

Infinite Automata 2025/26

Starred Exercises Series 2

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Submit solutions by emailing them to Henry. Please include “Infinite Automata” in the subject line.

Star Exercise 2.1. Let \mathcal{S} be a homogeneous system of integer linear equations over n natural variables $\mathbf{x} \in \mathbb{N}^n$ (it does not matter how many equations there are) and suppose that every coefficient of every equation in \mathcal{S} has absolute value at most M . Suppose that, for any minimal non-zero solution $\mathbf{x} \in \mathbb{N}^n$ satisfies $\|\mathbf{x}\|_1 \leq B$. Let \mathcal{S}' be the homogeneous system of integer linear equations that is \mathcal{S} with one additional linear equation $\mathbf{a} \cdot \mathbf{x} = 0$ where $\mathbf{a} \in \mathbb{Z}^n$ and $\|\mathbf{a}\|_\infty \leq M$. Let $\mathbf{y} \in \mathbb{N}^n$ be a minimal non-zero solution to \mathcal{S}' , show that $\|\mathbf{y}\|_1 \leq \text{poly}(B, M, n)$.

Hint. One can aim to prove the bound $\|\mathbf{y}\|_1 \leq B(2BMn + 1)$.

Star Exercise 2.2. Prove that reachability in binary-encoded bidirected 1-VASS is in P.

A bidirected 1-VASS $V = (Q, T)$ has the property that for every $(p, x, q) \in T$, the transition $(q, -x, p) \in T$ as well. Roughly speaking, this means that every transition is “bidirected”: you can both transition from p to q while adding x , and transition from q back to p while adding $-x$.

Star Exercise 2.3. Prove that there exists a $d \leq 10$ such that there exists a family of instances of reachability in unary-encoded d -VASS whose shortest runs have at least double exponential length.

Remark. For $d = 11$, one can obtain a family of instances of reachability whose shortest runs have Tower length (this will follow from the arguments presented in Tutorial 12 and Lecture 12). On the other hand, it is known that reachability in unary-encoded 2-VASS can always be witnessed by polynomial length runs. The minimal $d \in \{3, 4, \dots, 10\}$ for which this exercise holds is an interesting and challenging open question.

Star Exercise 2.4. Assuming that reachability in VASS is decidable, show that reachability in VASS with disequality tests is decidable.

A d -VASS (Q, T, D) with disequality tests is a d -VASS (Q, T) which also has disequality tests $D \subseteq Q \times \{1, \dots, d\} \times \mathbb{N}$. The disequality tests forbid specified counters from having a specified value at specified state (in the same kind of way that a B -bounded VASS is forbidden from having counters exceeding B). Precisely, if $(q, i, c) \in D$, then all configurations (q, \mathbf{x}) such that $\mathbf{x}[i] = c$ are forbidden. For example, if $(p, 2, 10) \in D$ then the second counter cannot equal 10 at state q .

Hint. Without loss of generality, one can assume that there is at most one disequality test (q, i, \mathbf{x}) for every state $q \in Q$. This is because one can introduce a chain of states, each with one disequality test, and are connected with zero-effect transitions.