Infinite Automata 2025/26

Exercise Sheet 9

Wojciech Czerwiński and Henry Sinclair-Banks

Exercise 9.1. Show that semilinear sets are closed under complement. In other words, if $S \subseteq \mathbb{N}^d$ is a semilinear set, then $\mathbb{N}^d \setminus S$ is a semilinear set.

Hint 1. It is suffices to prove that the complement of a linear sets with linearly independent periods is a semilinear set.

Hint 2. Let C be the closure of a given linear set L with linearly independent periods (i.e. C is the smallest integer cone that contains L and does not have any gap inside. The complement of $\mathbb{N}^d \setminus L$ can be described as $(\mathbb{N}^d \setminus C) \cup (C \setminus L)$.

Hint 3. Notice that the set $C \setminus L$ is a linear set with the same periods as L but different bases.

Exercise 9.2. Show that semilinear sets are closed under difference. In other words, if $S_1, S_2 \subseteq \mathbb{N}^d$ are semilinear sets then $S_1 \setminus S_2$ is a semilinear set.

Exercise 9.3. Show that the coverability problem for binary-encoded 1-VASS is in PSPACE.

Hint. Show that if there is a run witnessing coverability, then there must be a run witnessing coverability that has at most exponential length.

Exercise 9.4. Show that the coverability problem for binary-encoded 1-VASS is in NP.

Hint. Prove that if there is a run witnessing coverability, then there must be a run witnessing coverability that take the form: a short path, followed by a short positive cycle taken at most exponentially many times, followed by a short path.

Exercise 9.5. Show that the coverability problem for binary-encoded 1-VASS is in P.

Hint. Prove that, actually the witnesses consisting of a short path, followed by a short positive cycle, followed by a short path can be checked by a polynomial time dynamic programming algorithm.