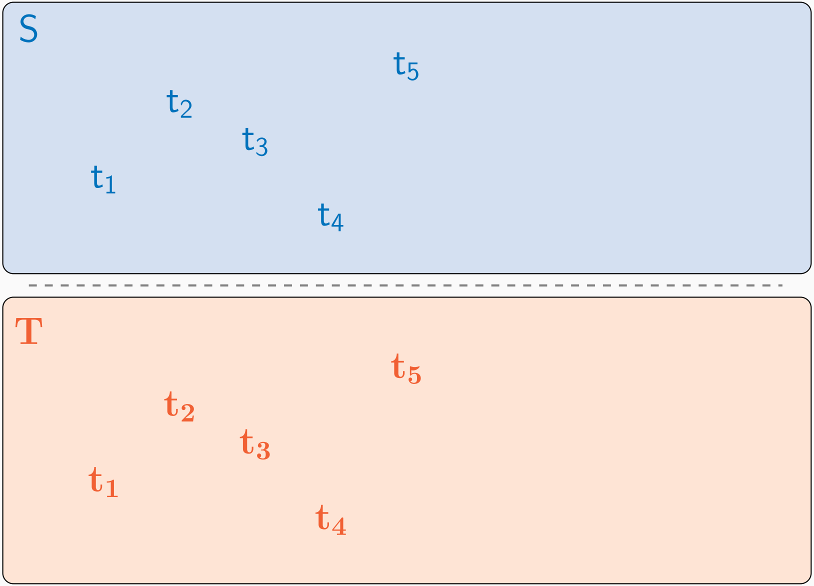

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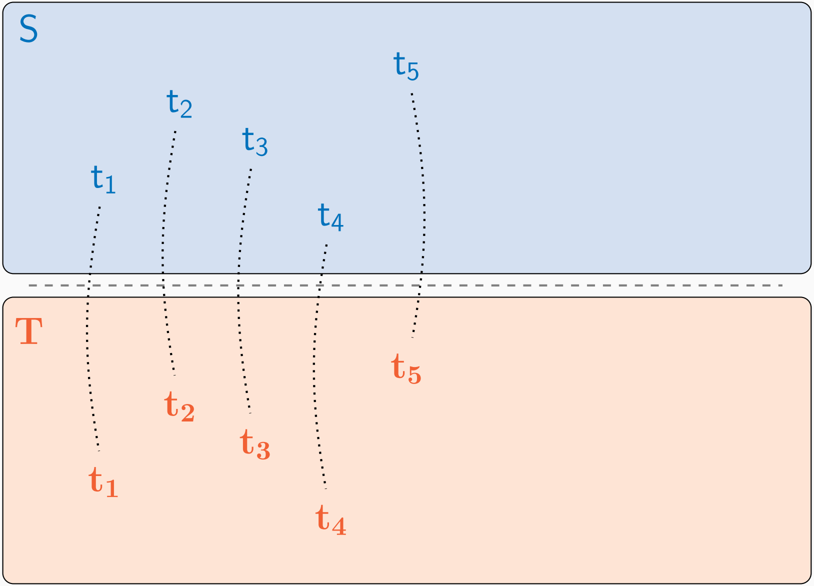

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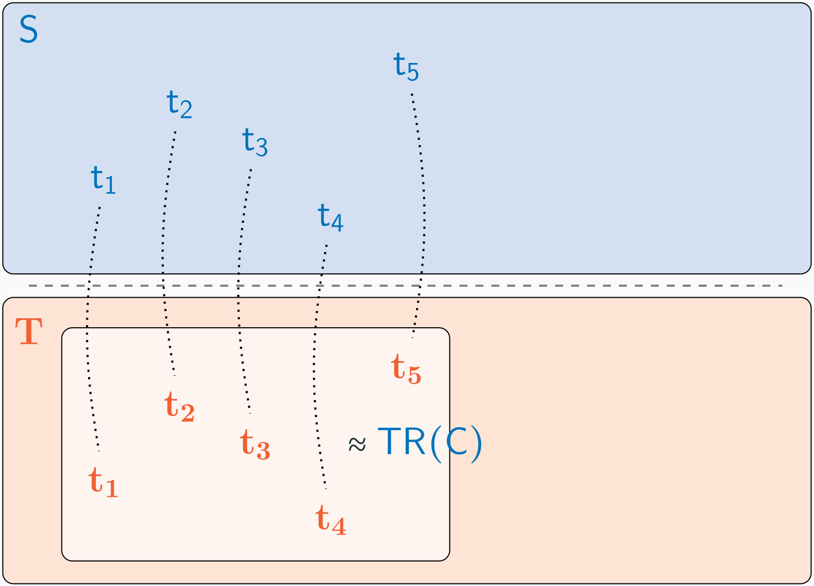

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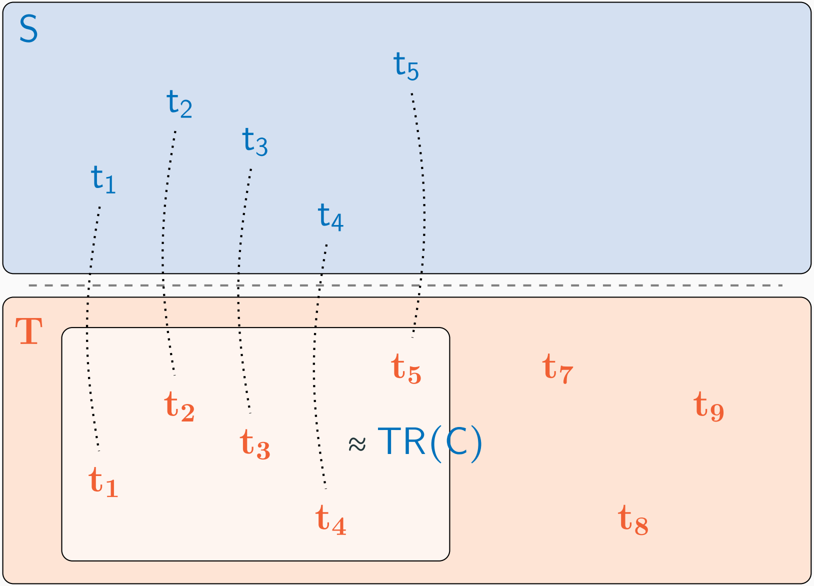

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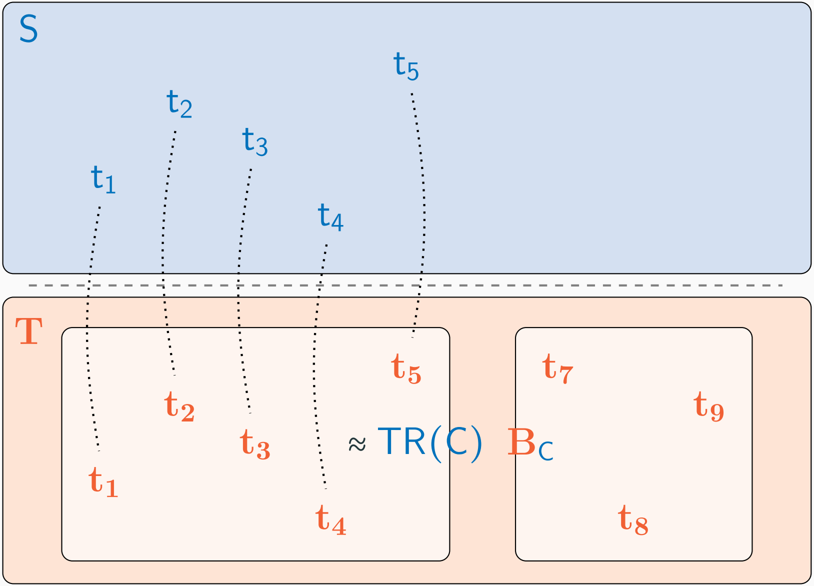

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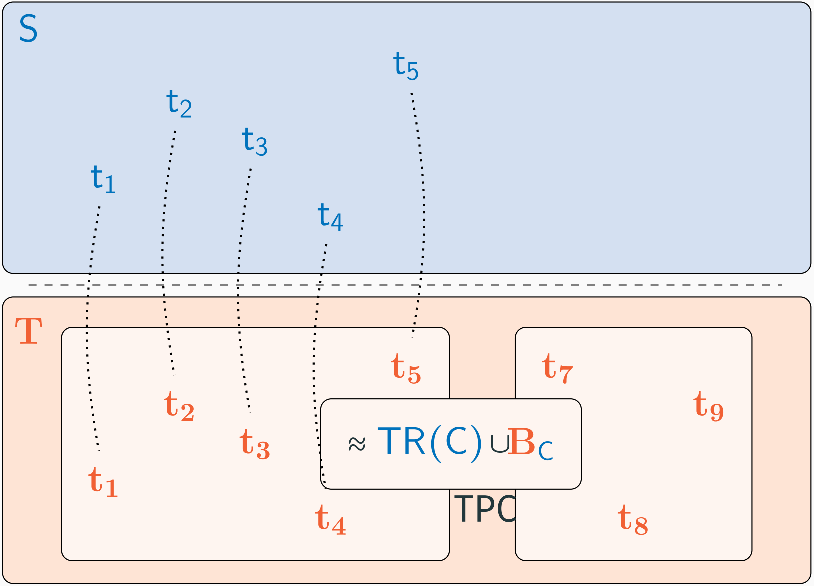

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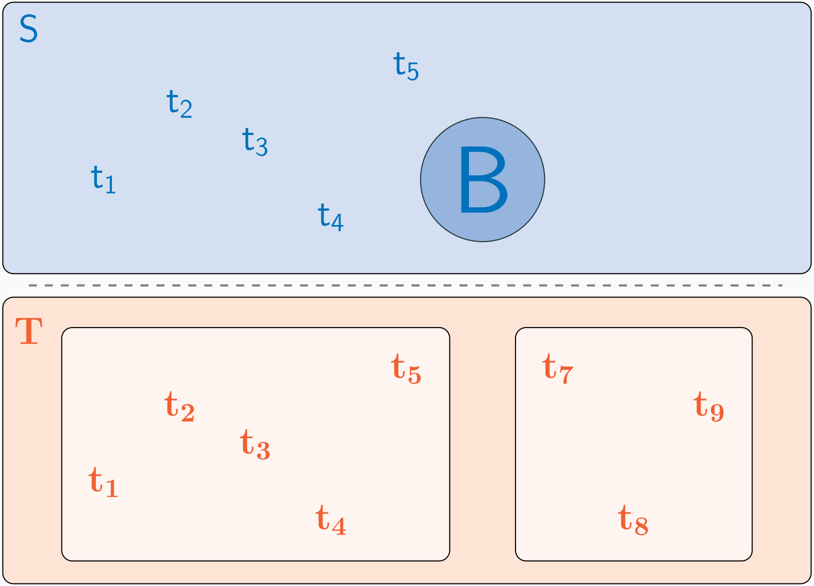

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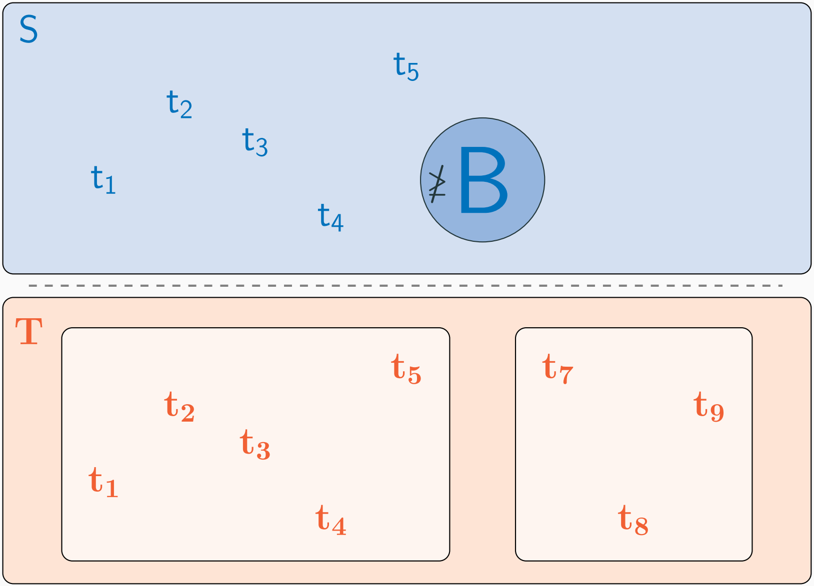

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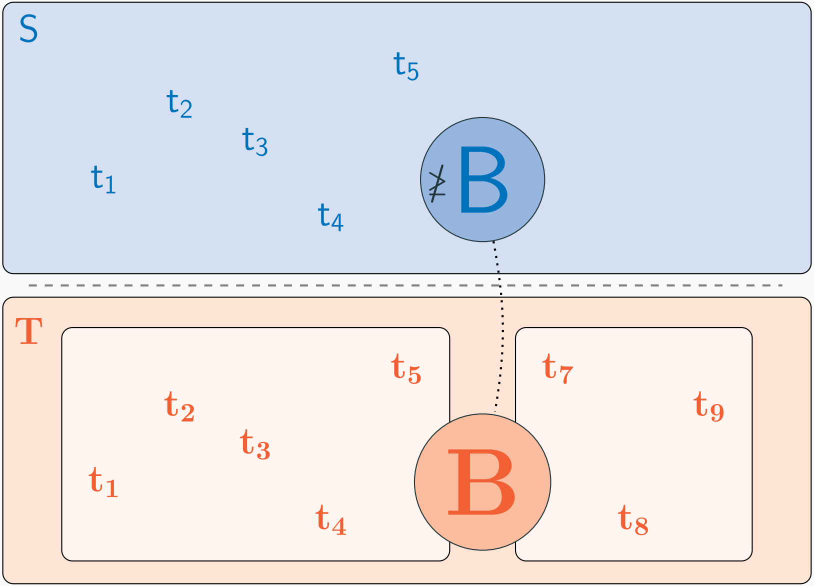

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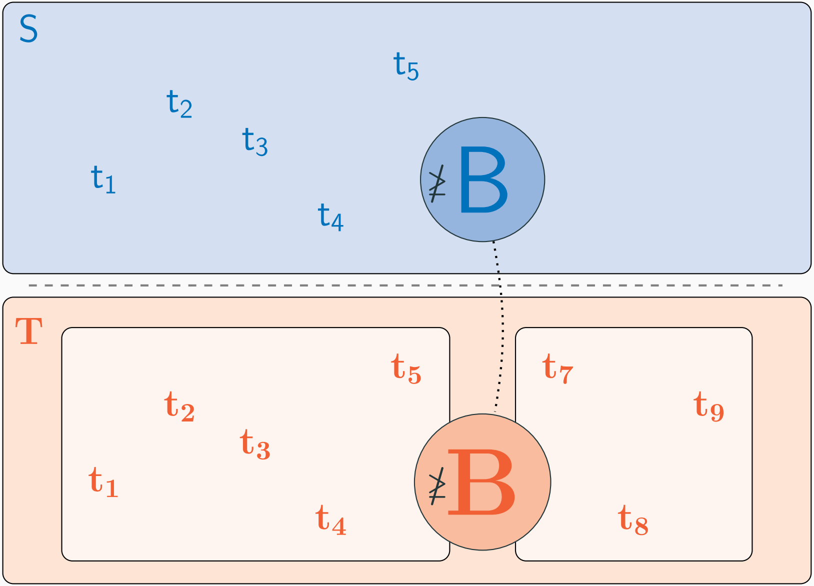

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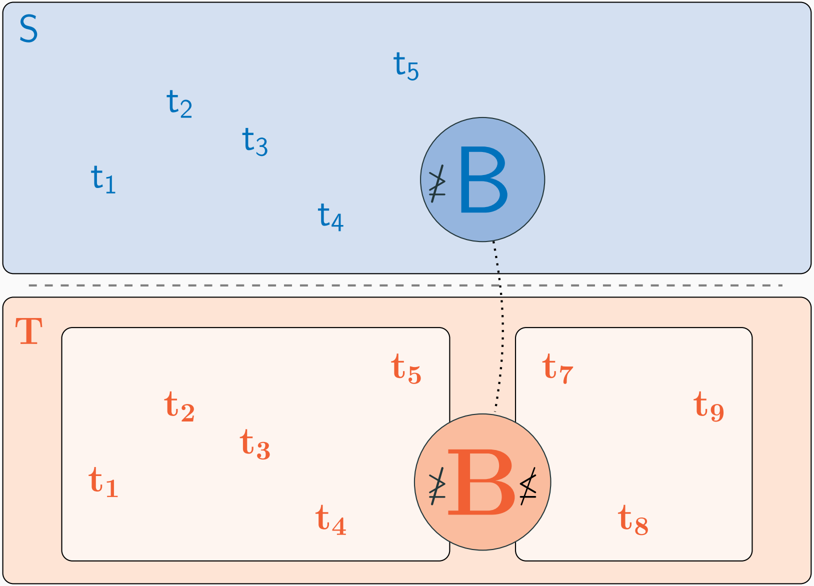

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- all are robust (TPC implicitly quantifies over all $\mathfrak{C}$)
- RC criteria (except for RSC and RHS) deal with properties
- TPC cannot handle properties