Infinite Automata 2025/26

Exercise Sheet 1

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Exercise 1.1. Given multisets A and B each comprising of at least n^2 positive integers not greater than n, show that there exist subsets $A' \subseteq A$ and $B' \subseteq B$ such that $A', B' \neq \emptyset$ and

$$\sum_{a \in A'} a = \sum_{b \in B'} b.$$

Exercise 1.2. (Strengthening of Exercise 1.1.) Given multisets A and B each comprising of at least $2n$ positive integers not greater than n, show that there exist subsets $A' \subseteq A$ and $B' \subseteq B$ such that $A', B' \neq \emptyset$ and

$$\sum_{a \in A'} a = \sum_{b \in B'} b.$$

Exercise 1.3. (Generalisation of Exercise 1.2.) Given a multiset A of n positive integers not greater than m and a multiset B of m positive integers not greater than n, show that there exist subsets $A' \subseteq A$ and $B' \subseteq B$ such that $A', B' \neq \emptyset$ and

$$\sum_{a \in A'} a = \sum_{b \in B'} b.$$

Exercise 1.4. Show that the reachability problem for pushdown automata is decidable in polynomial time. We shall assume that the automaton starts in a distinguished initial state and with an empty stack and we ask whether it can reach a distinguished final state also with an empty stack.

Hint 1. Compute an equivalent context-free grammar.

Hint 2. For each pair of states (p, q), let $X_{p,q}$ be a nonterminal deriving the words which can be accepted starting from state p with an empty stack and ending in state q with an empty stack.

Exercise 1.5. Prove that reachability problem for two-stack pushdown automata is undecidable.

Hint. Reduce from the reachability problem (the halting problem) for Turing machines with one tape.

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