

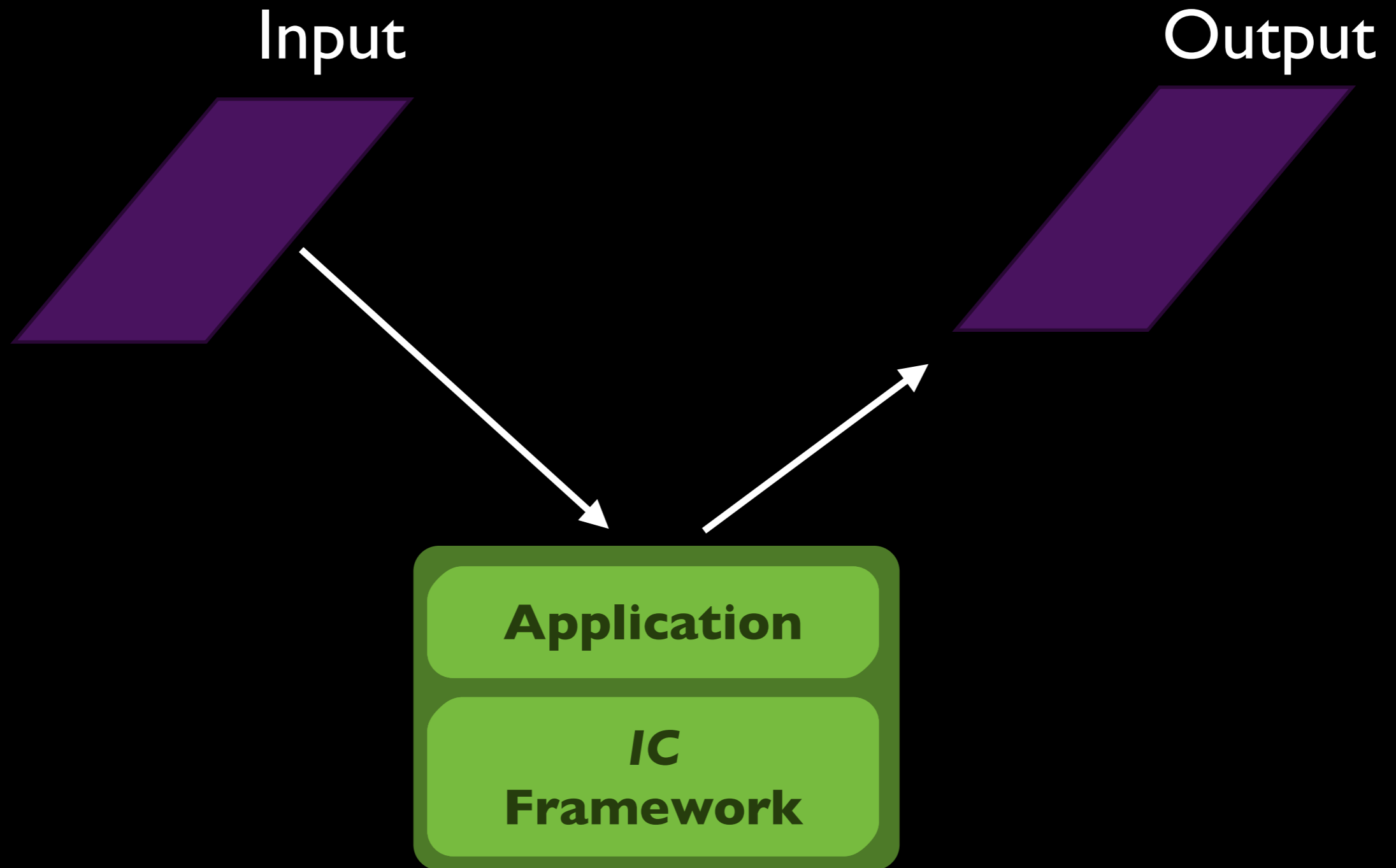
Adapton:
*Composable Demand-Driven
Incremental Computation*

Matthew A. Hammer

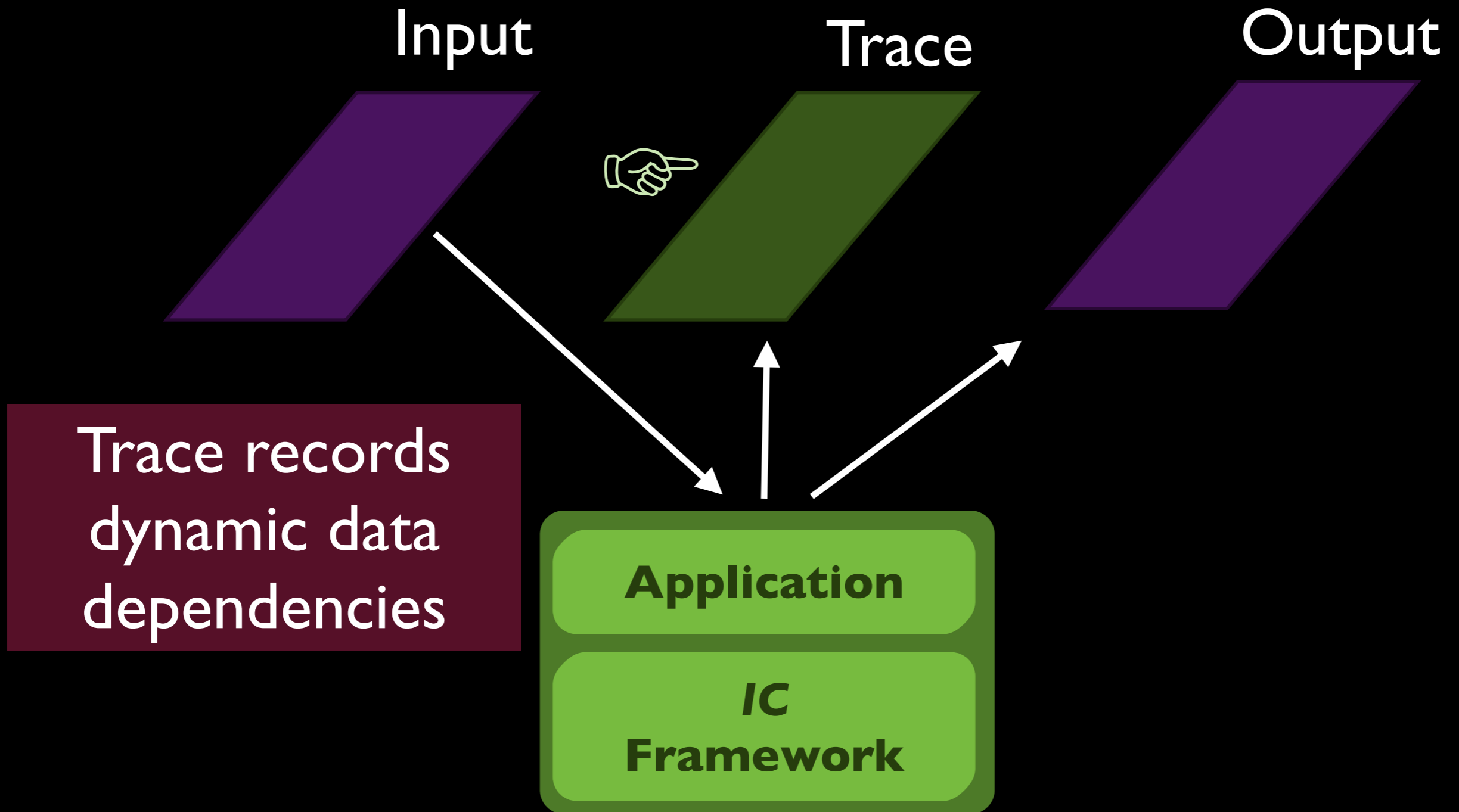
Khoo Yit Phang, Michael Hicks *and*
Jeffrey S. Foster



Incremental Computation



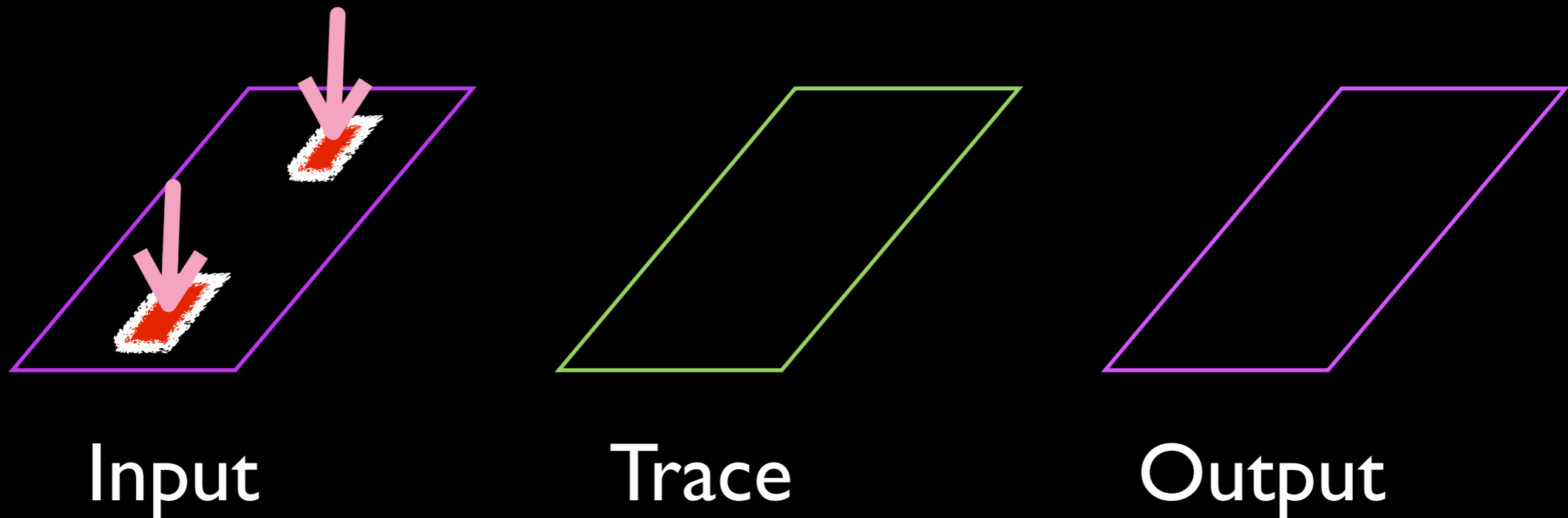
Incremental Computation



Incremental Computation



 **Mutations**



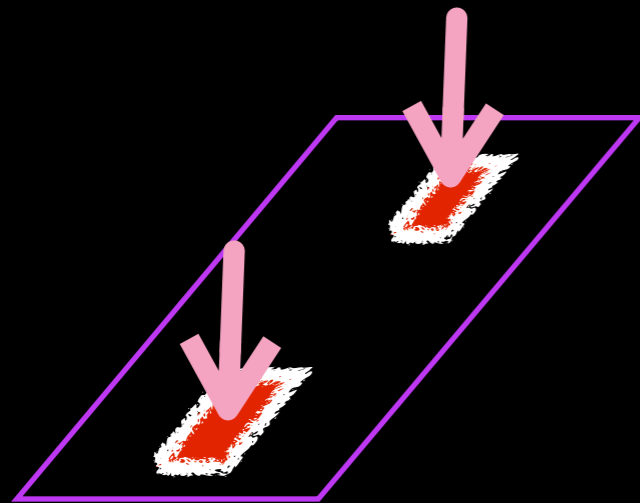
Incremental Computation



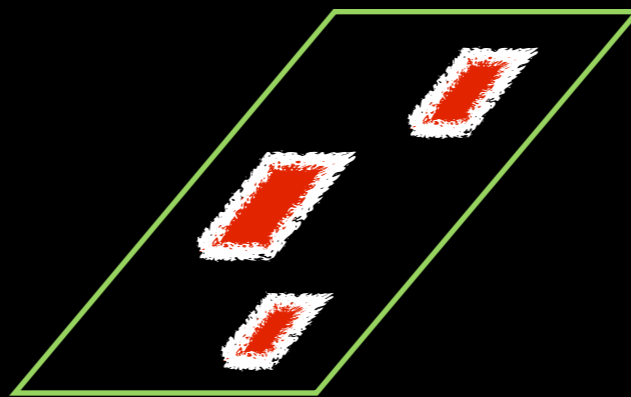
Mutations



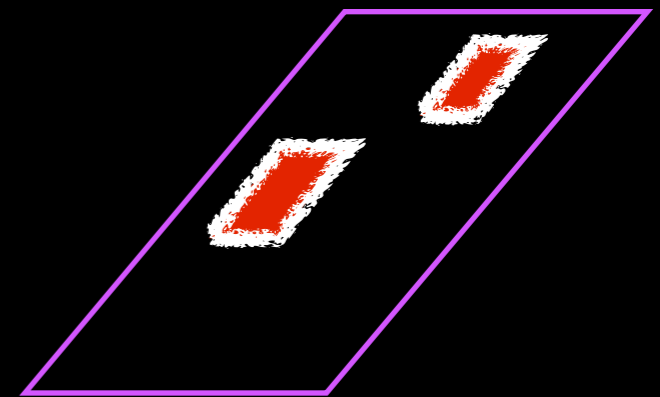
Inconsistencies



Input

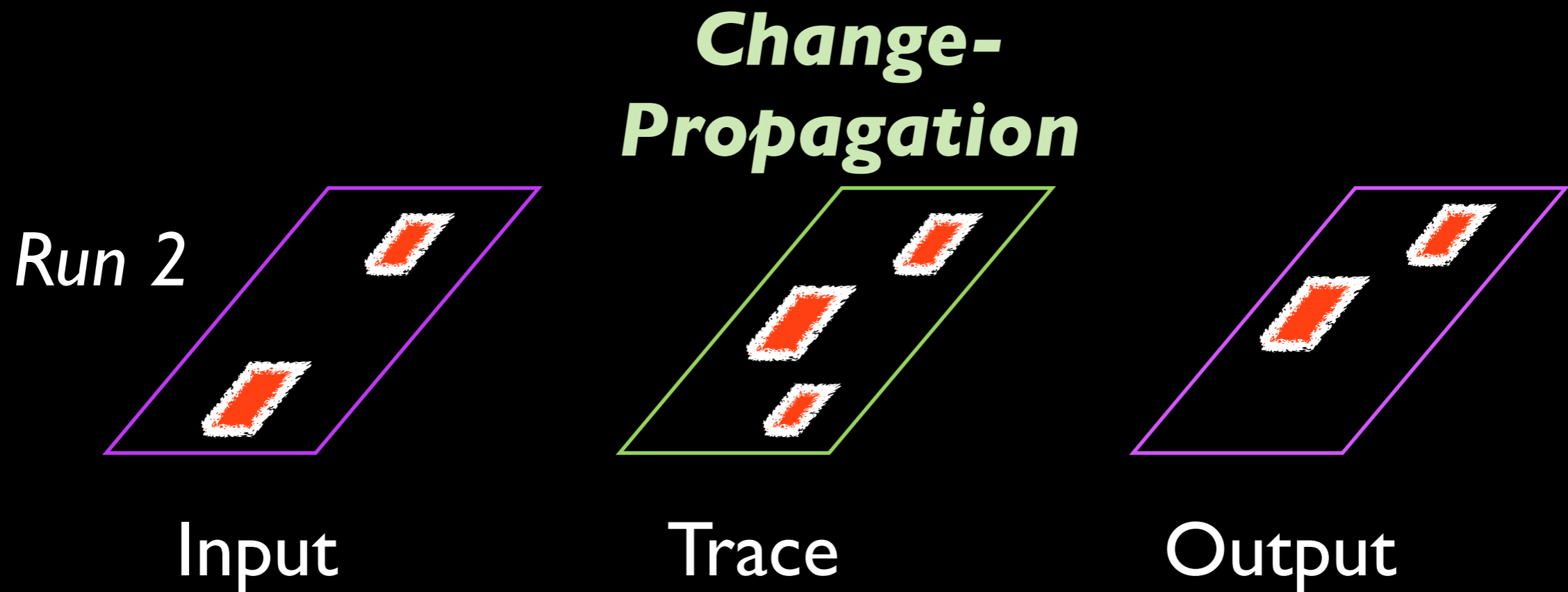


Trace

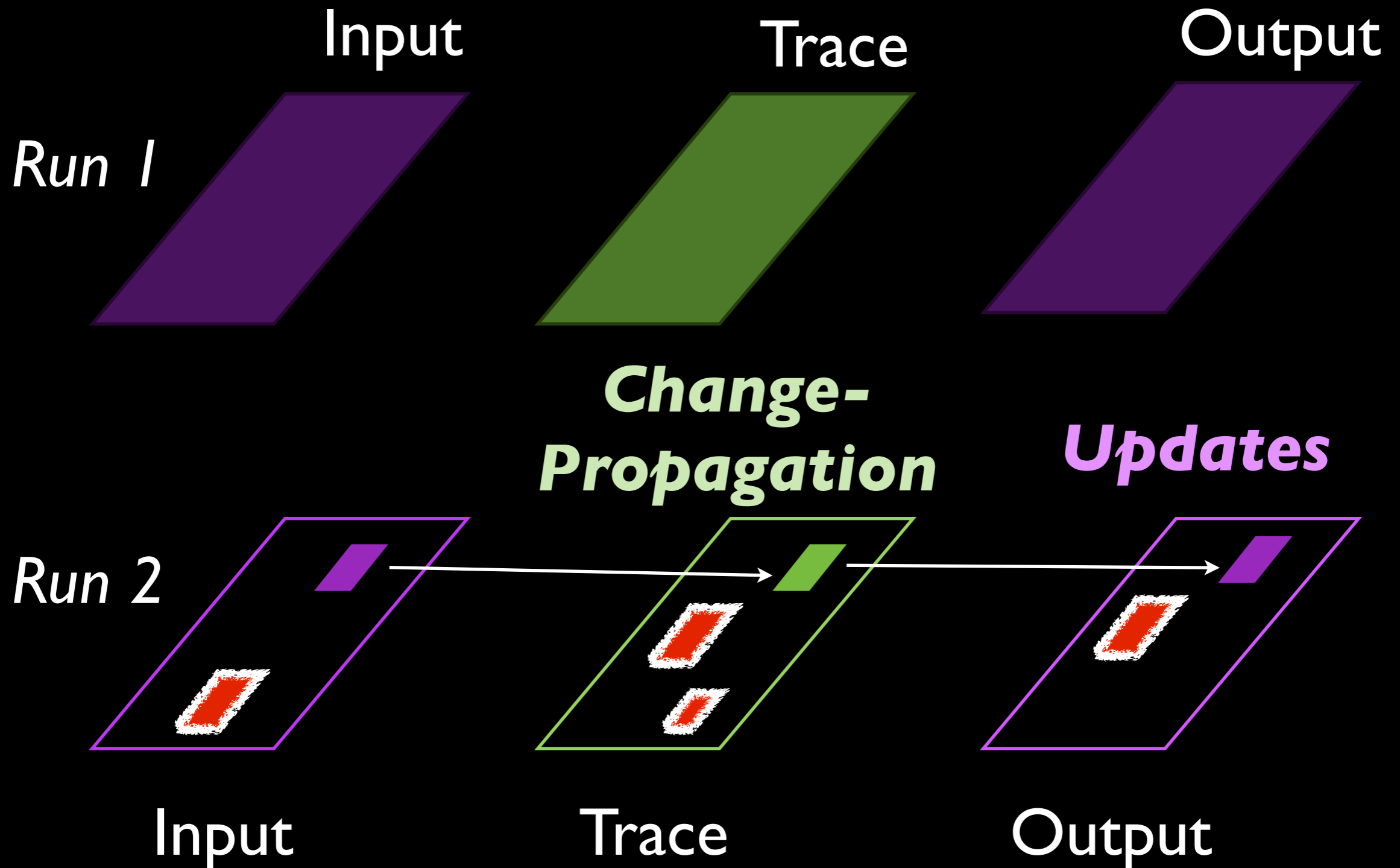


Output

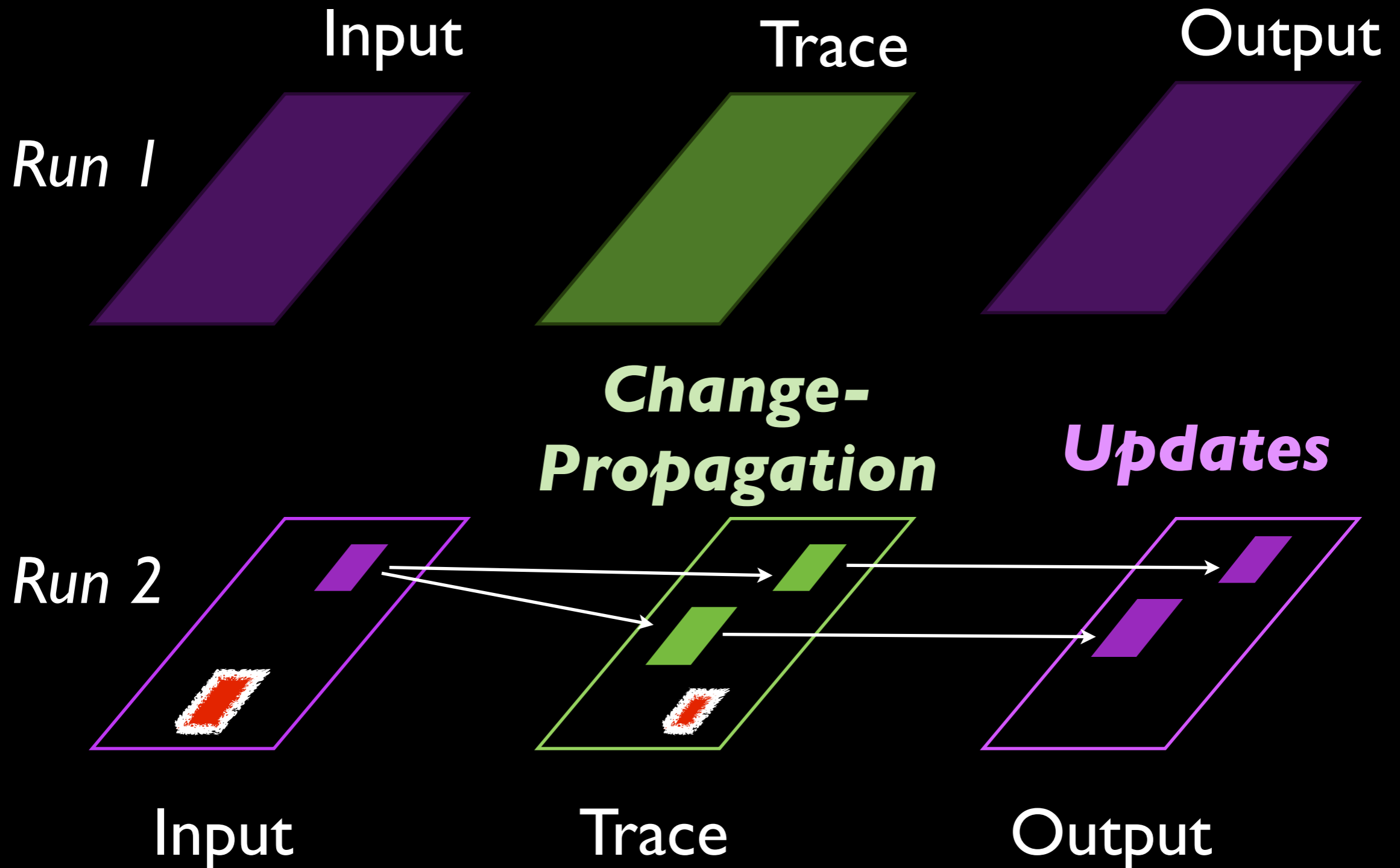
Incremental Computation



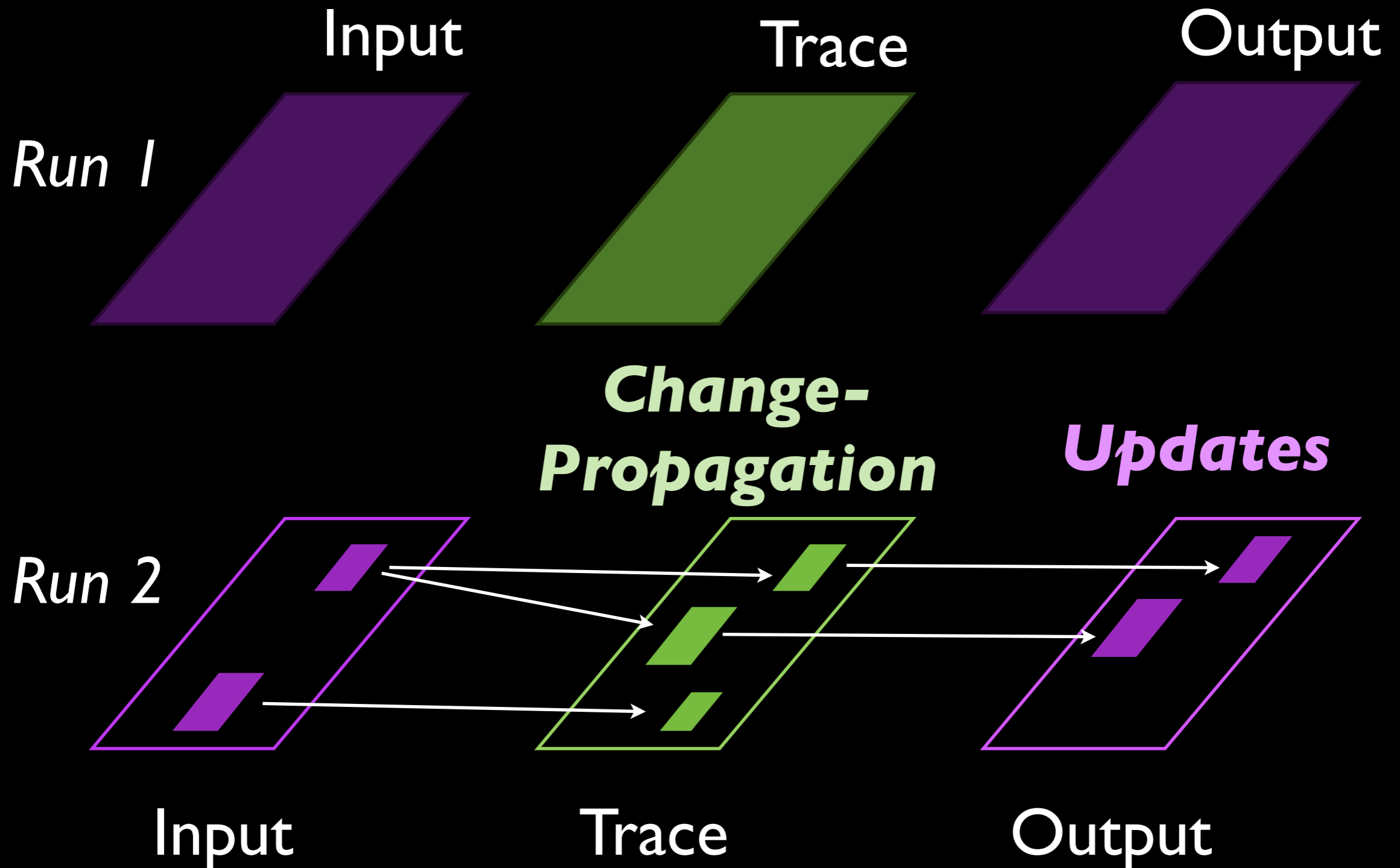
Incremental Computation



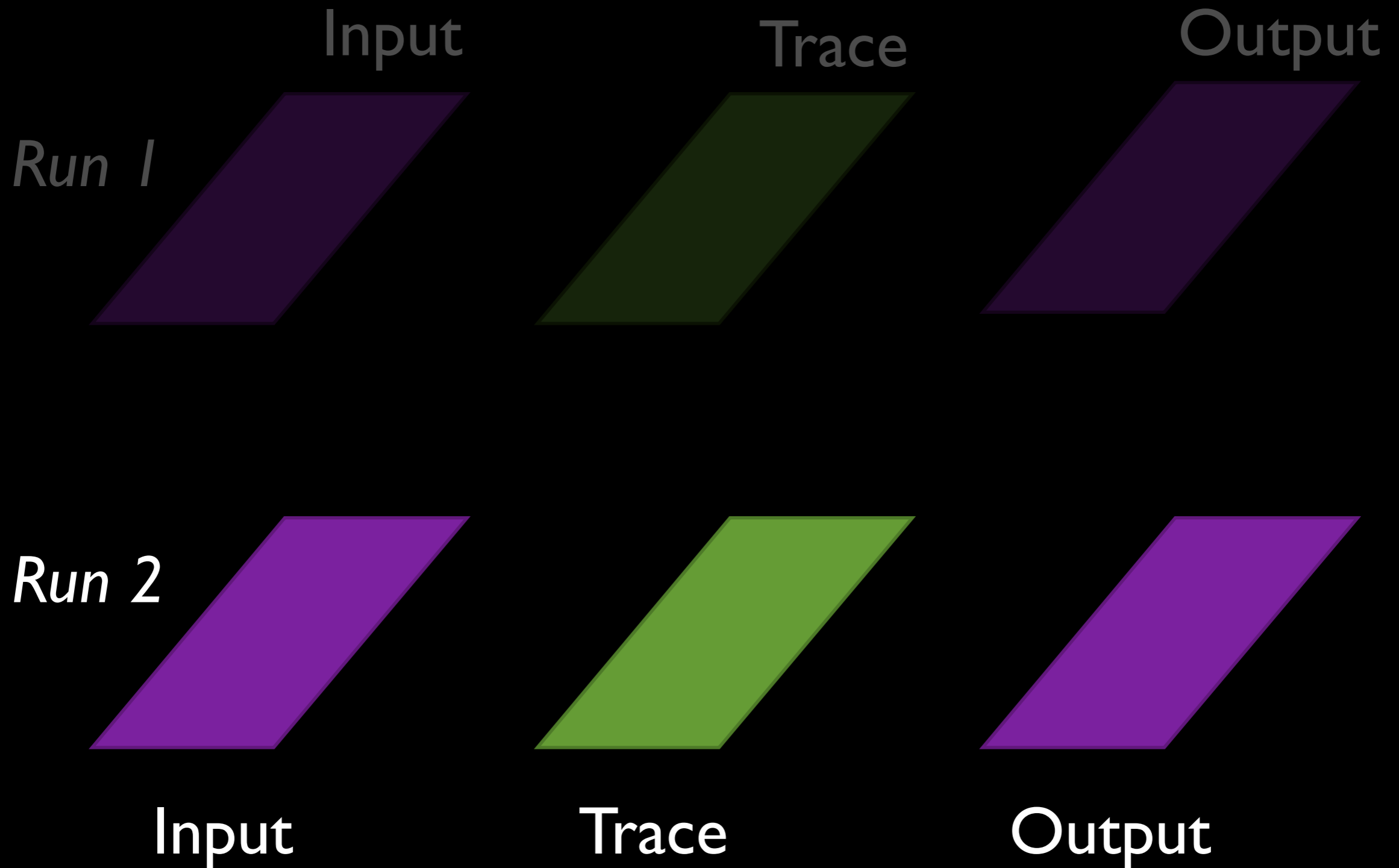
Incremental Computation



Incremental Computation



Incremental Computation

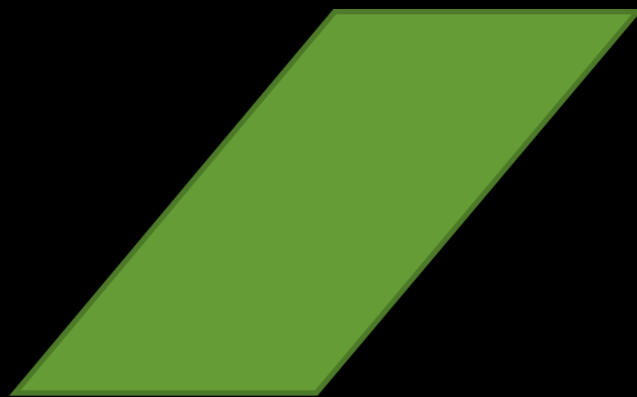


Incremental Computation

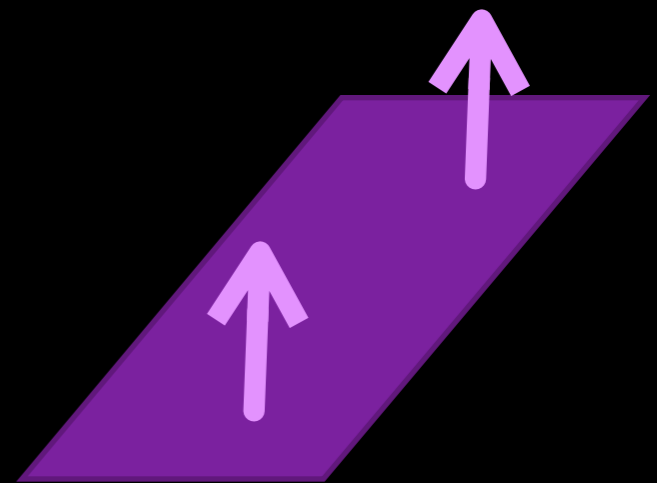
Run 2



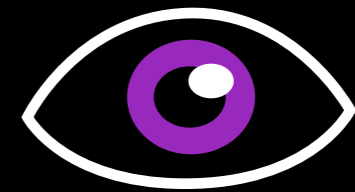
Input



Trace



Output

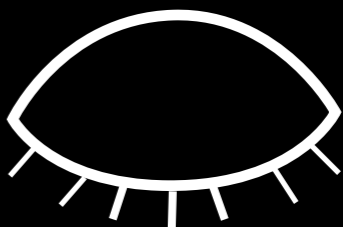


Observations

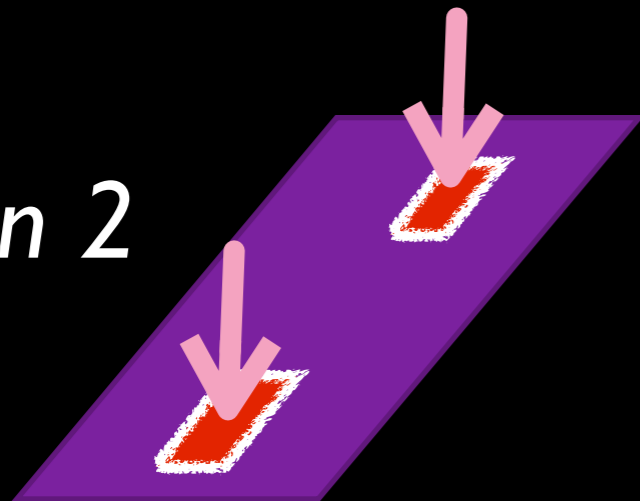
Incremental Computation

loop..

 **Mutations**


Observations

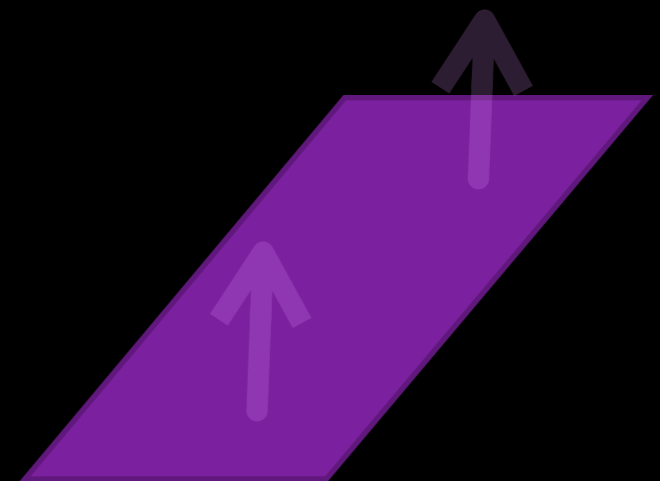
Run 2



Input



Trace



Output

Incremental Computation

Propagation respects program semantics:

Theorem:
Trace and output are
**“from-scratch”-
consistent**

Equivalently:
Change propagation is
**History
independent**

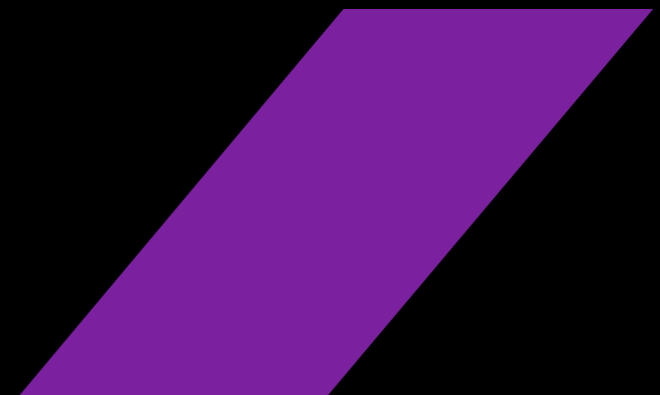
Run 2



Input



Trace



Output

Existing Limitations

(self-adjusting computation)

▶ **Change propagation is eager**

Not driven by output observations



▶ **Trace representation**

= **Total ordering**

Limits reuse, excluding certain patterns

Interactive settings suffer in particular

Adapton: Composable, Demand-Driven IC

- Key concepts:

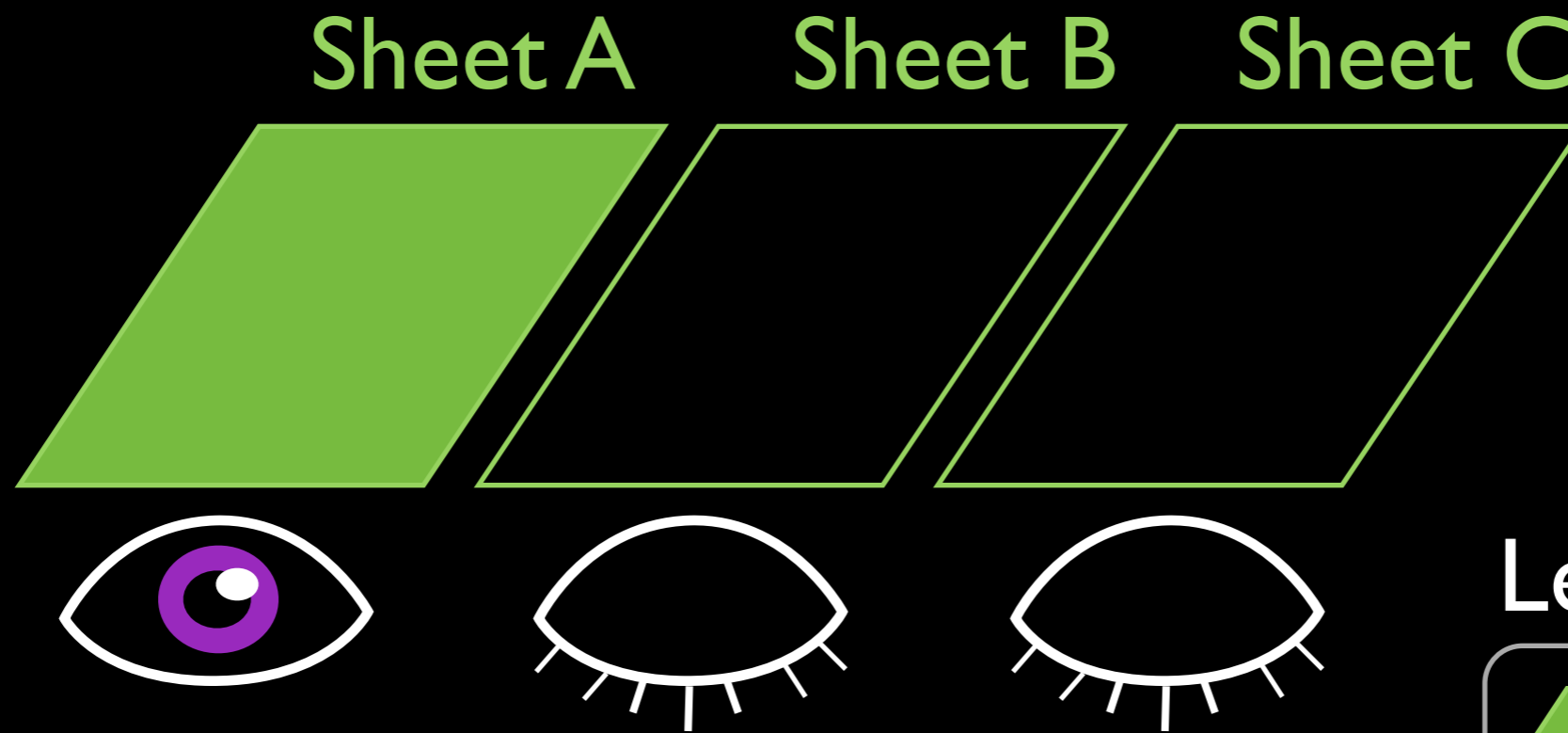
 - **Lazy thunks**: programming interface

 - ***Demanded Computation Graph***
(DCG): represents execution trace

- Formal semantics, proven sound
- Implemented in OCaml (and Python)
- Speedups for all patterns (unlike SAC)
- Freely available at <http://ter.ps/adapton>




Interaction Pattern: **Laziness**

Do not (re)compute obscured sheets



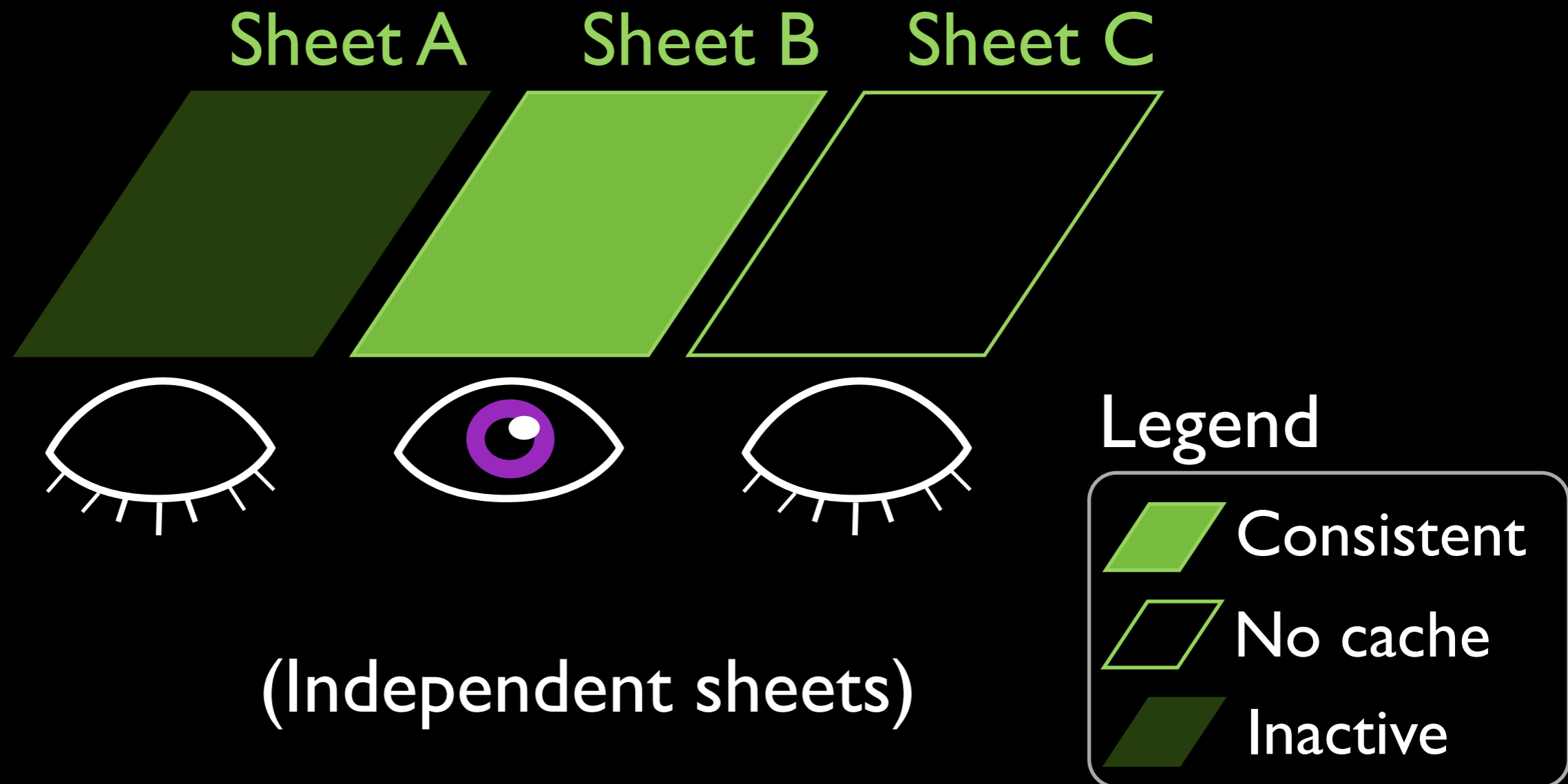
(Independent sheets)

Legend

-  Consistent
-  No cache
-  Inactive

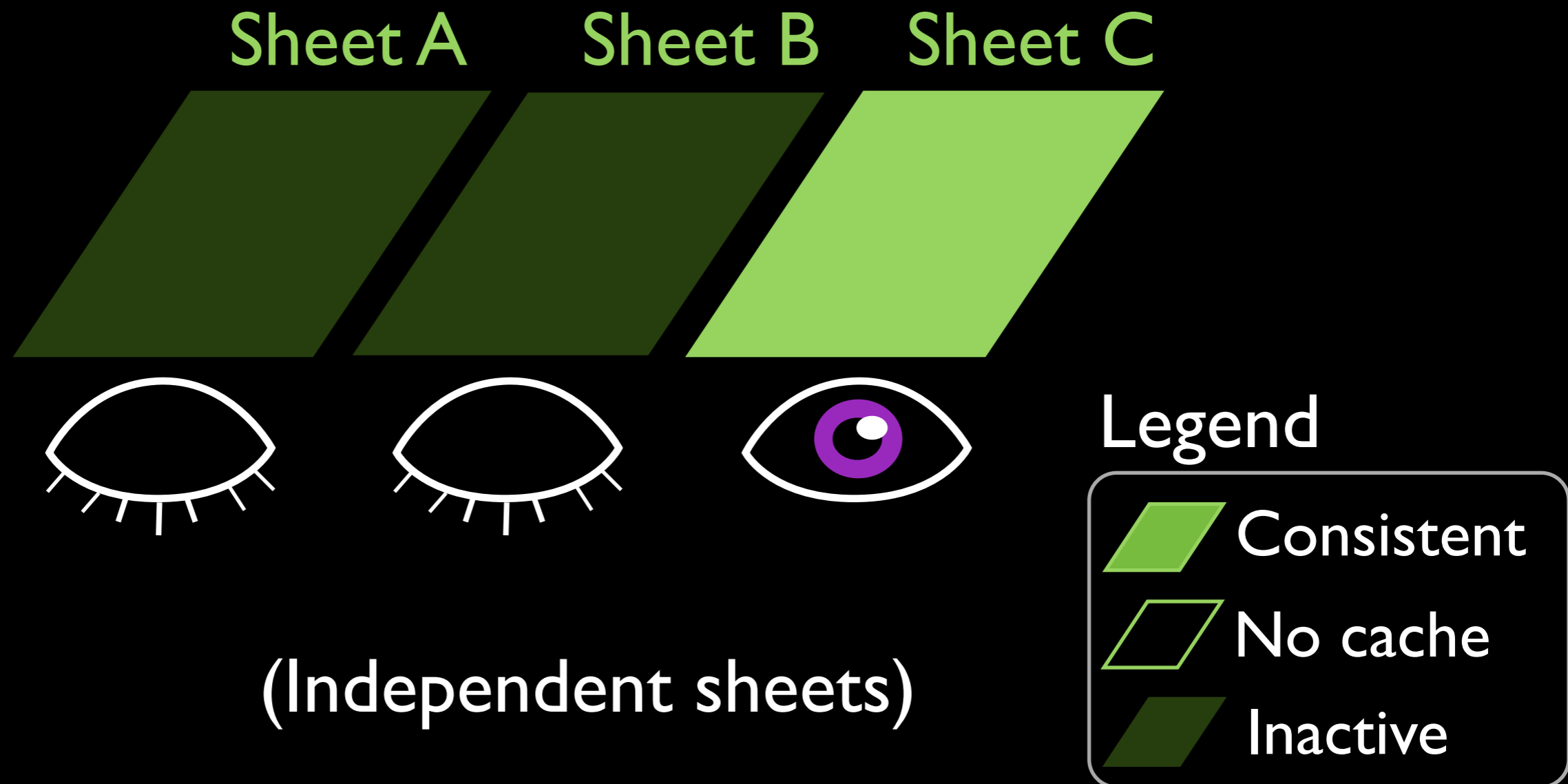
Interaction Pattern: **Laziness**

Do not (re)compute obscured sheets



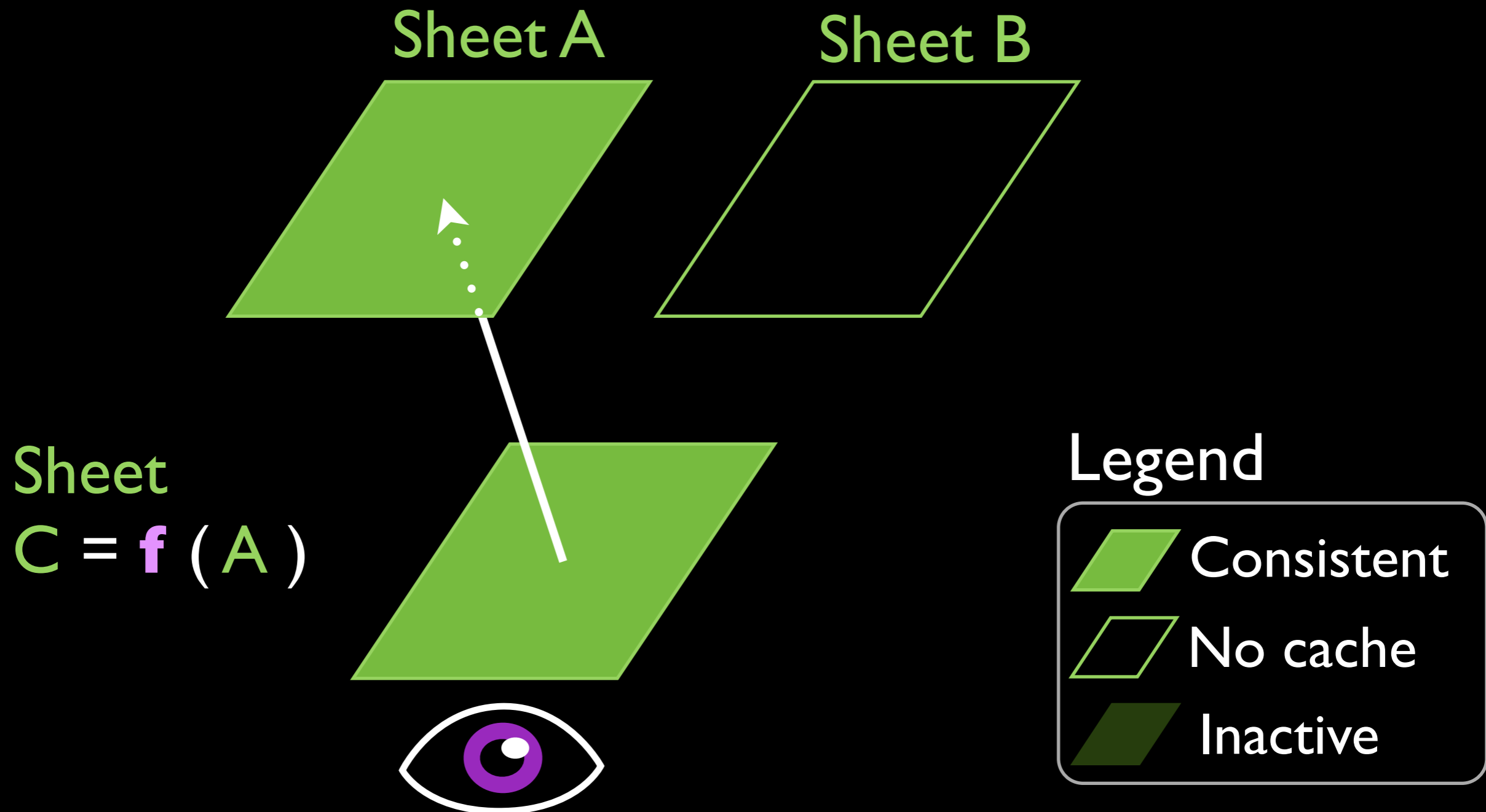
Interaction Pattern: **Laziness**

Do not (re)compute obscured sheets



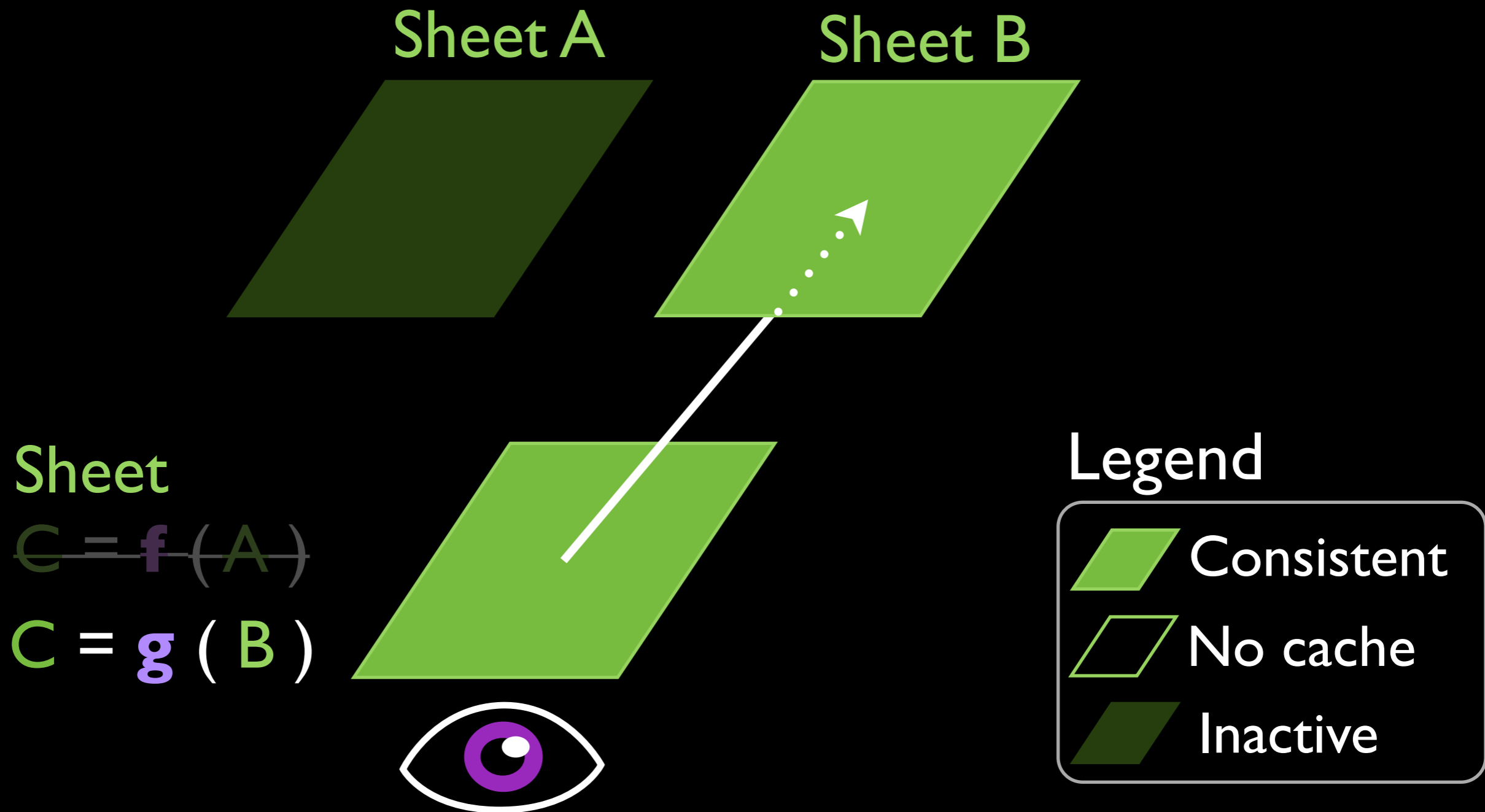
Interactive Pattern: **Switching**

Demand / control-flow change



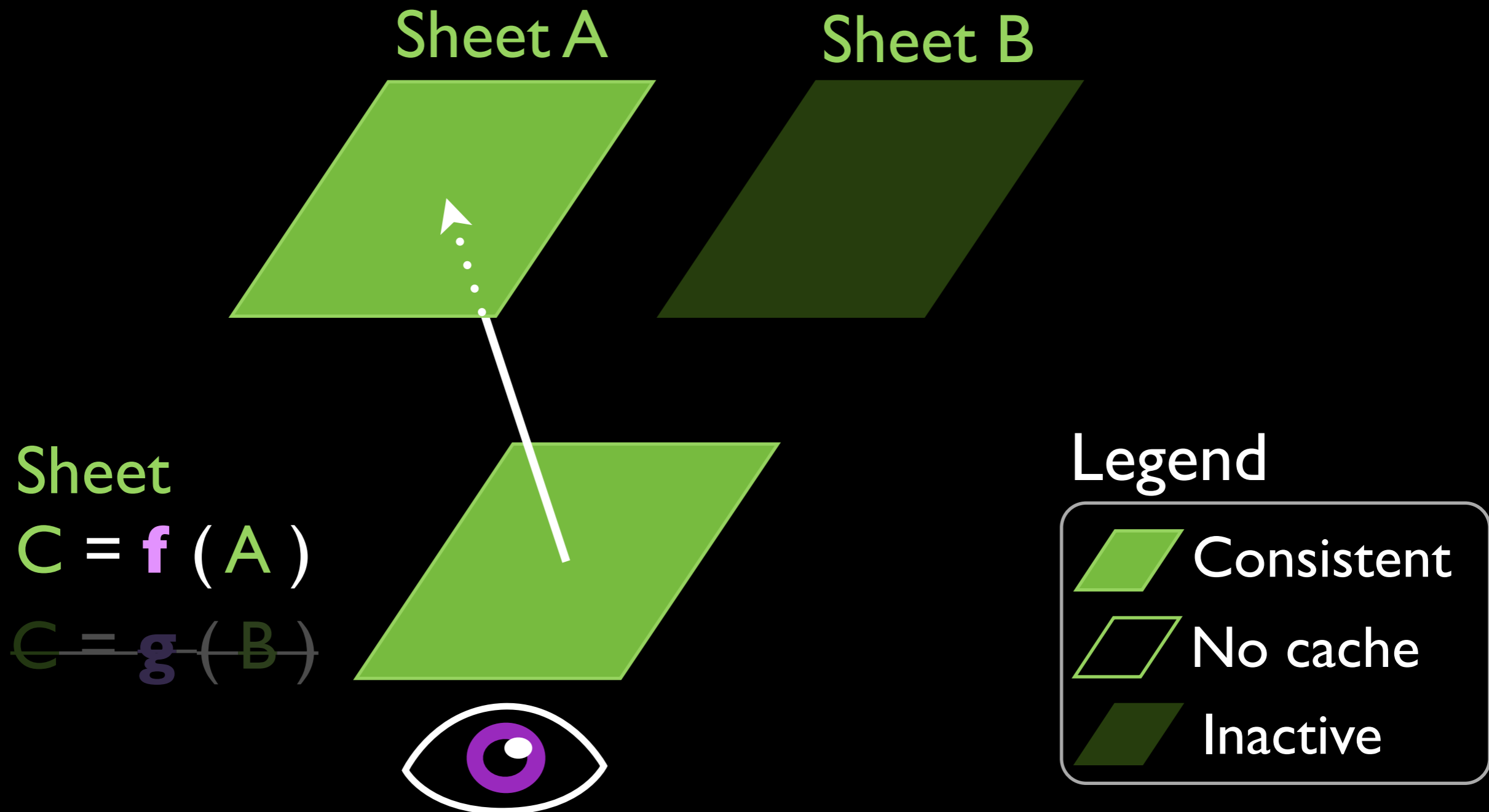
Interactive Pattern: **Switching**

Demand / control-flow change



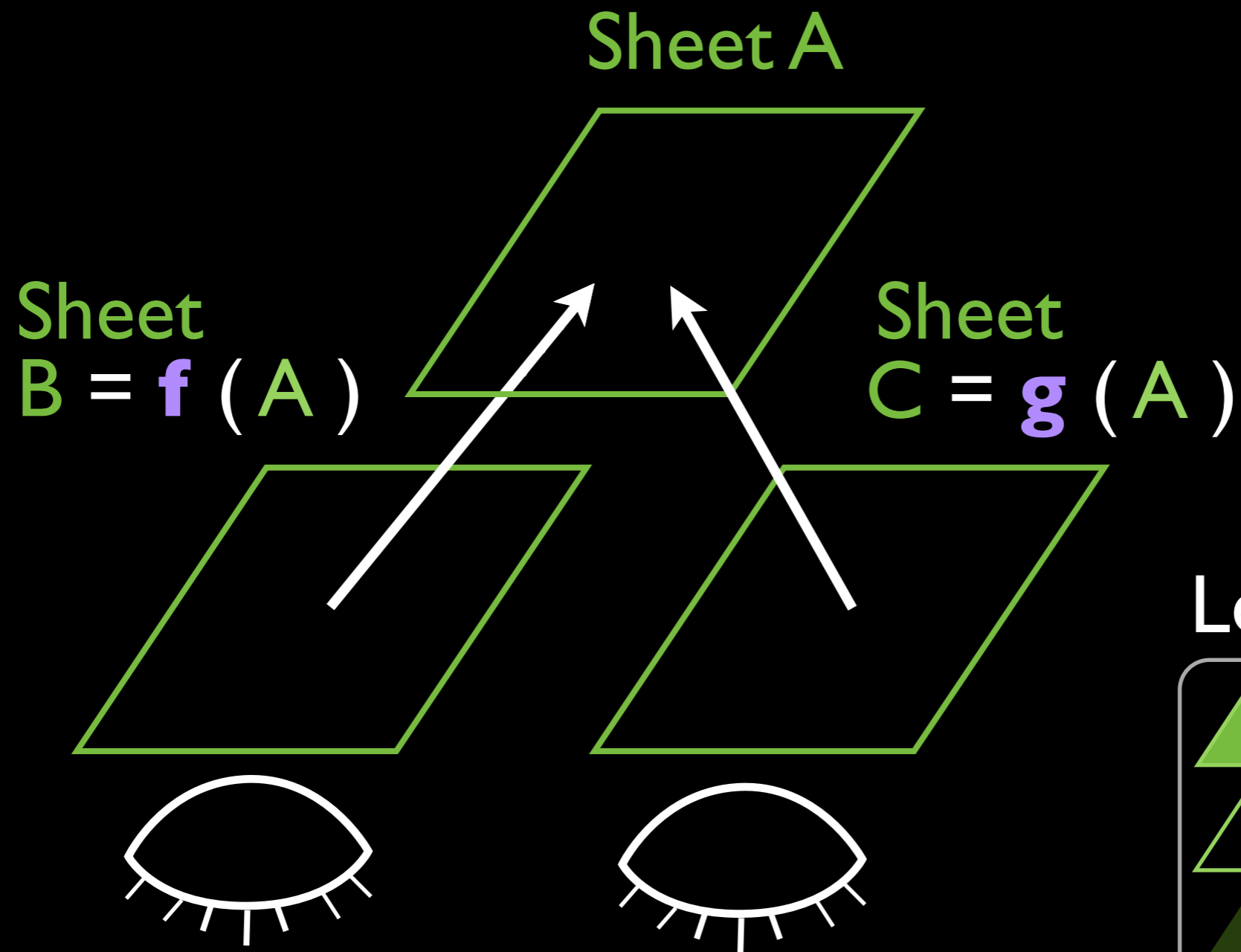
Interactive Pattern: **Switching**

Demand / control-flow change



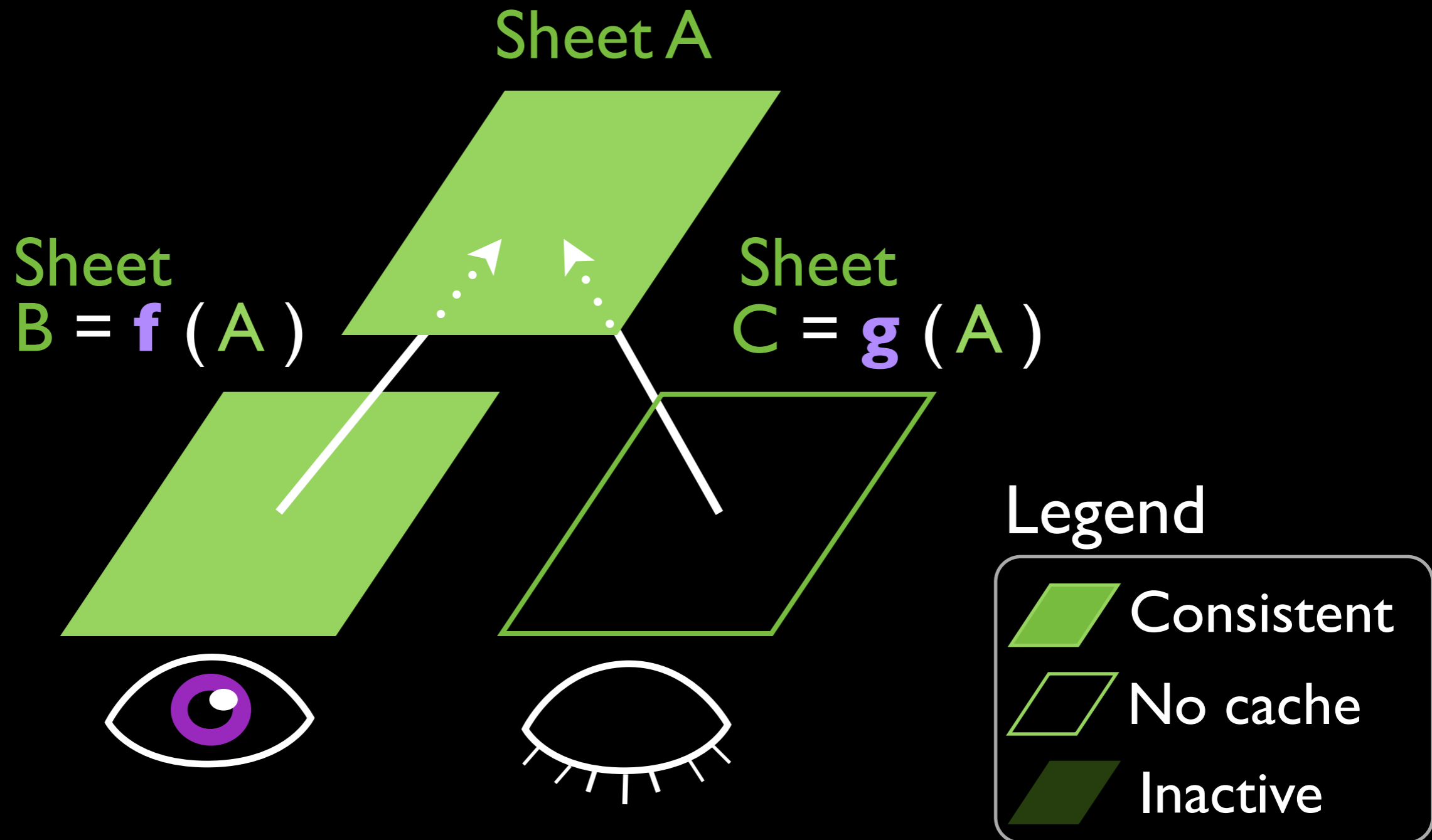
Interaction Pattern: **Sharing**

B and C share work for A



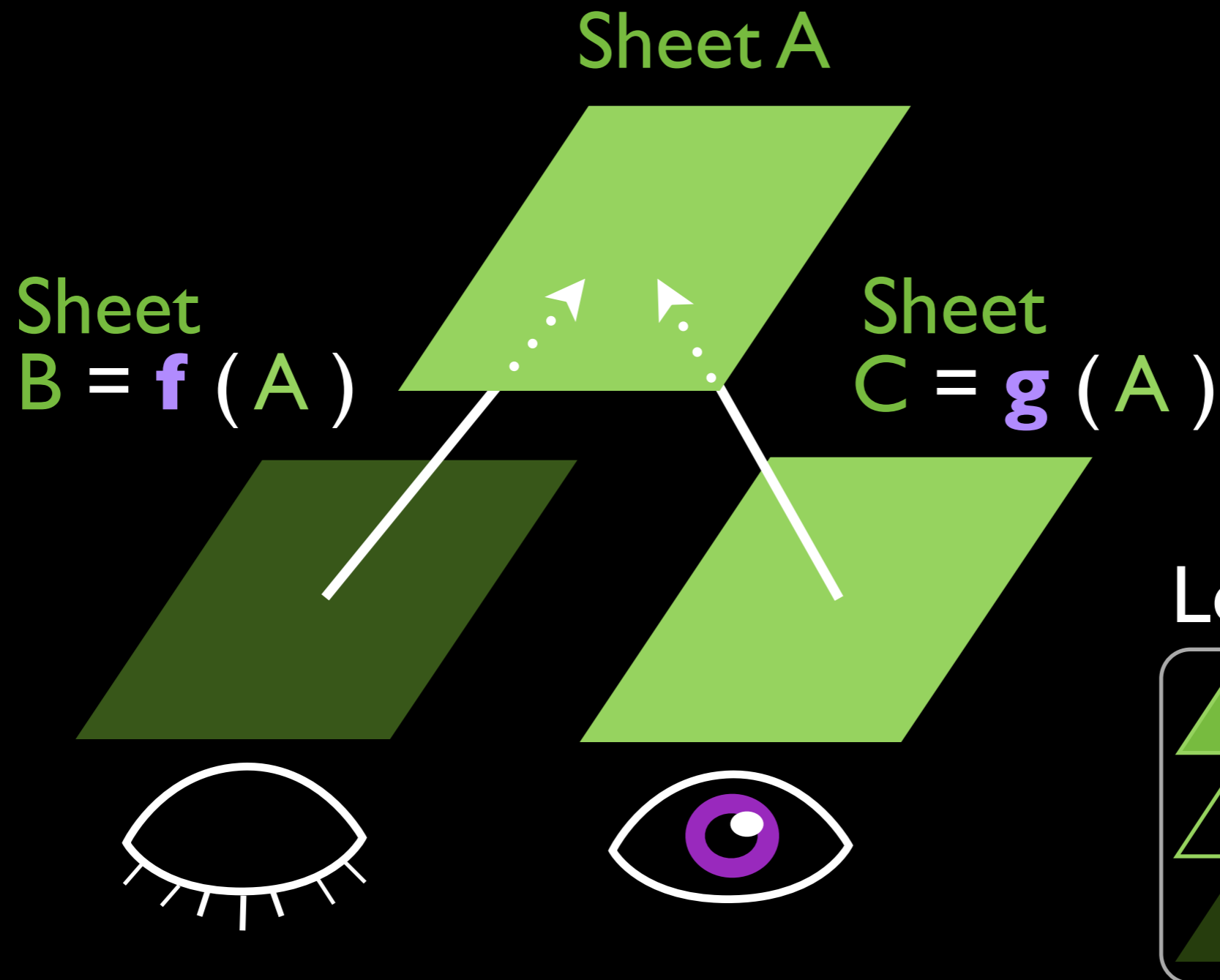
Interaction Pattern: **Sharing**

B and C share work for A



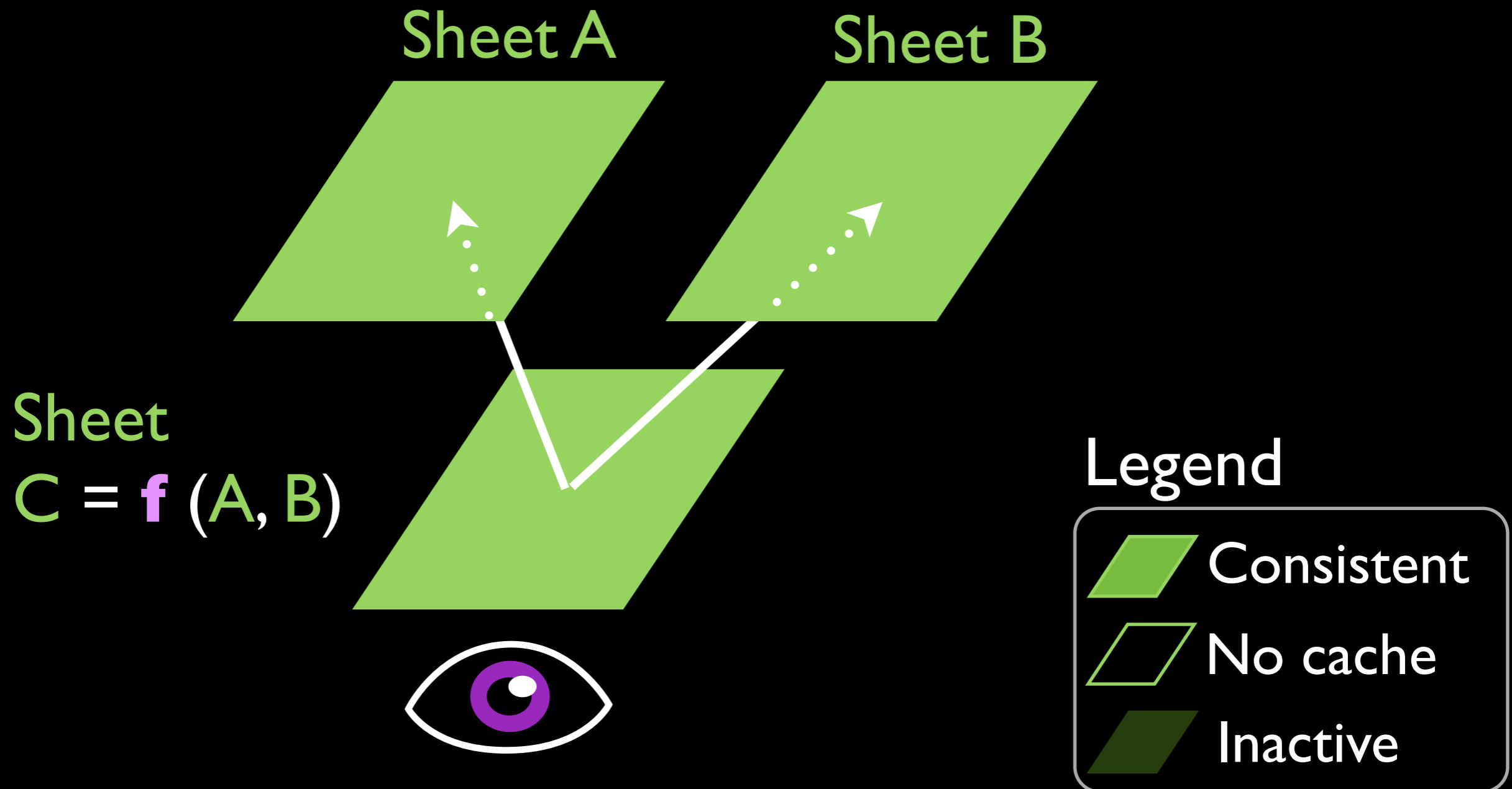
Interaction Pattern: **Sharing**

B and C share work for A



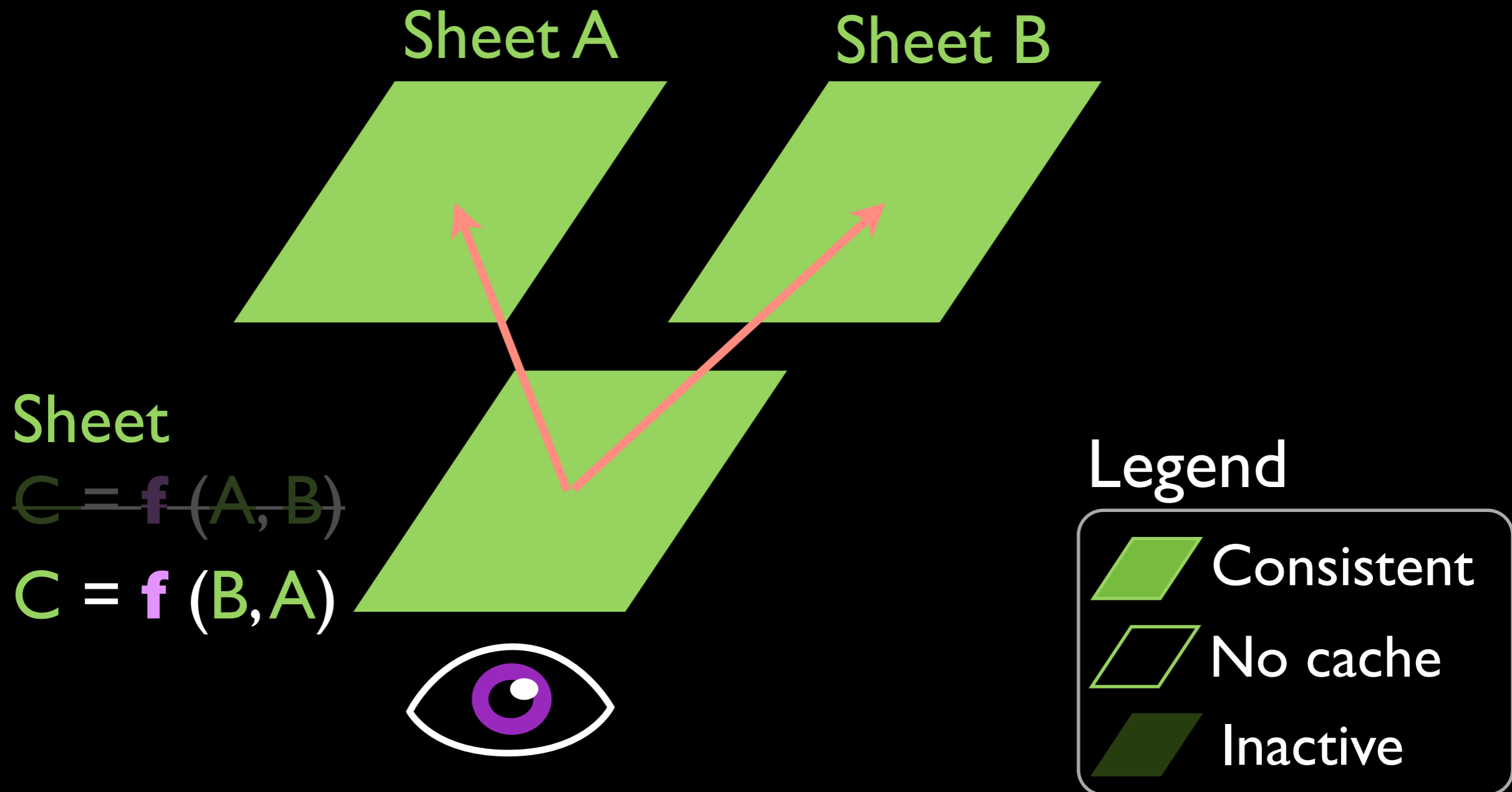
Interactive Pattern: **Swapping**

Swaps input / evaluation order



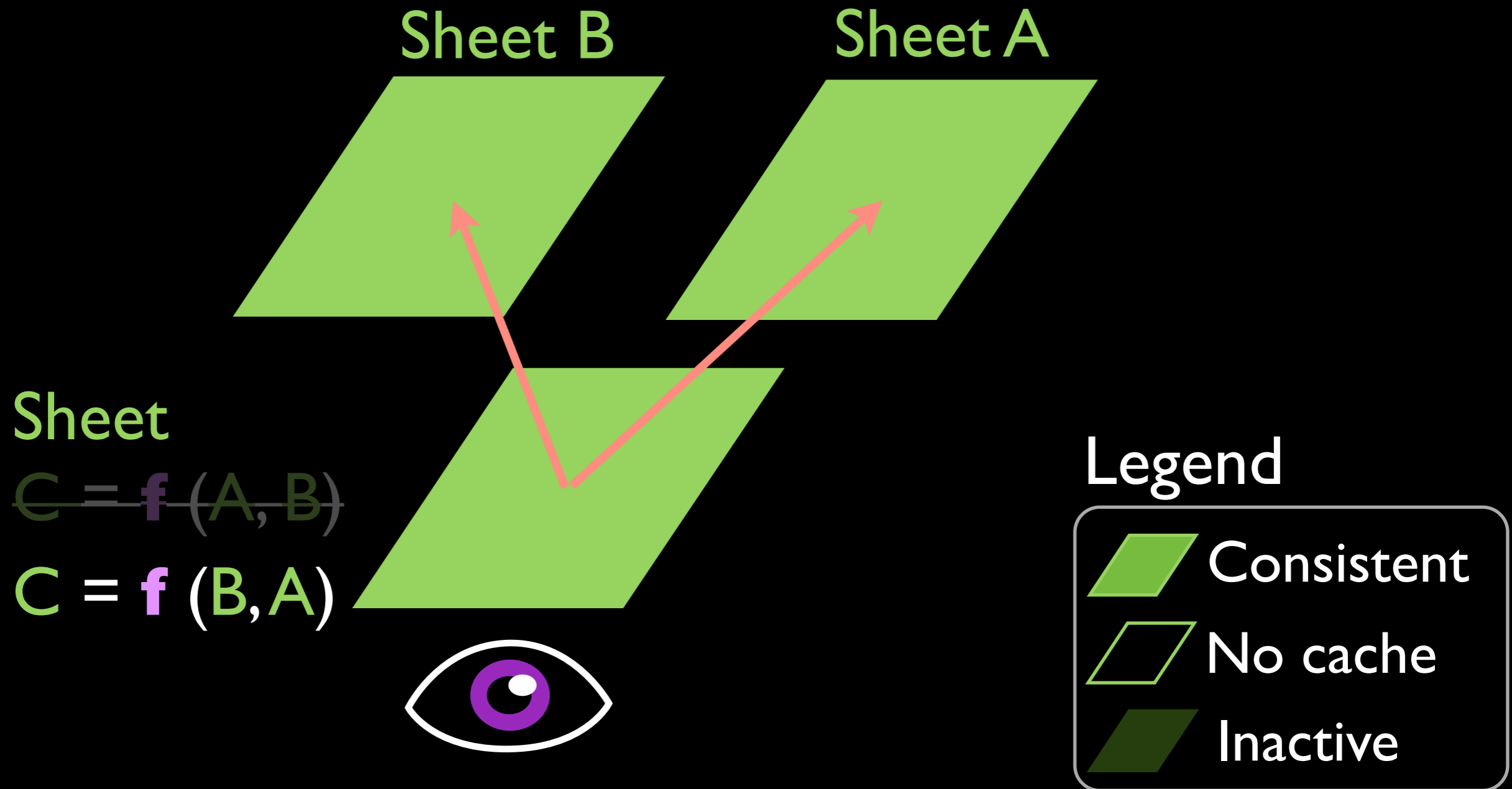
Interactive Pattern: **Swapping**

Swaps input / evaluation order



Interactive Pattern: **Swapping**

Swaps input / evaluation order



Adapton's Approach

- When we **mutate** an **input**, we mark dependent computations as **dirty**
- When we **demand a thunk**:
 - **Memo-match** equivalent thunks
 - **Change-propagation** repairs inconsistencies, **on demand**

Spread Sheet Evaluator

type cell = formula ref

and formula =

| Leaf of int

| Plus of cell * cell

Spread Sheet Evaluator

Mutable

```
type cell = formula ref
```

```
and formula =
```

```
| Leaf of int
```

```
| Plus of cell * cell
```

Depends
on cells

Spread Sheet Evaluator

Example

```
type cell = formula ref
```

```
and formula =
```

```
| Leaf of int
```

```
| Plus of cell * cell
```

```
let n1 = ref (Leaf 1)
```

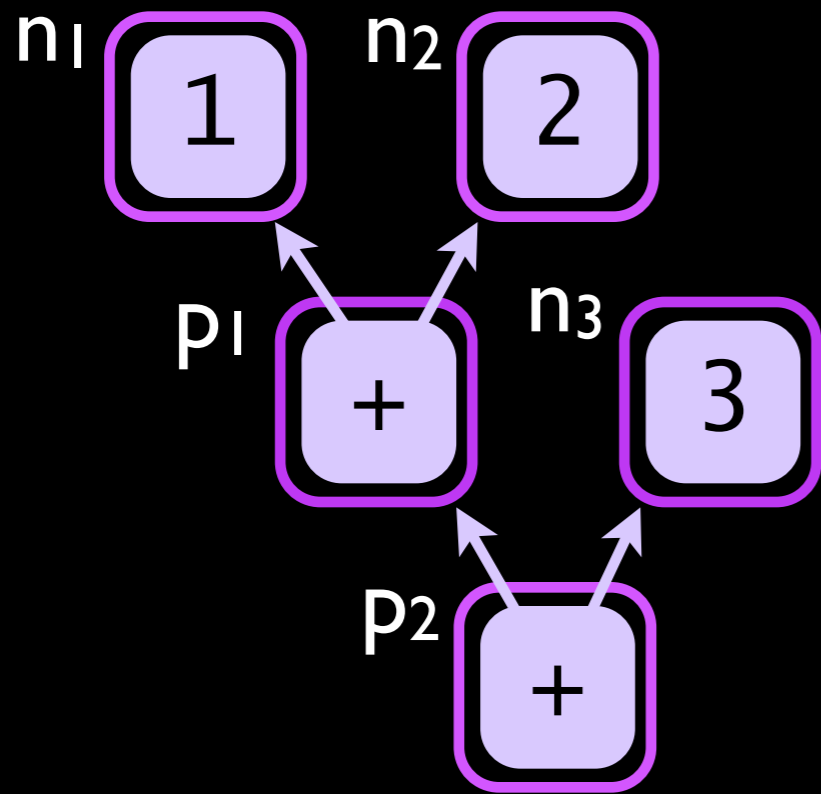
```
let n2 = ref (Leaf 2)
```

```
let n3 = ref (Leaf 3)
```

```
let p1 = ref (Plus (n1, n2))
```

```
let p2 = ref (Plus (p1, n3))
```

Spread Sheet Evaluator



Example

```
let n1 = ref (Leaf 1)
```

```
let n2 = ref (Leaf 2)
```

```
let n3 = ref (Leaf 3)
```

```
let p1 = ref (Plus (n1, n2))
```

```
let p2 = ref (Plus (p1, n3))
```

type cell = formula ref

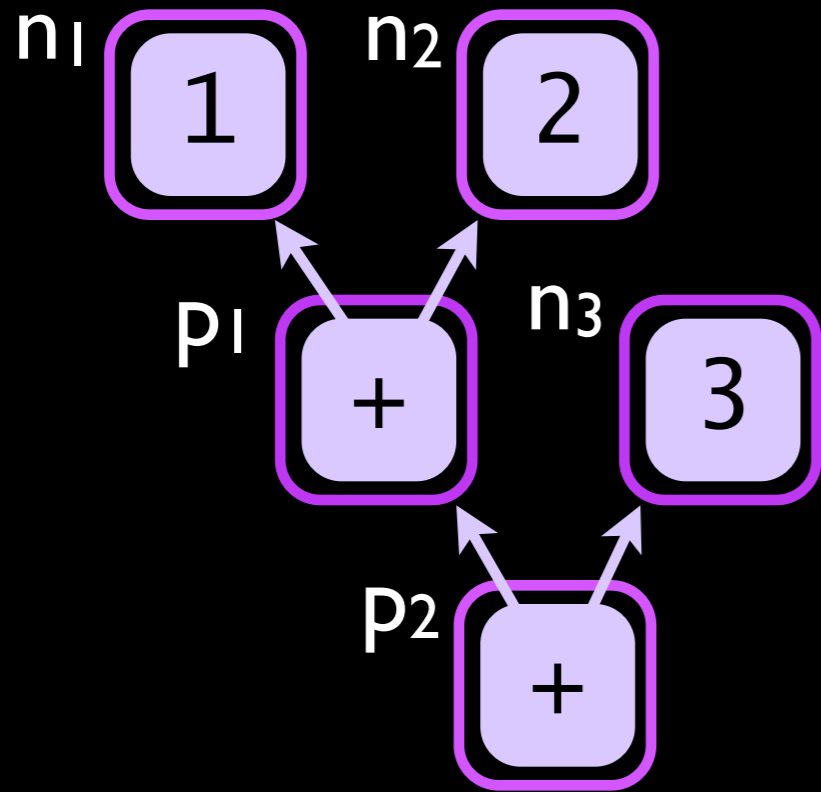
and formula =

| Leaf of int

| Plus of cell * cell

“User interface” (REPL)

Spread Sheet Evaluator

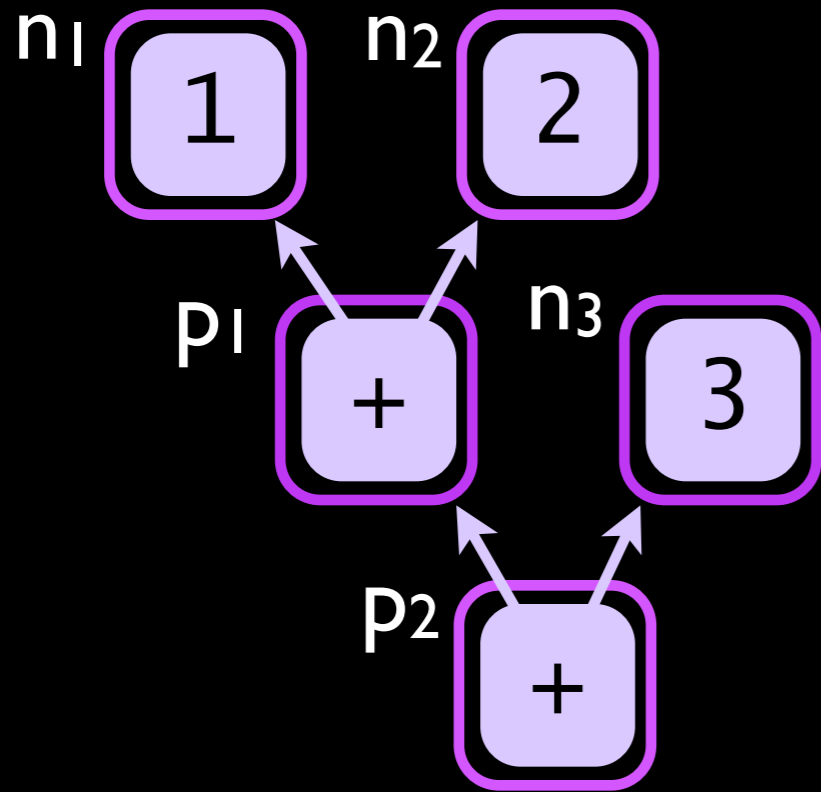


```
type cell = formula ref
and formula =
  | Leaf of int
  | Plus of cell * cell
```

Evaluator logic

```
eval : cell → (int thunk)
eval c = thunk ((
  case (get c) of
  | Leaf n ⇒ n
  | Plus(c1, c2) ⇒
    force (eval c1) +
    force (eval c2)
  ))
```

Spread Sheet Evaluator



type cell = formula ref

and formula =

| Leaf of int

| Plus of cell * cell

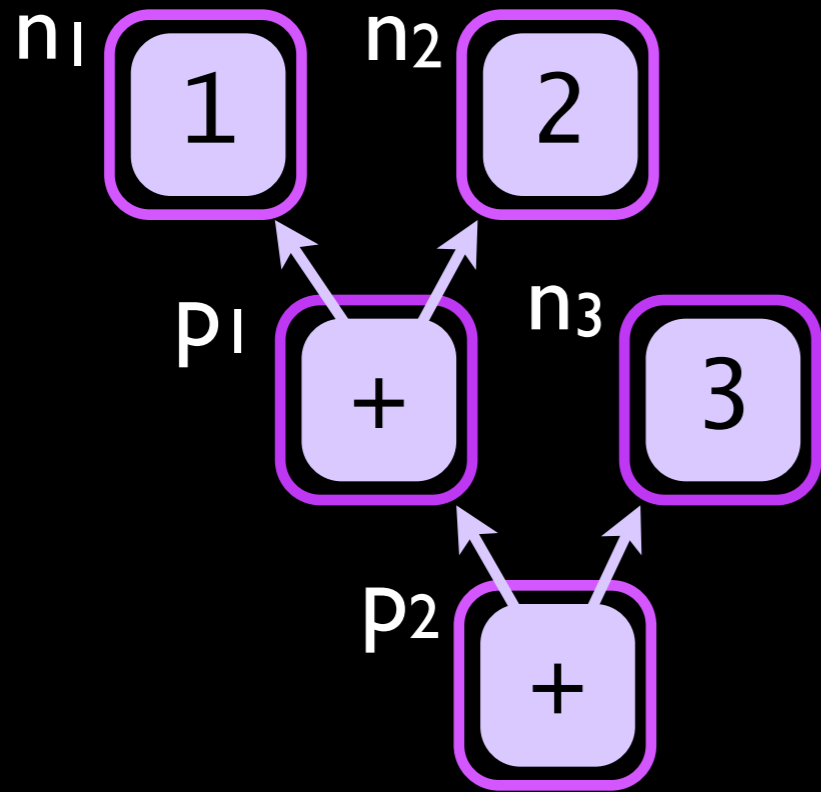
set : cell x formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

Spread Sheet Evaluator



set : cell x formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User Interface” (REPL)

Demands
evaluation

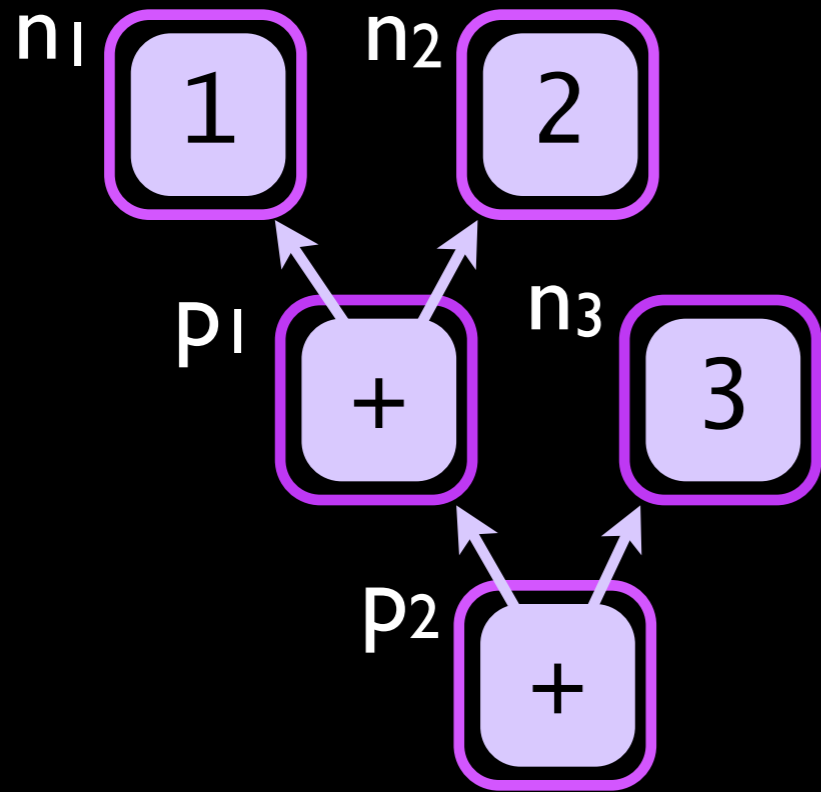
type cell = formula ref

and formula =

| Leaf of int

| Plus of cell * cell

Spread Sheet Evaluator



type cell = formula ref

and formula =

| Leaf of int

| Plus of cell * cell

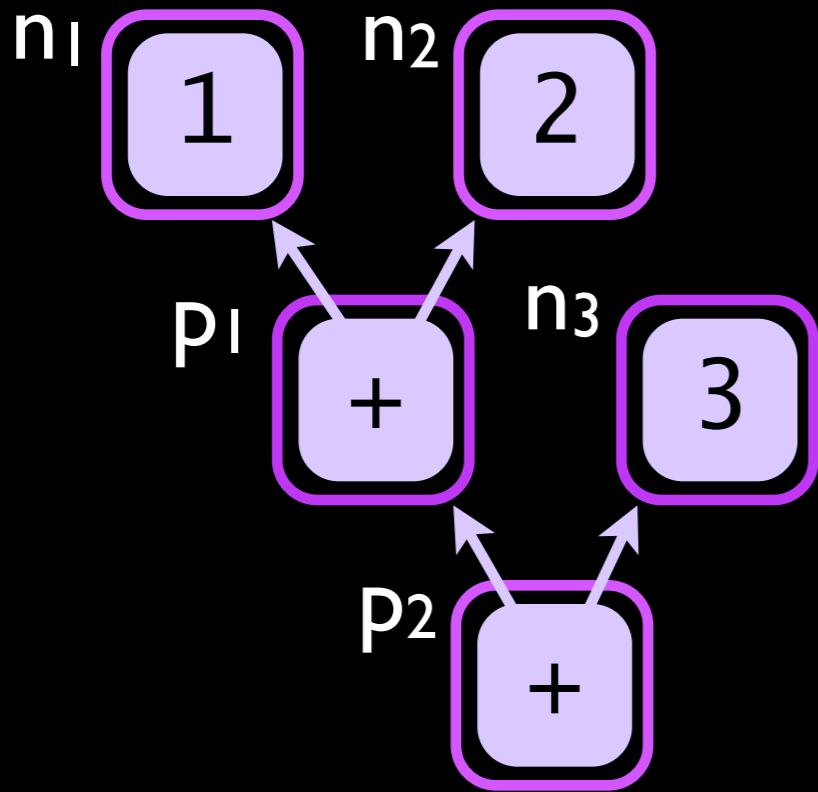
set : cell x formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit


“User interface” (REPL)

type cell = formula ref

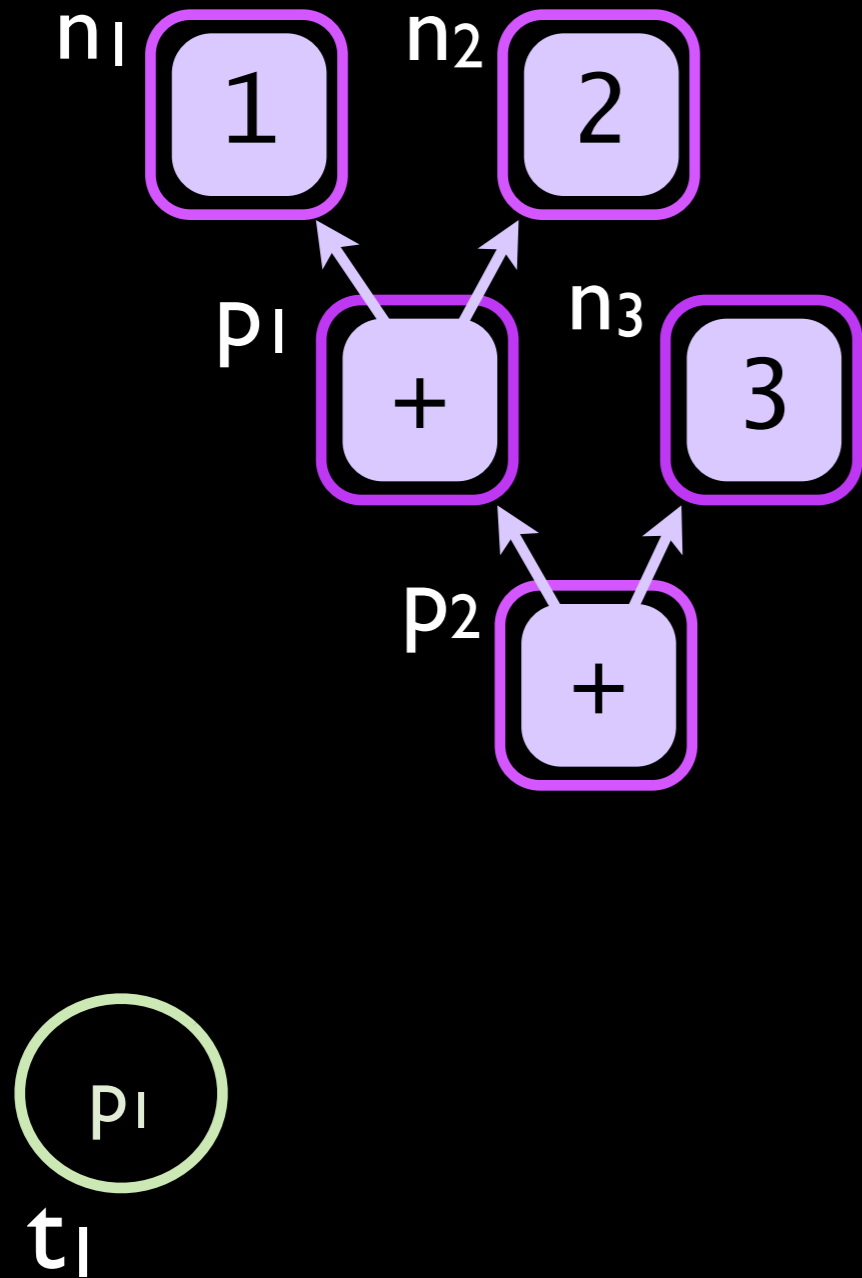
and formula =

| Leaf of int

| Plus of cell * cell

 let t₁ = eval p₁

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

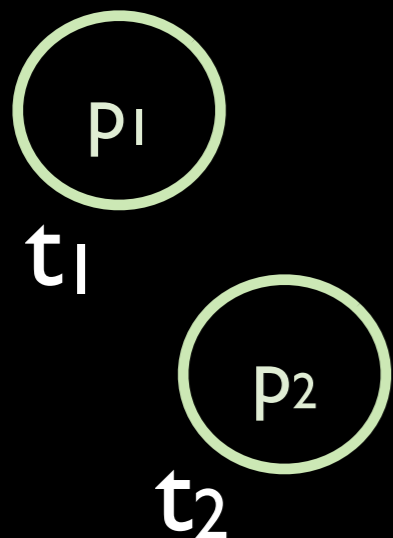
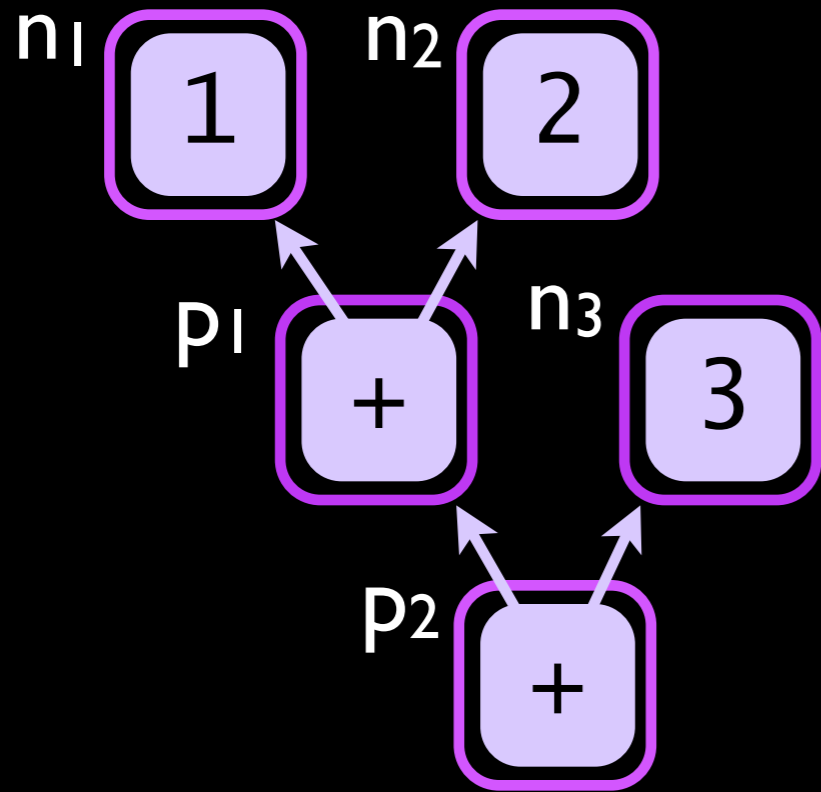
display : (int thunk) \rightarrow unit

“User interface” (REPL)

 let $t_1 = \text{eval } p_1$



Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

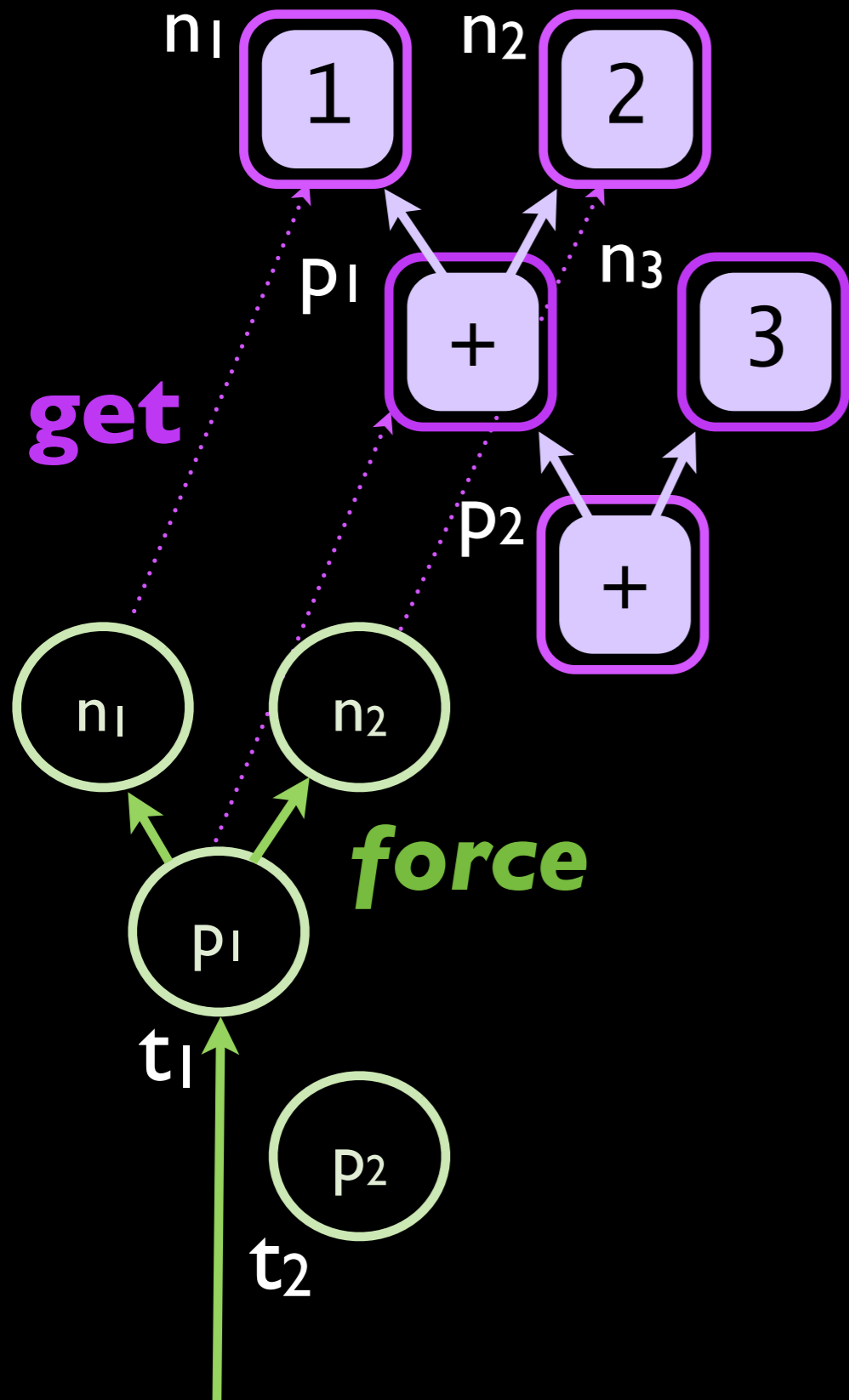
“User interface” (REPL)

👉 let $t_1 = \text{eval } p_1$

👉 let $t_2 = \text{eval } p_2$

👉

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

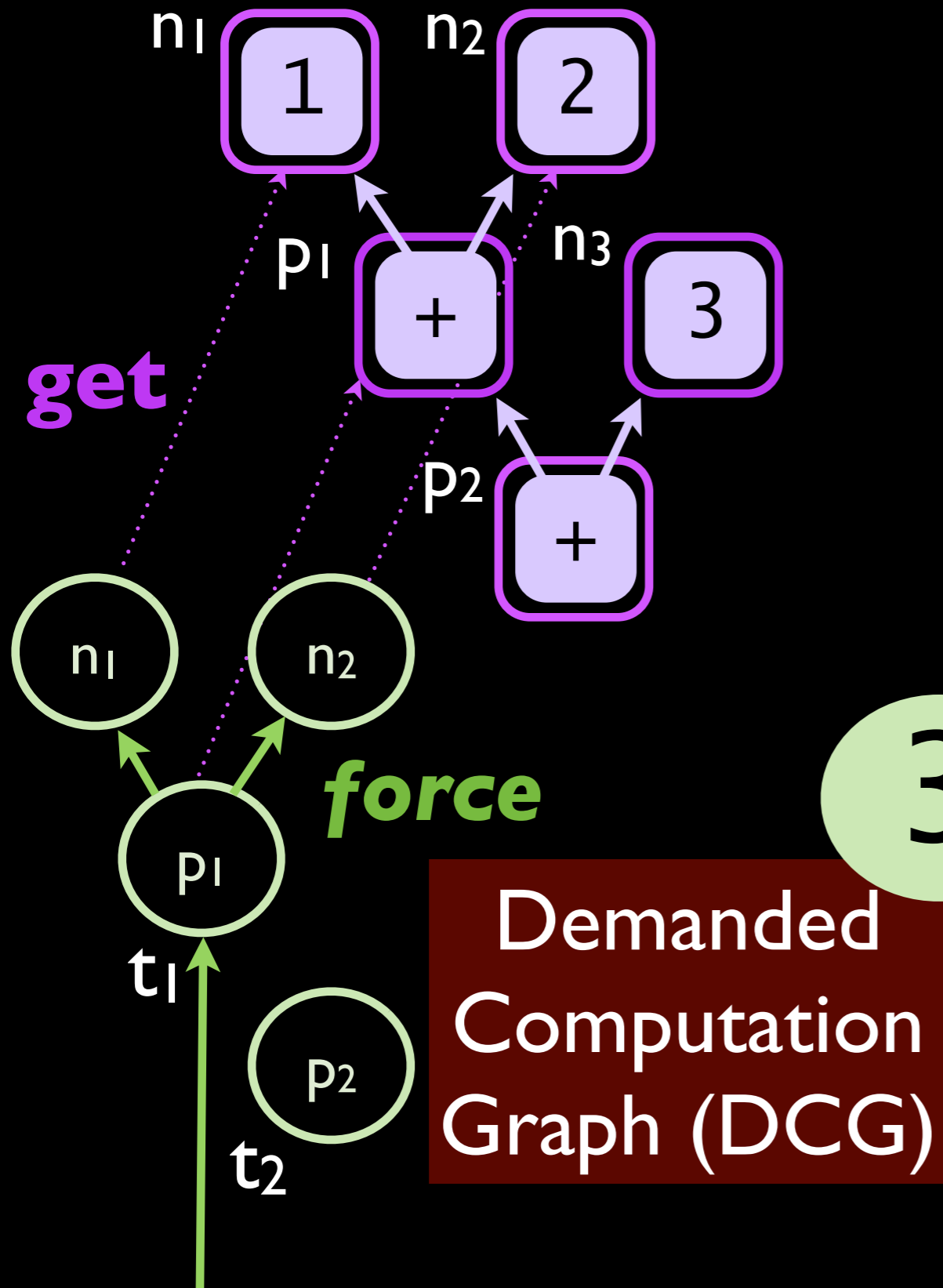
let $t_1 = \text{eval } p_1$

let $t_2 = \text{eval } p_2$

display t_1

demand!

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

let $t_1 = \text{eval } p_1$

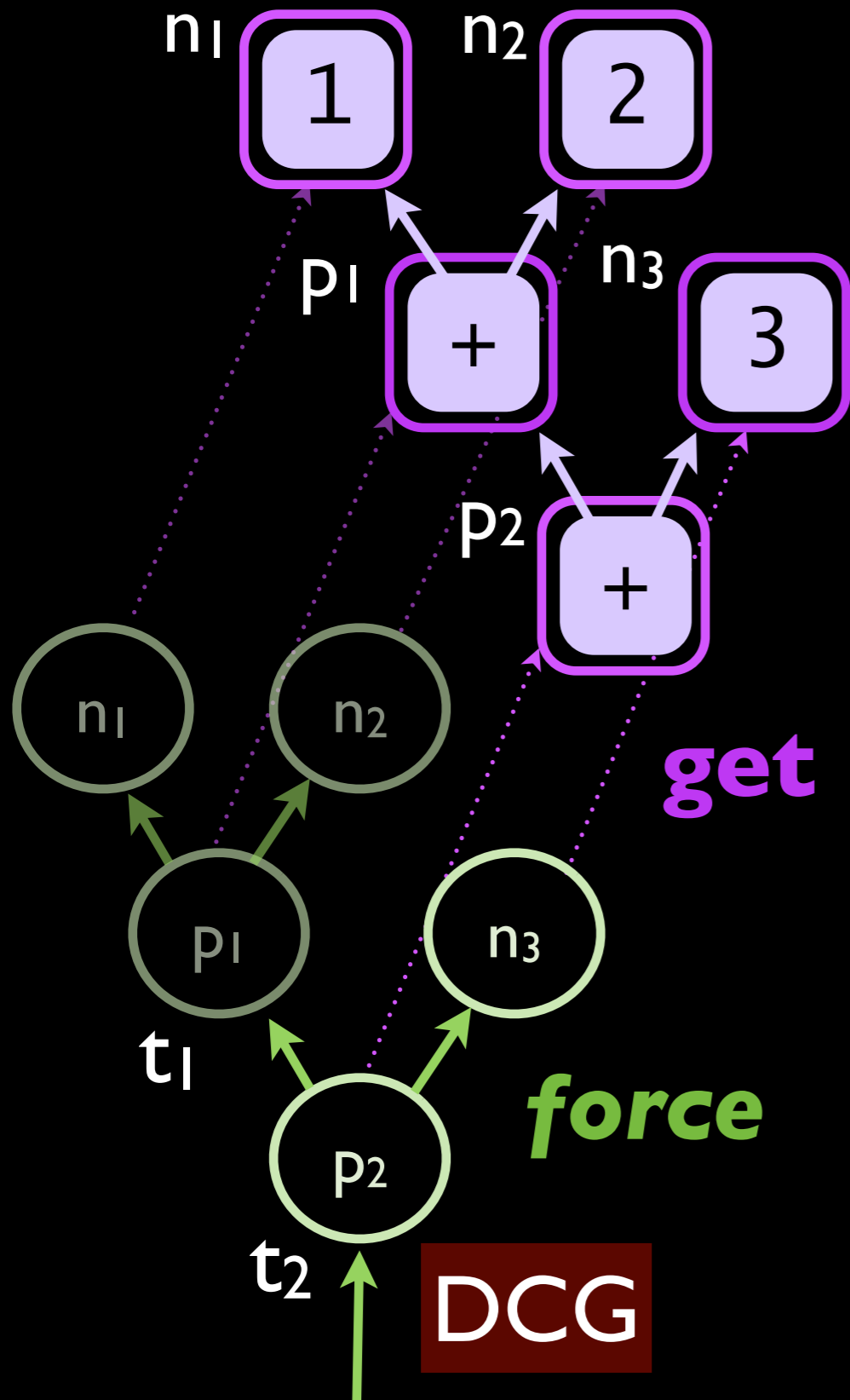
let $t_2 = \text{eval } p_2$

display t_1



Demanded
Computation
Graph (DCG)

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

let $t_1 = \text{eval } p_1$

let $t_2 = \text{eval } p_2$

display t_1

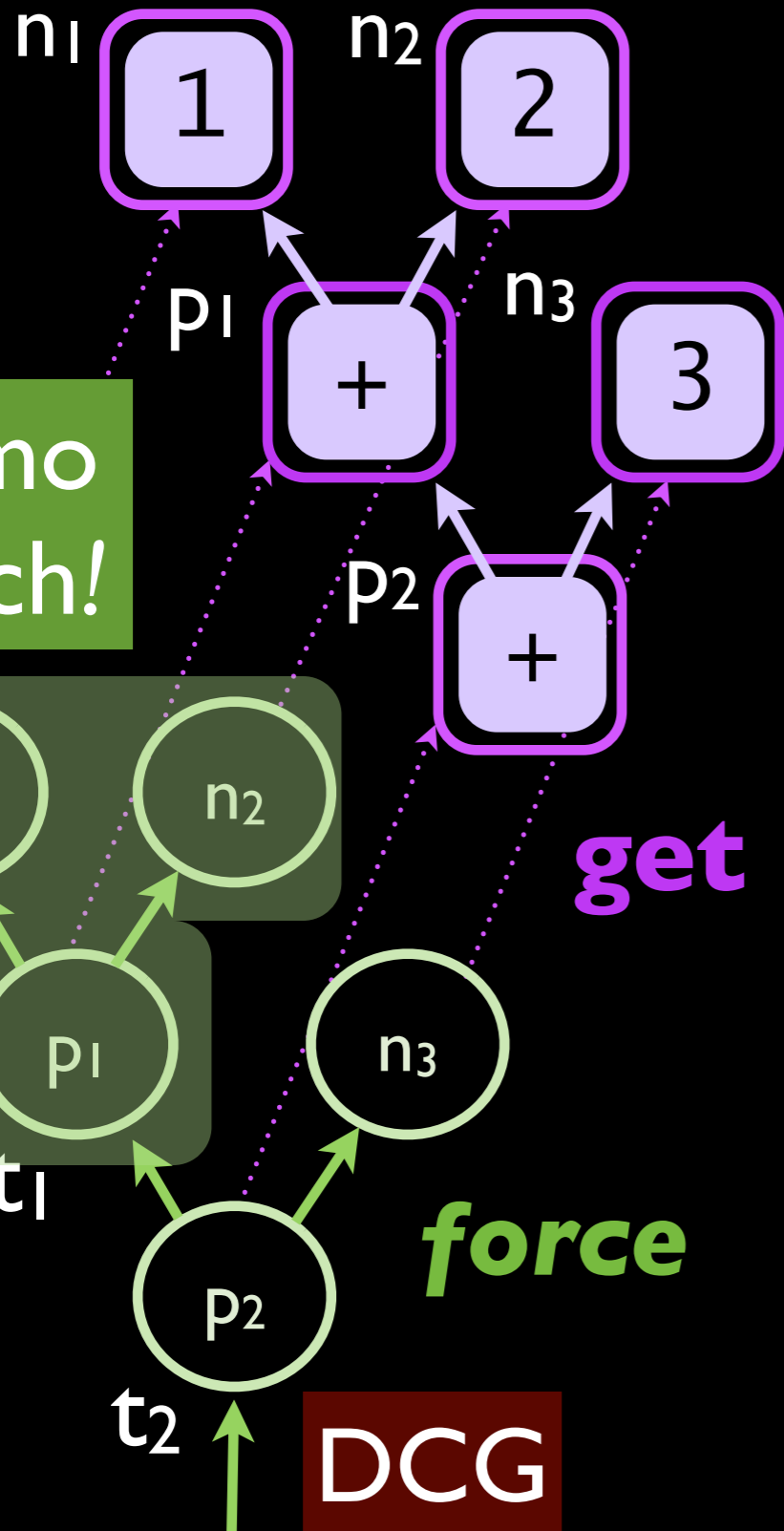
display t_2



6

demand!

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

let $t_1 = \text{eval } p_1$

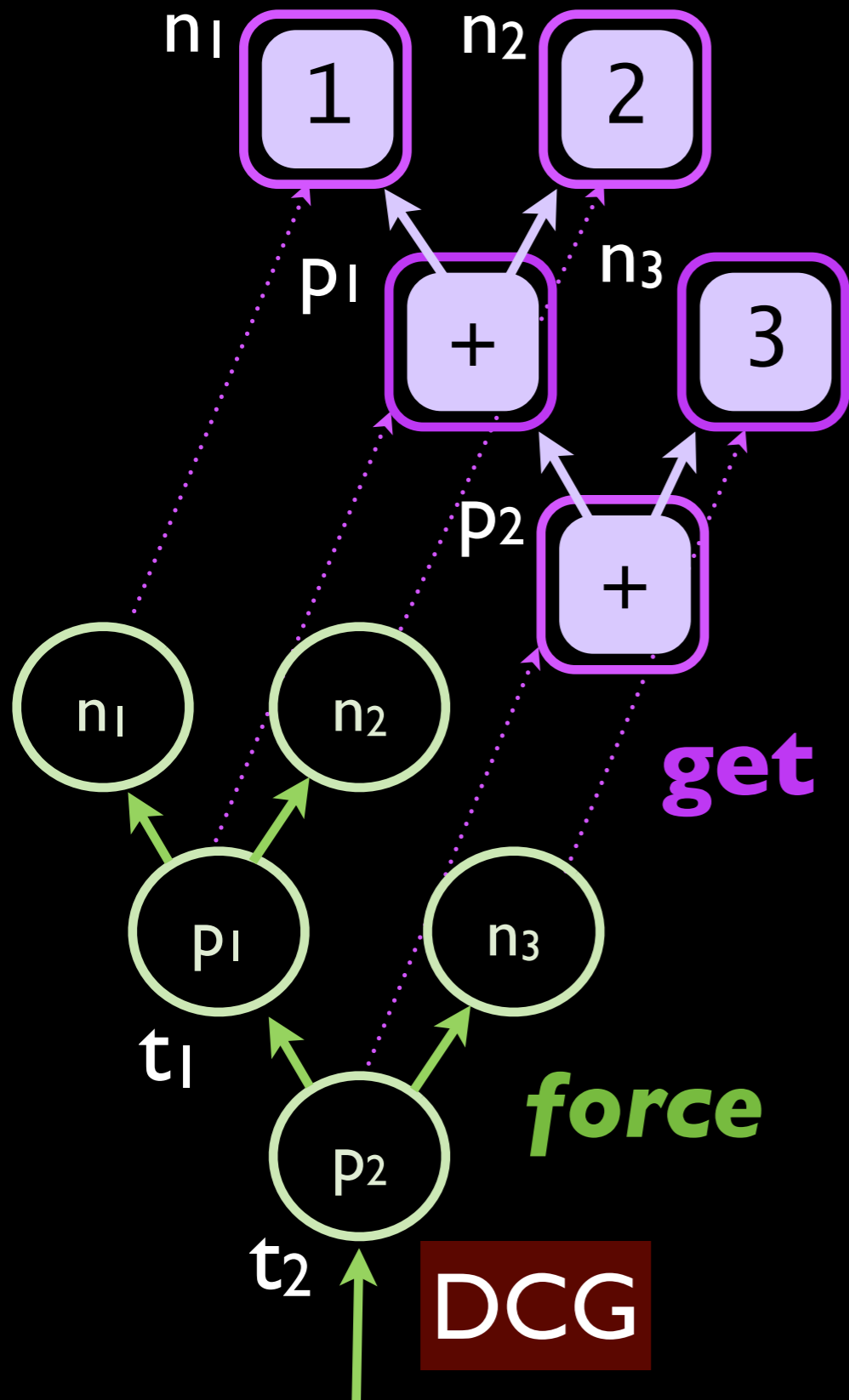
let $t_2 = \text{eval } p_2$

display t_1

display t_2

Memo match!
Sharing

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

let $t_1 = \text{eval } p_1$

let $t_2 = \text{eval } p_2$

display t_1

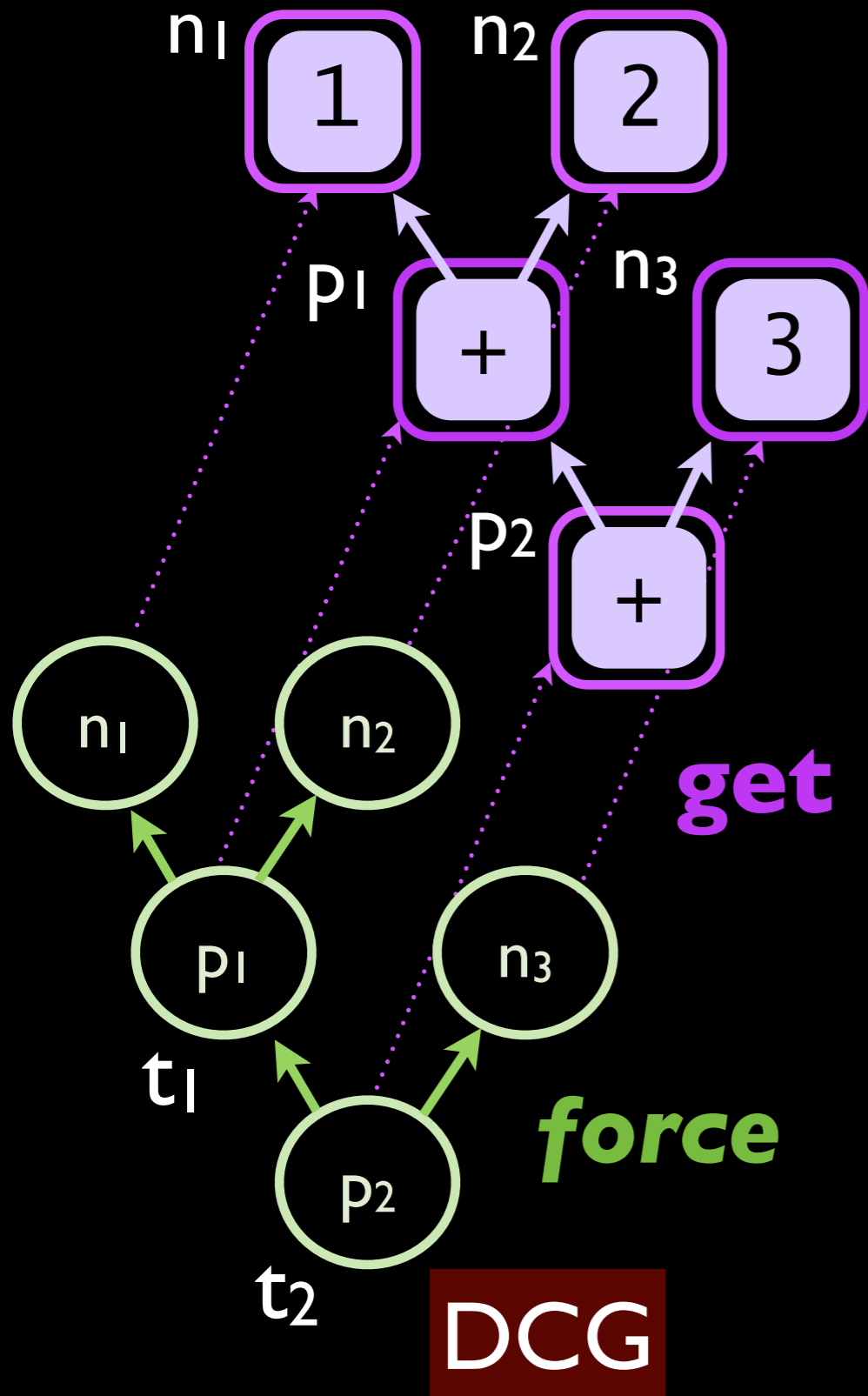
display t_2

clear

6

DCG

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

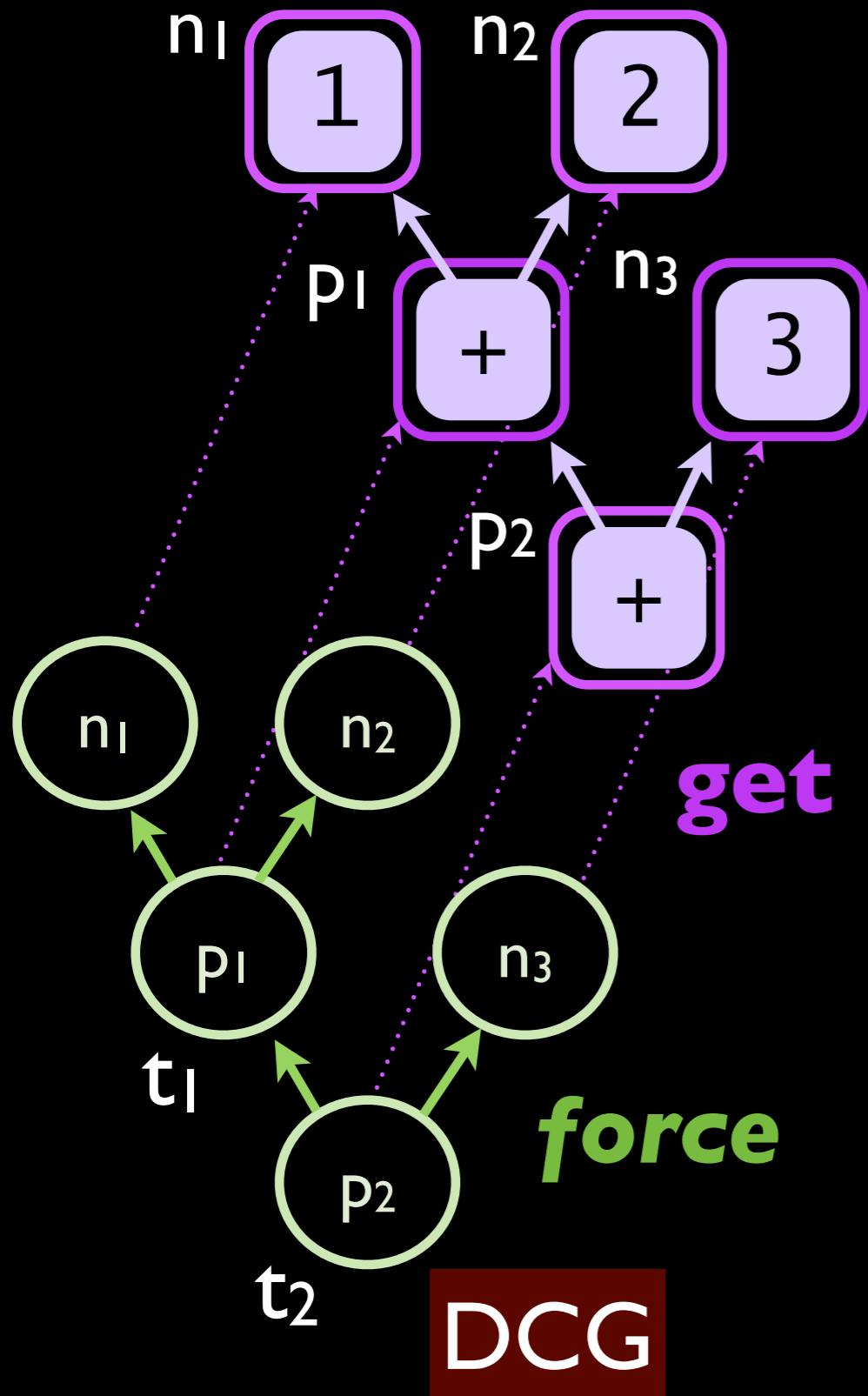
display : (int thunk) \rightarrow unit

“User interface” (REPL)



DCG

Spread Sheet Evaluator



$\text{set} : \text{cell} \times \text{formula} \rightarrow \text{unit}$

$\text{eval} : \text{cell} \rightarrow (\text{int} \text{ thunk})$

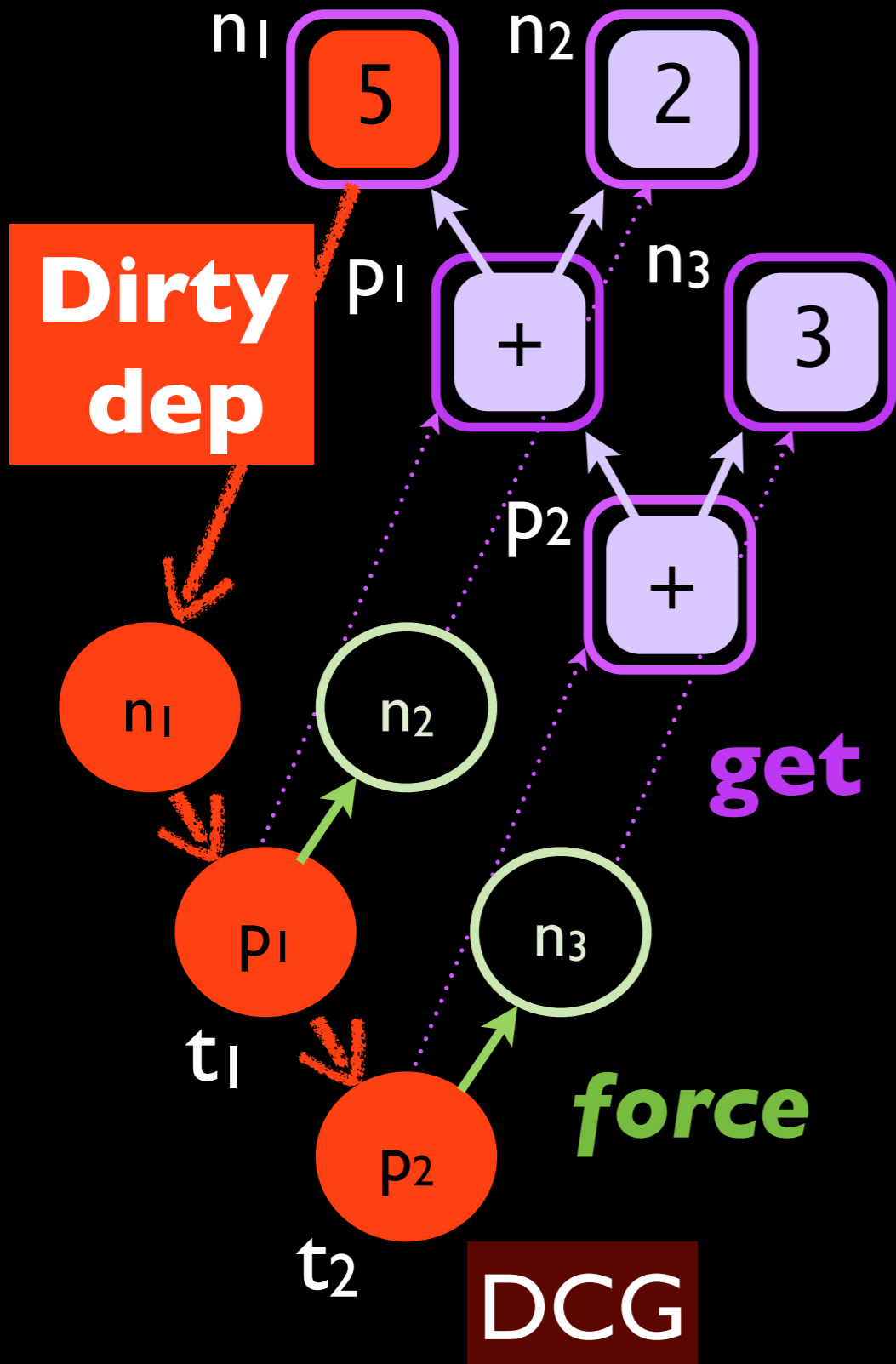
$\text{display} : (\text{int} \text{ thunk}) \rightarrow \text{unit}$

“User interface” (REPL)

 $\text{set } n_1 \leftarrow \text{Leaf } 5$

DCG

Spread Sheet Evaluator



set : cell x formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

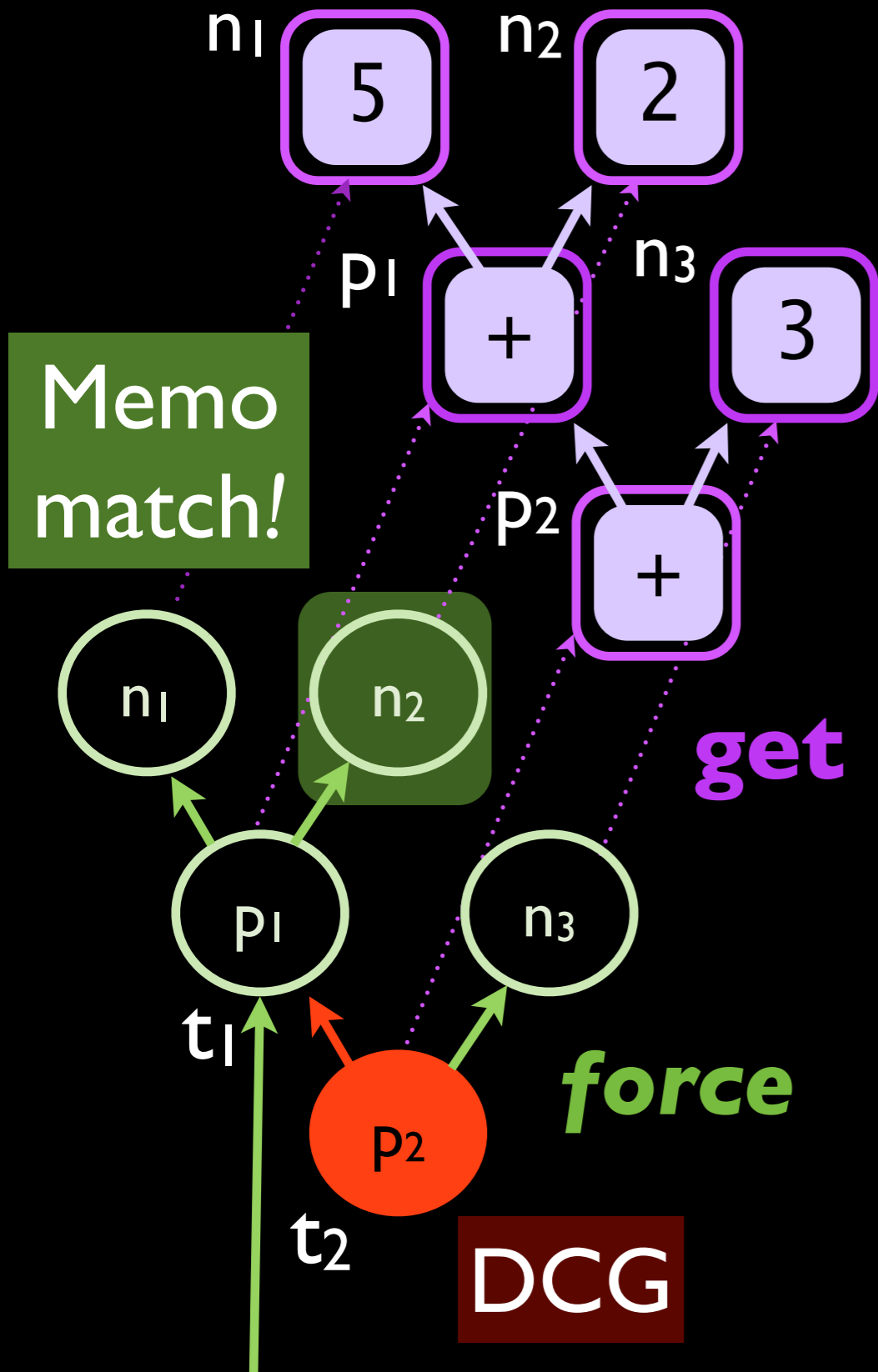
“User interface” (REPL)

👉 set n₁ ← Leaf 5



**Dirty
phase**

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

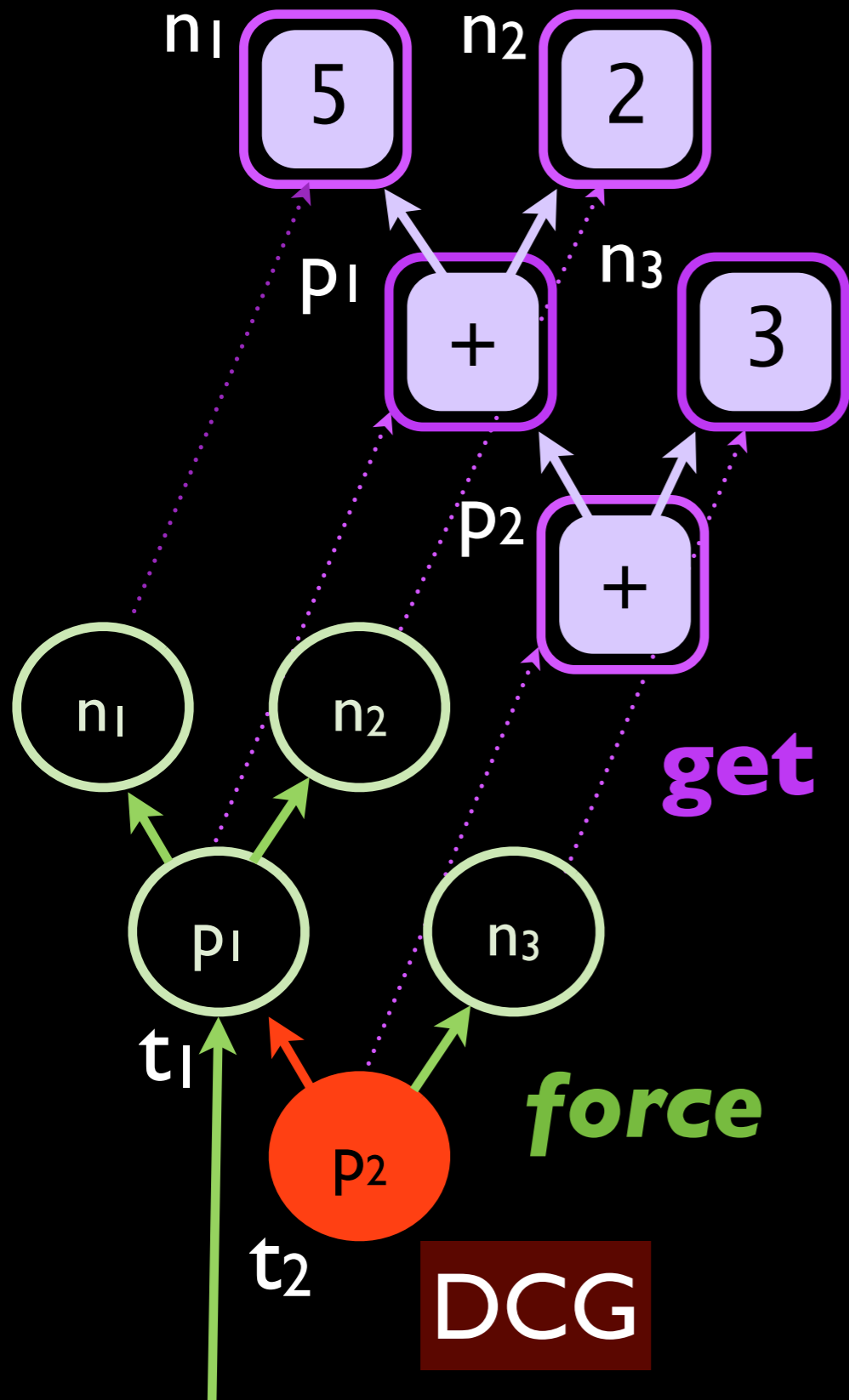
“User interface” (REPL)

7

 set n_1 \leftarrow Leaf 5

 display t_1

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

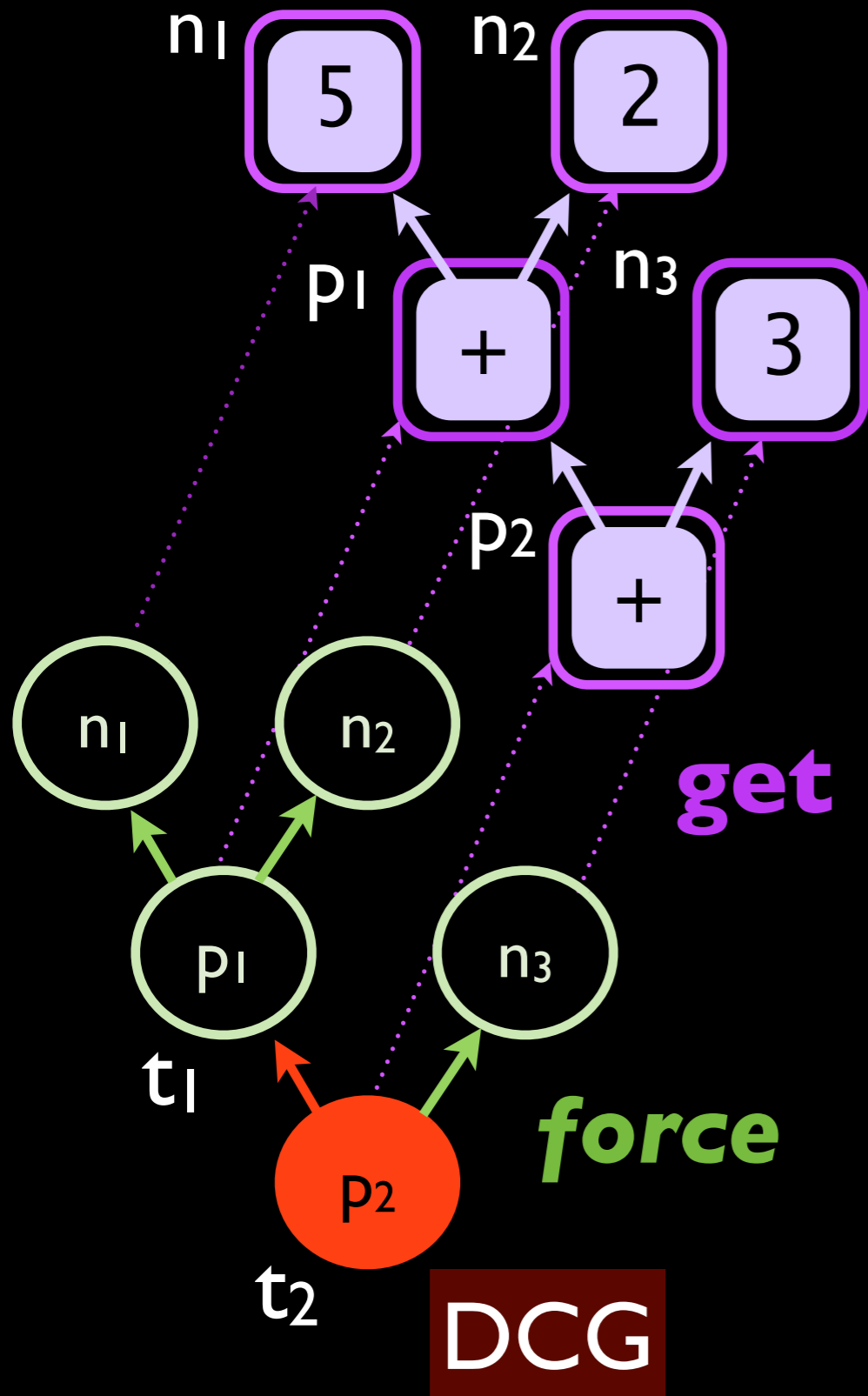
7

set n_1 \leftarrow Leaf 5

display t_1

DCG

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

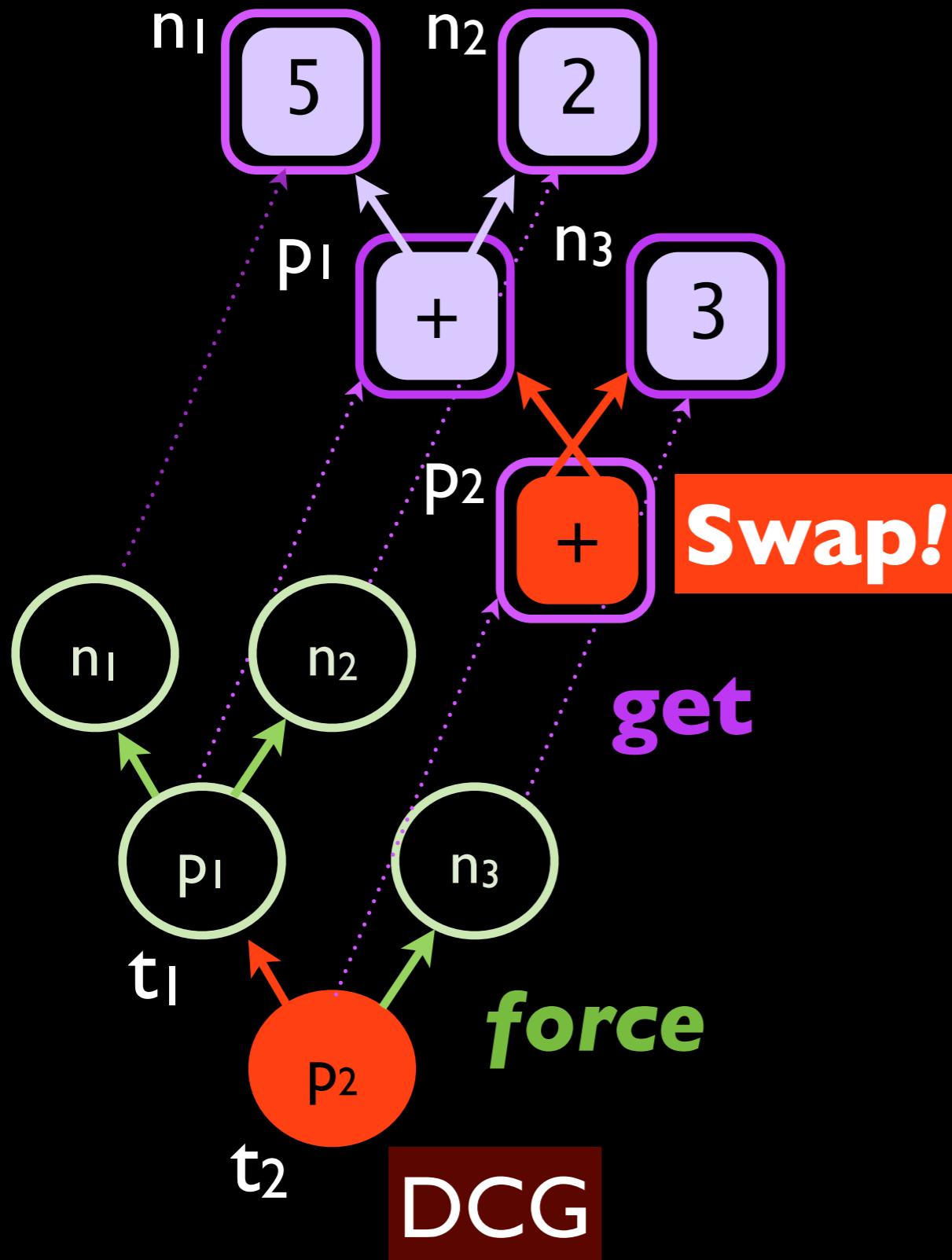
set n_1 \leftarrow Leaf 5

display t_1

set p_2 \leftarrow Plus(n_3, p_1)

DCG

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

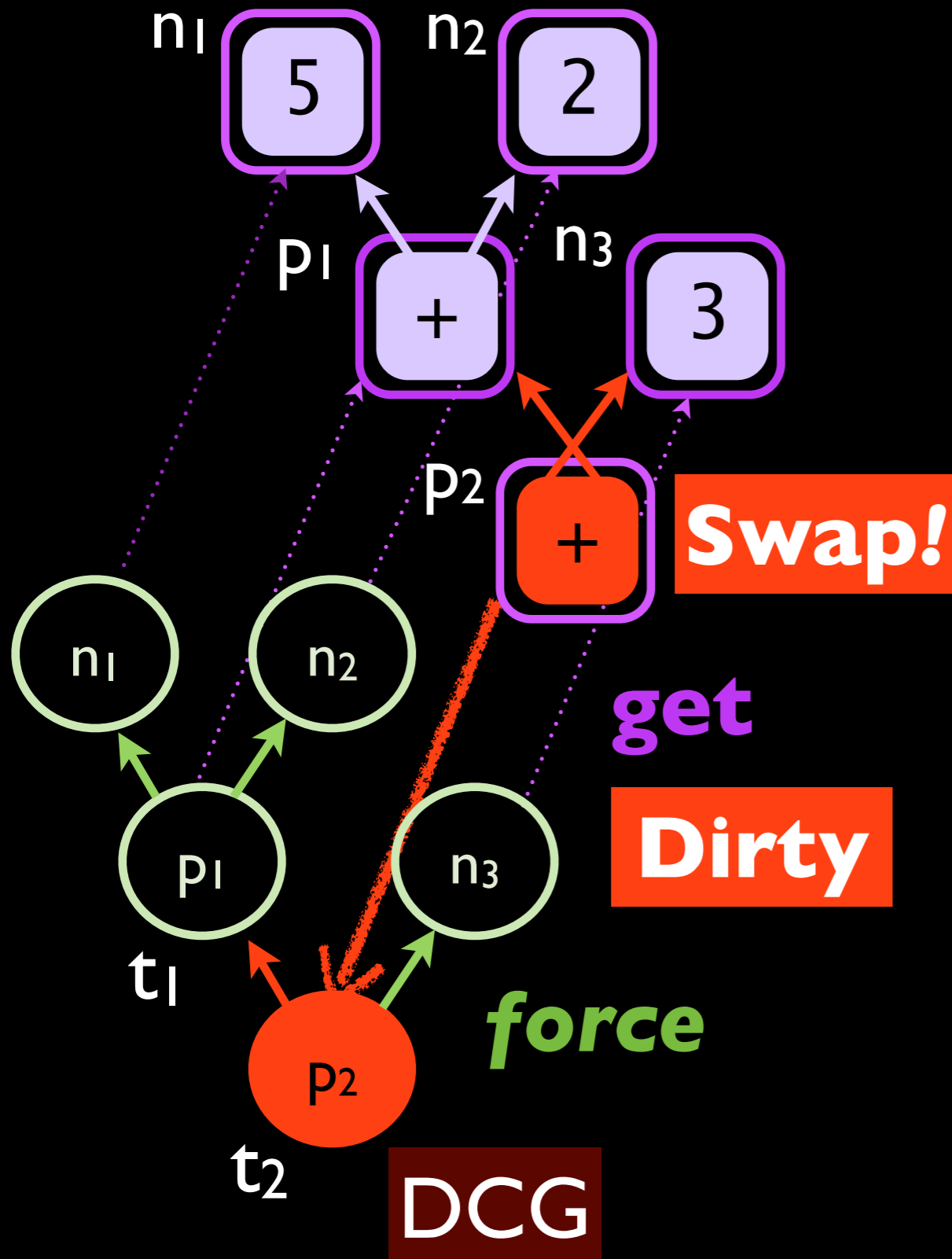
☞ set n_1 \leftarrow Leaf 5

☞ display t_1

☞ set p_2 \leftarrow Plus(n_3 , p_1)

☞

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

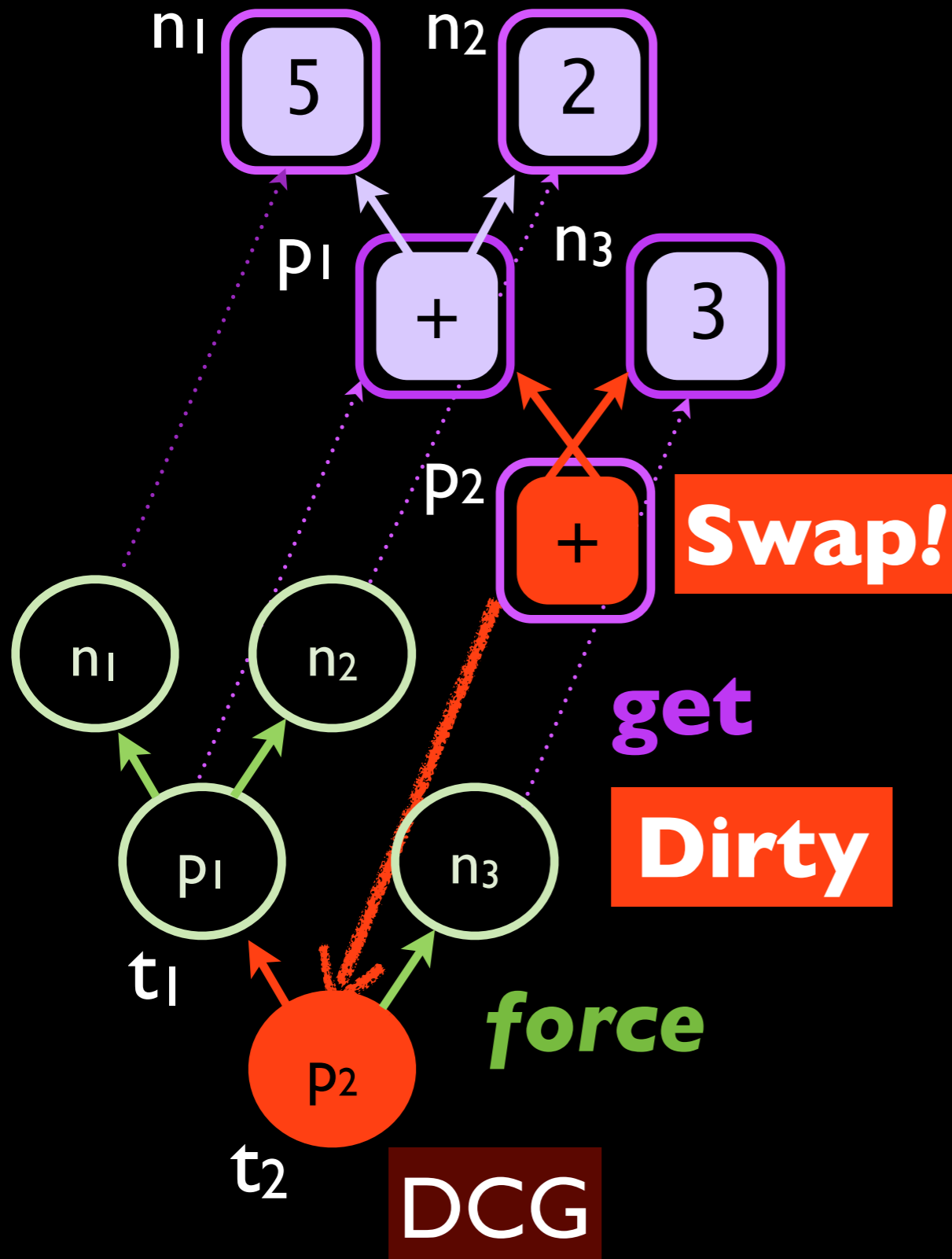
 set n_1 \leftarrow Leaf 5

 display t_1

 set p_2 \leftarrow Plus(n_3 , p_1)



Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

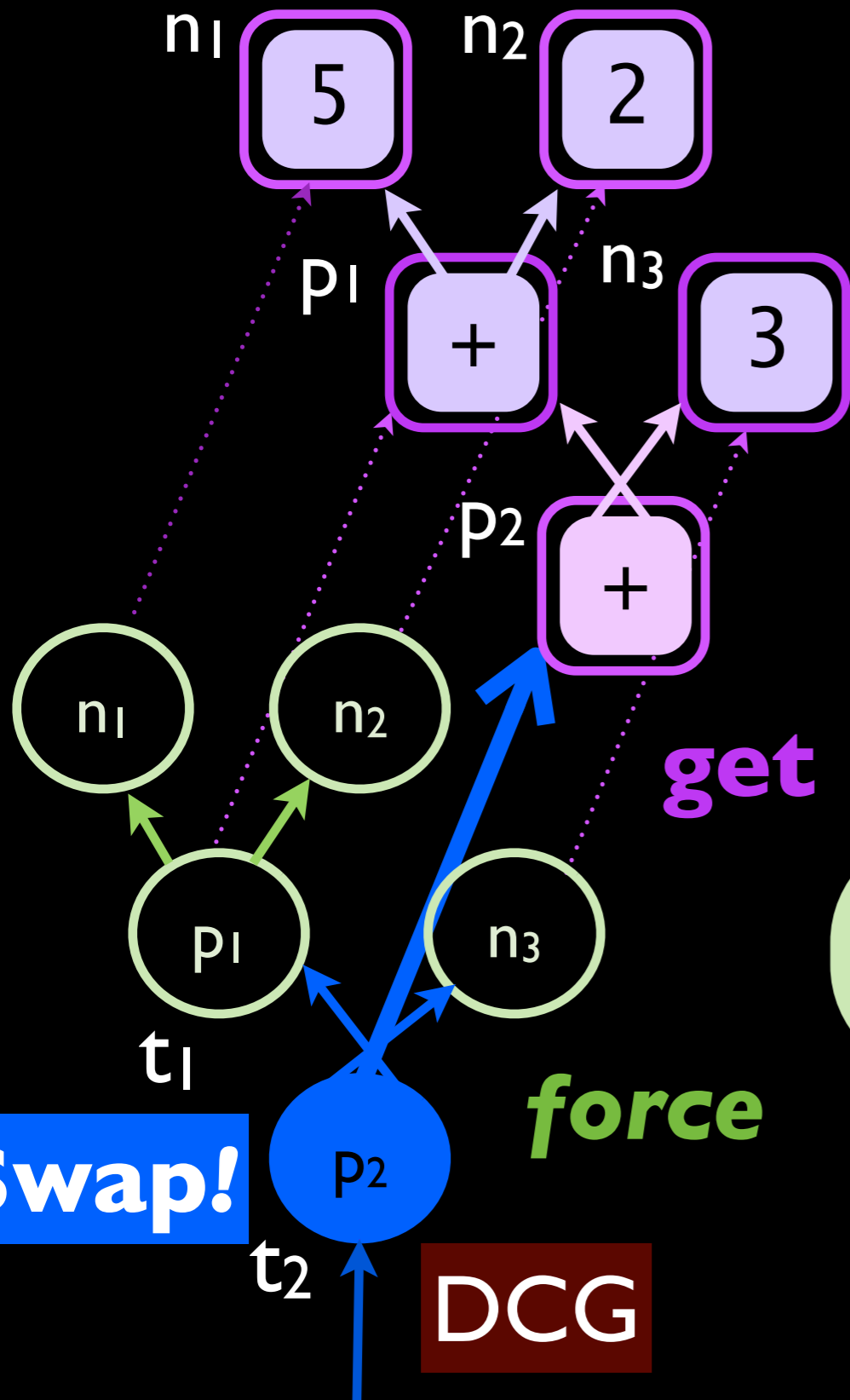
set $n_1 \leftarrow$ Leaf 5

display t_1

set $p_2 \leftarrow$ Plus(n_3, p_1)

display t_2

Spread Sheet Evaluator



set : cell x formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

“User interface” (REPL)

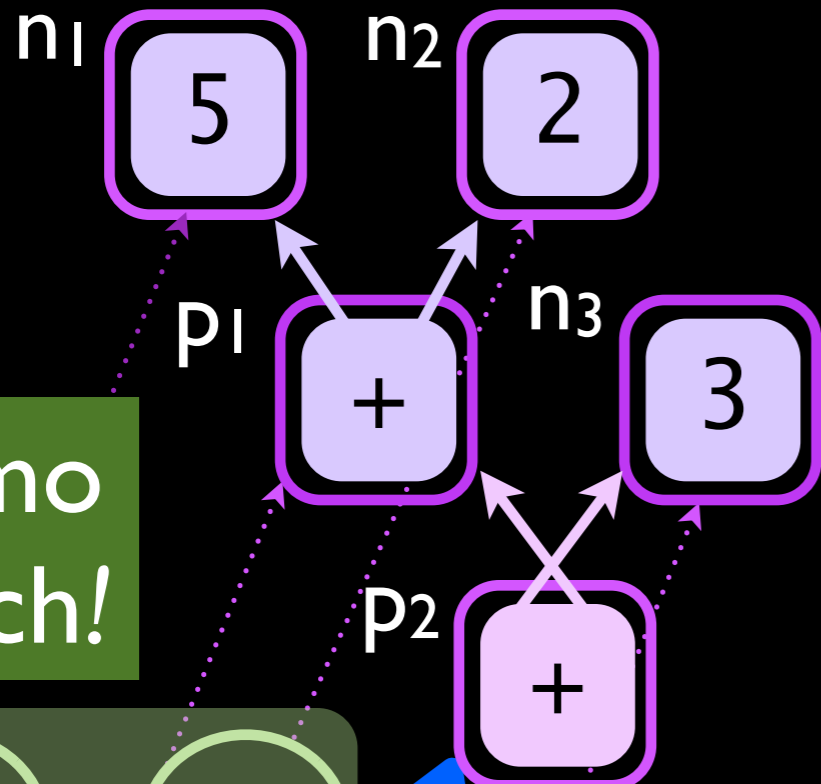
set n_1 \leftarrow Leaf 5

display t_1

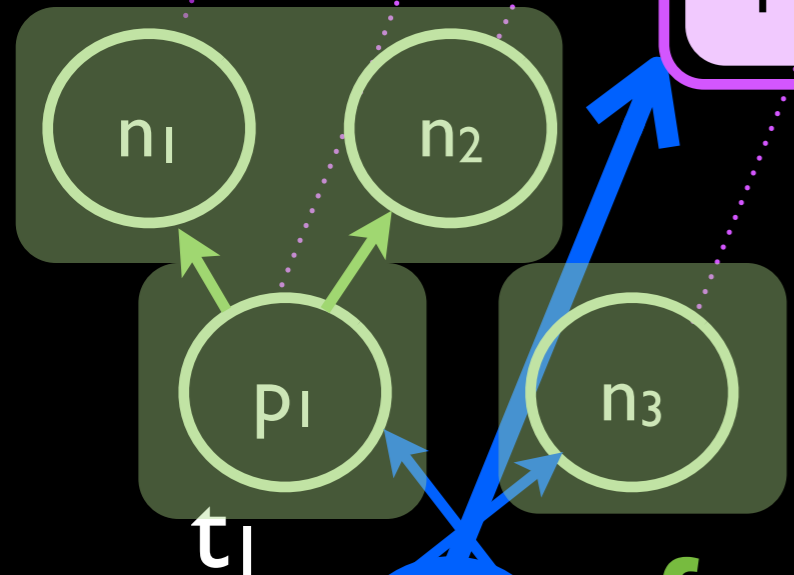
set p_2 \leftarrow Plus(n_3 , p_1)

display t_2

Spread Sheet Evaluator



Memo match!



Swap!

DCG

10

set : cell x formula → unit
eval : cell → (int thunk)
display : (int thunk) → unit

“User interface” (REPL)

- 👉 set n₁ ← Leaf 5
- 👉 display t₁
- 👉 set p₂ ← Plus(n₃, p₁)
- 👉 display t₂
- 👉

Lazy Structures

Laziness generalizes *beyond scalars*

Recursive structures: **lists, trees and graphs**

```
type 'a lzlist =
```

```
| Nil
```

```
| Cons of 'a * ('a lzlist) thunk
```

Recursive
lazy structure

Merging Lazy Lists

As in conventional lazy programming

```
let rec merge l1 l2 = function
```

```
| l1, Nil ⇒ l1
```

```
| Nil, l2 ⇒ l2
```

```
| Cons(h1,t1), Cons(h2,t2) ⇒
```

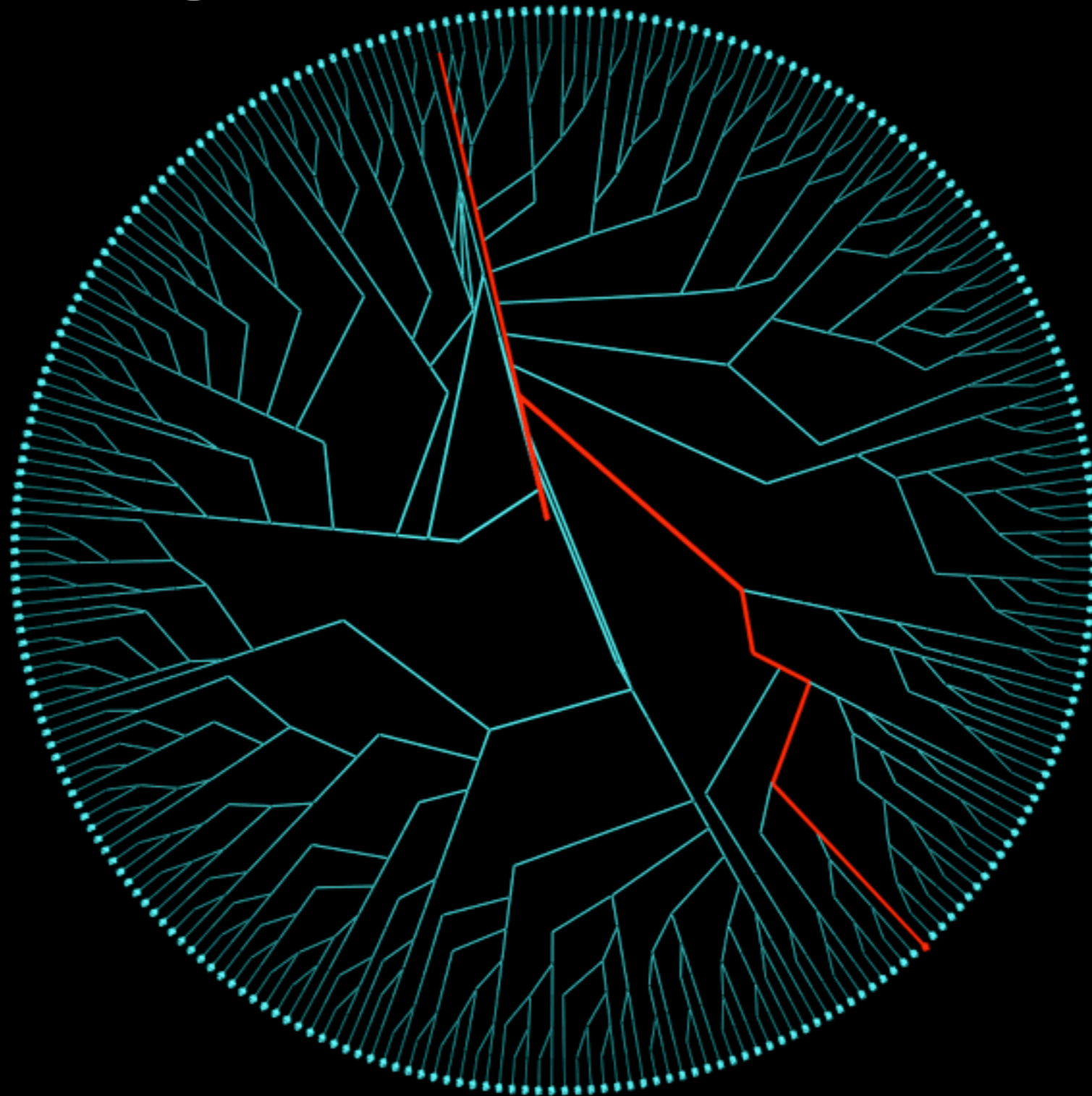
```
  if h1 <= h2 then
```

```
    Cons(h1, think(merge (force t1) l2))
```

```
  else
```

```
    Cons(h2, think(merge l1 (force t2)))
```

Mergesort **DCG** Viz.



Micro Benchmarks

List and tree applications:

filter, map

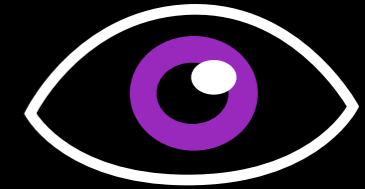
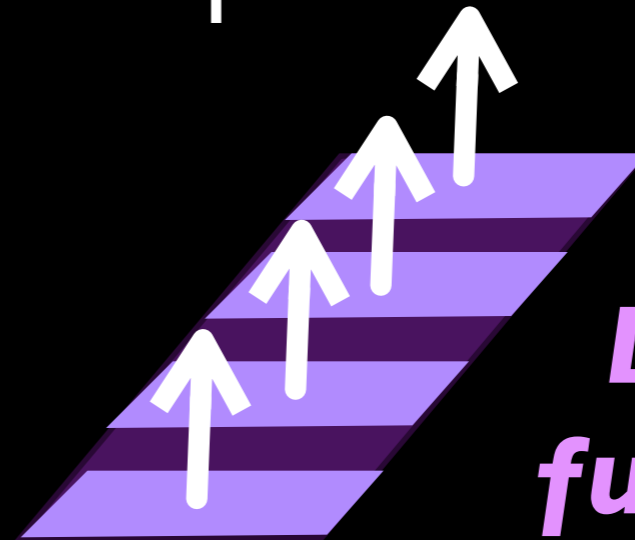
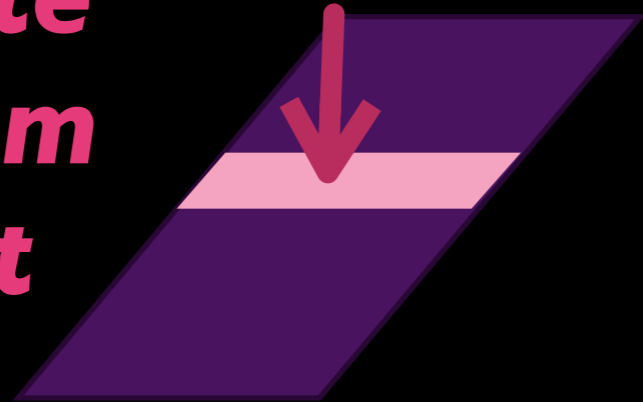
fold{min,sum}

quicksort, mergesort

expression tree evaluation

Batch Pattern: Experimental procedure:

*Mutate
random
input*

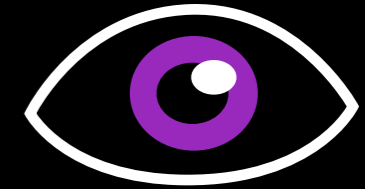
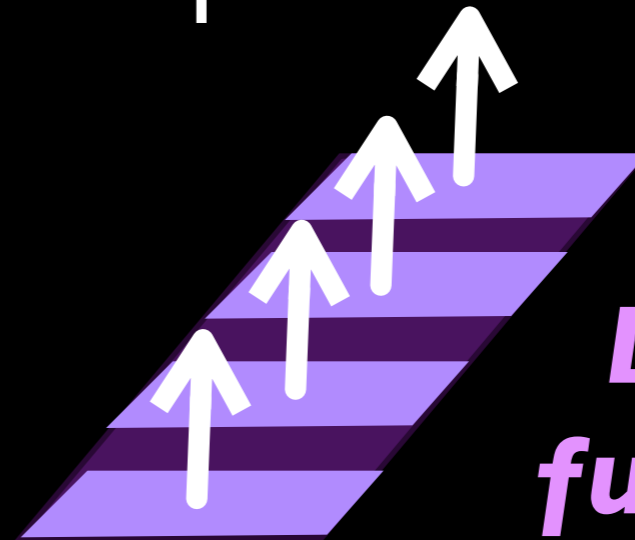
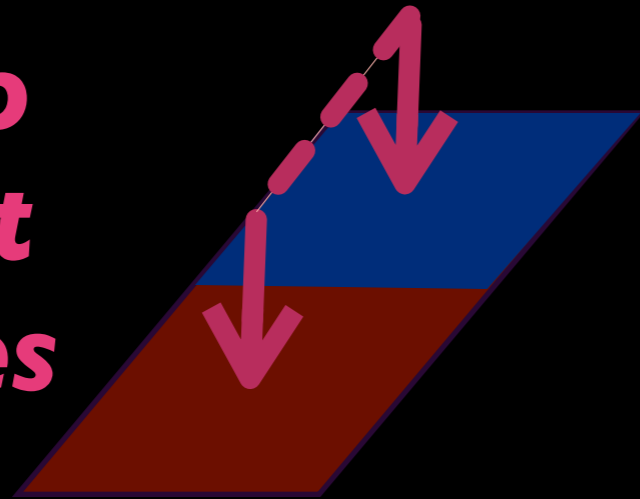


*Demand
full output*

Batch	Baseline time (s)	Adapton speedup	SAC speedup
filter	0.6	2.0	4.11
map	1.2	2.2	3.32
fold min	1.4	4350	3090
fold sum	1.5	1640	4220
exptree	0.3	497	1490

Swap Pattern: Experimental procedure:

*Swap
input
halves*

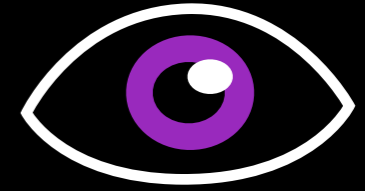
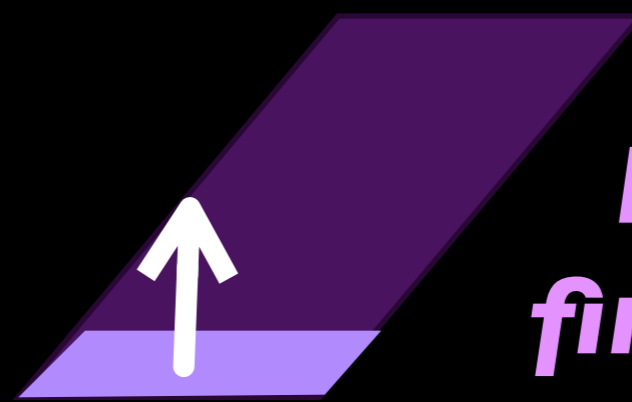
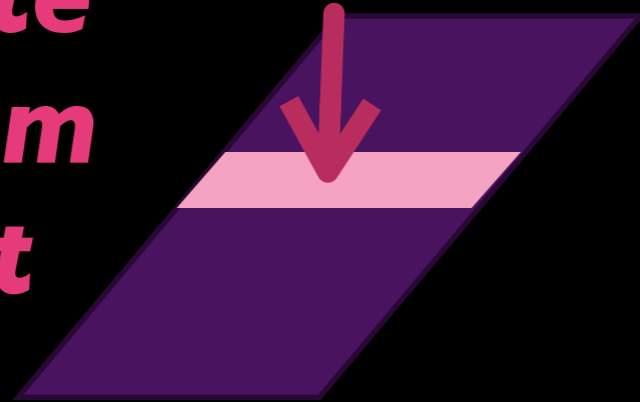


*Demand
full output*

<i>Swap</i>	Baseline time (s)	<i>Adapton speedup</i>	SAC speedup
filter	0.5	2.0	0.14
map	0.9	2.4	0.25
fold min	1.0	472	0.12
fold sum	1.1	501	0.13
exptree	0.3	667	10

Lazy Pattern: Experimental procedure:

*Mutate
random
input*



*Demand
first output*

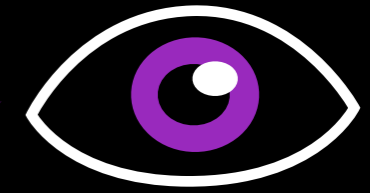
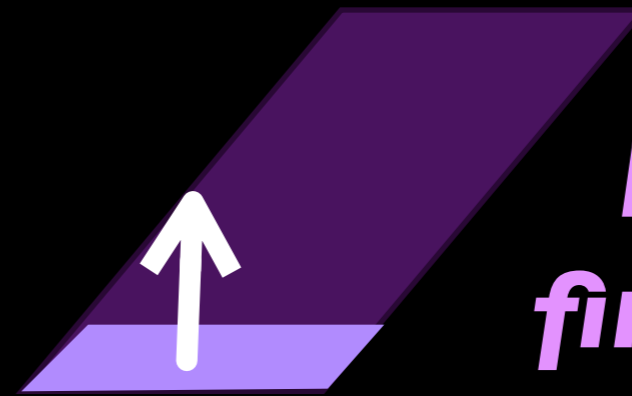
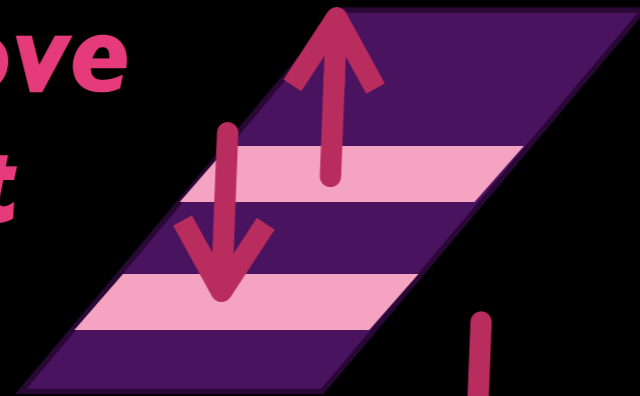
<i>Lazy</i>	Baseline time (s)	<i>Adapton</i> speedup	<i>SAC</i> speedup
filter	1.16E-05	12.8	2.2
map	6.86E-06	7.8	1.5
quicksort	7.41E-02	2020	22.9
mergesort	3.46E-01	336	0.148

Switch Pattern: Experimental procedure:

1. Remove

2. Insert

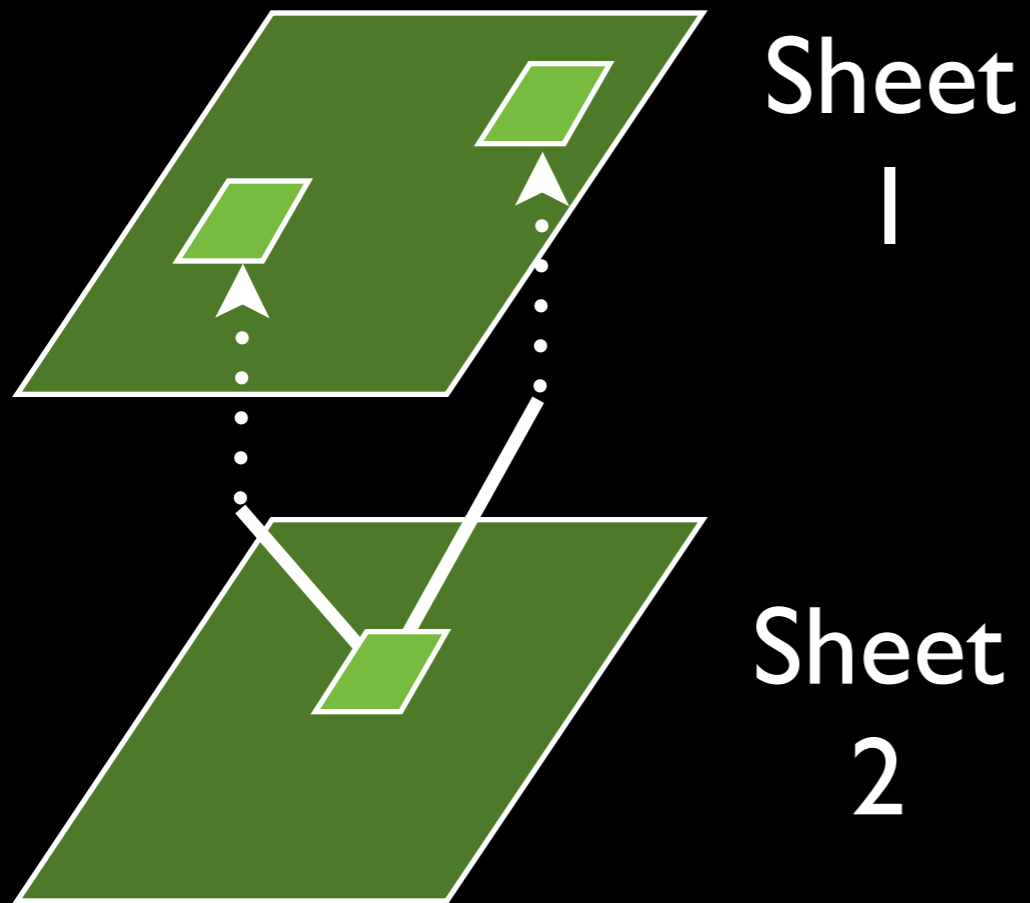
3. Toggle order



**Demand
first output**

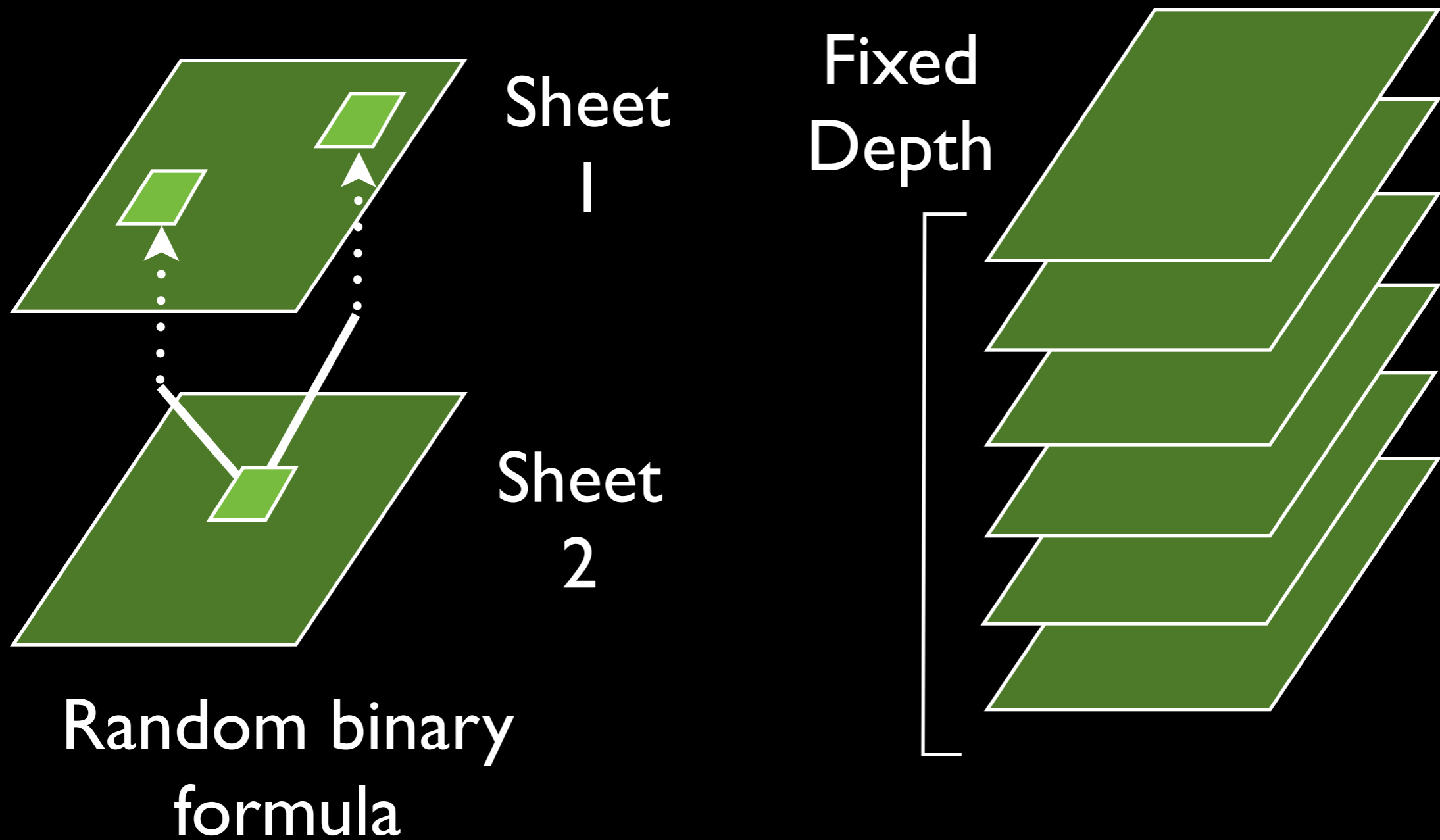
Switch	Baseline time (s)	Adapton speedup	SAC speedup
updown1	3.28E-02	22.4	2.47E-03
updown2	3.26E-02	24.7	4.28

Spreadsheet Experiments

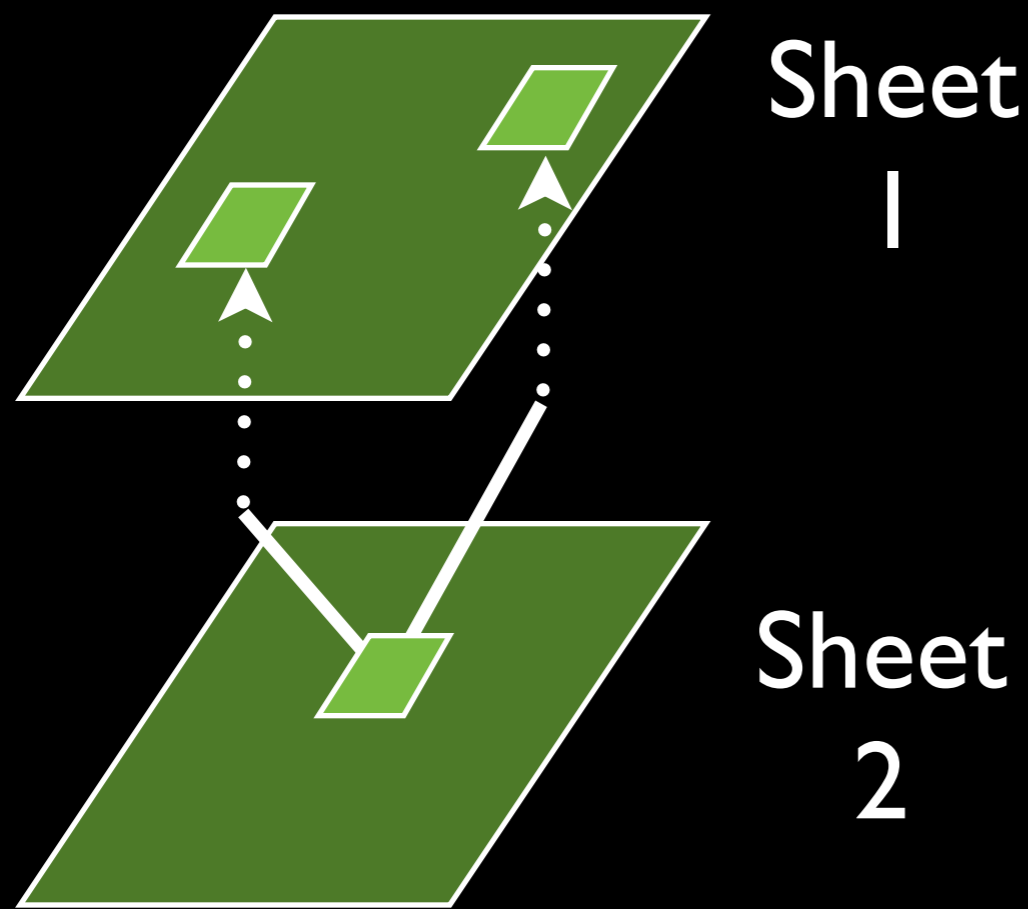


Random binary
formula

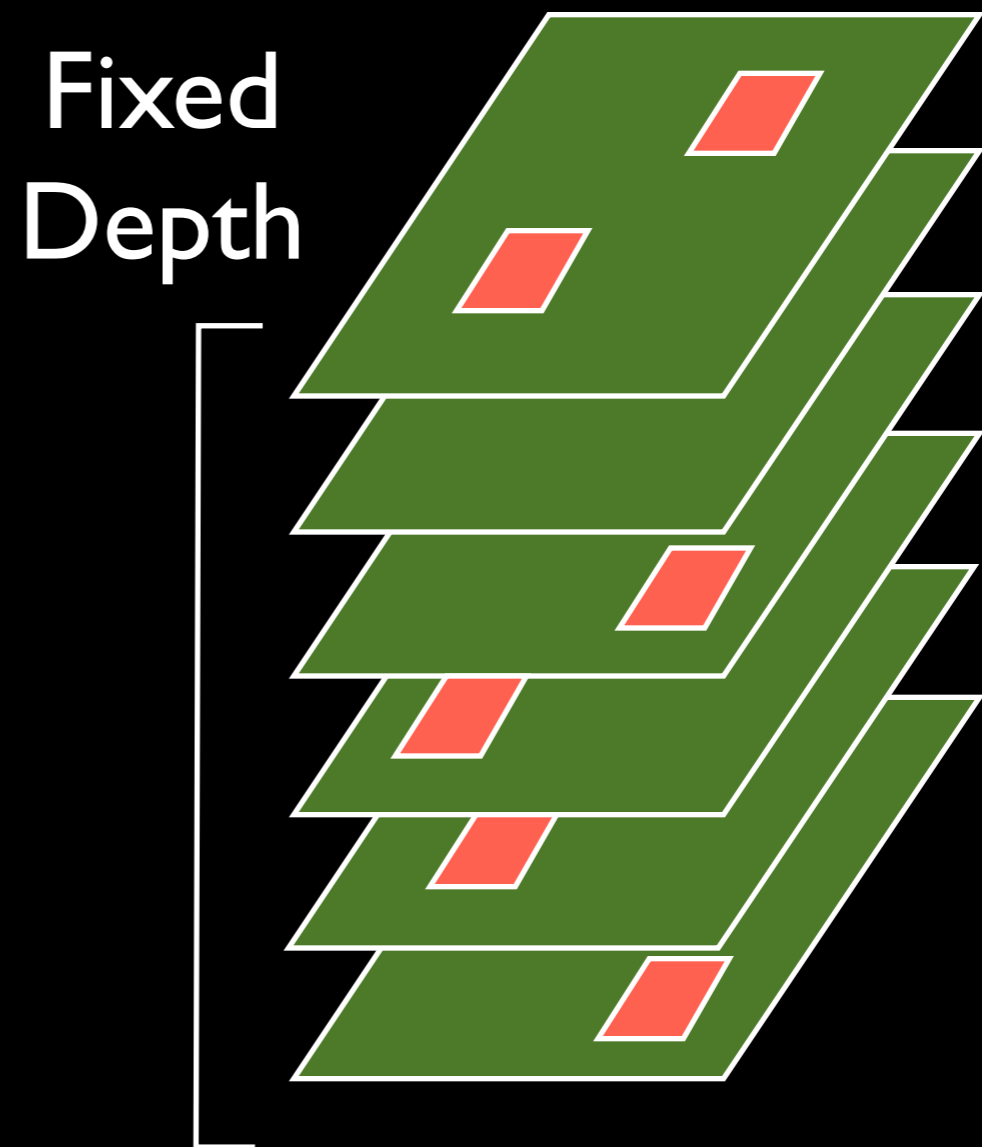
Spreadsheet Experiments



Spreadsheet Experiments

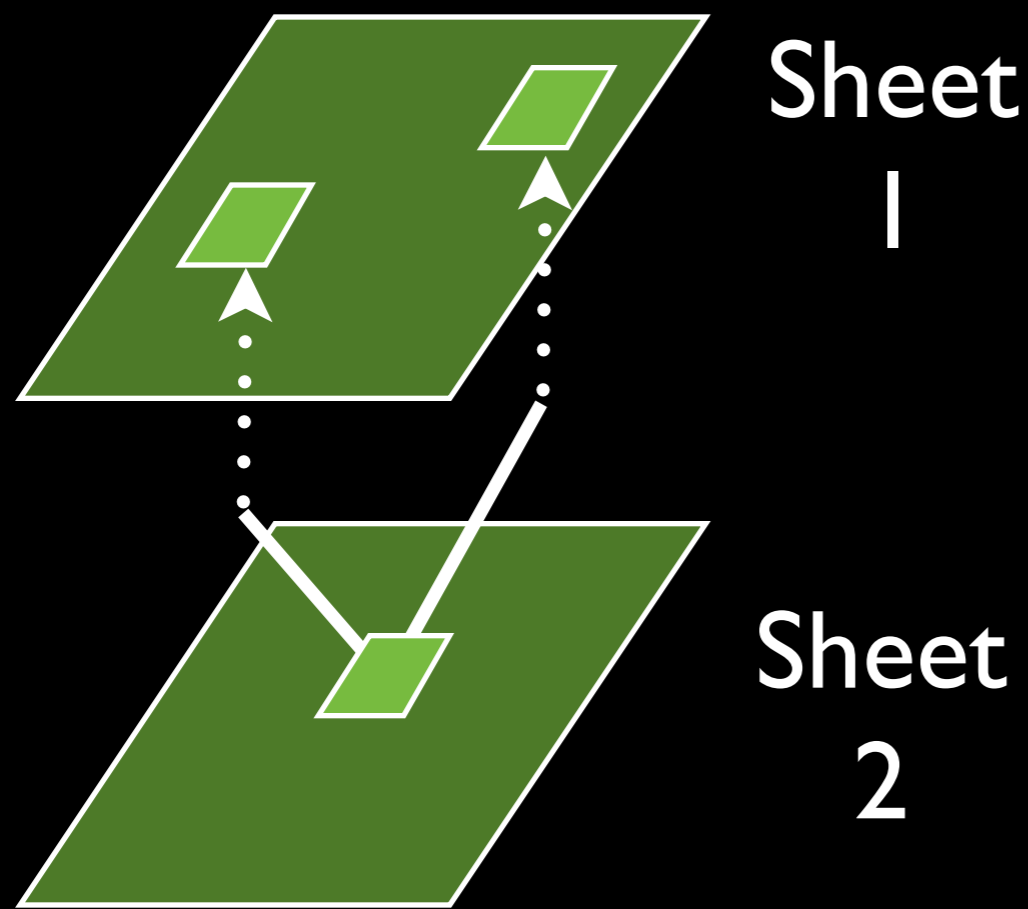


Random binary
formula

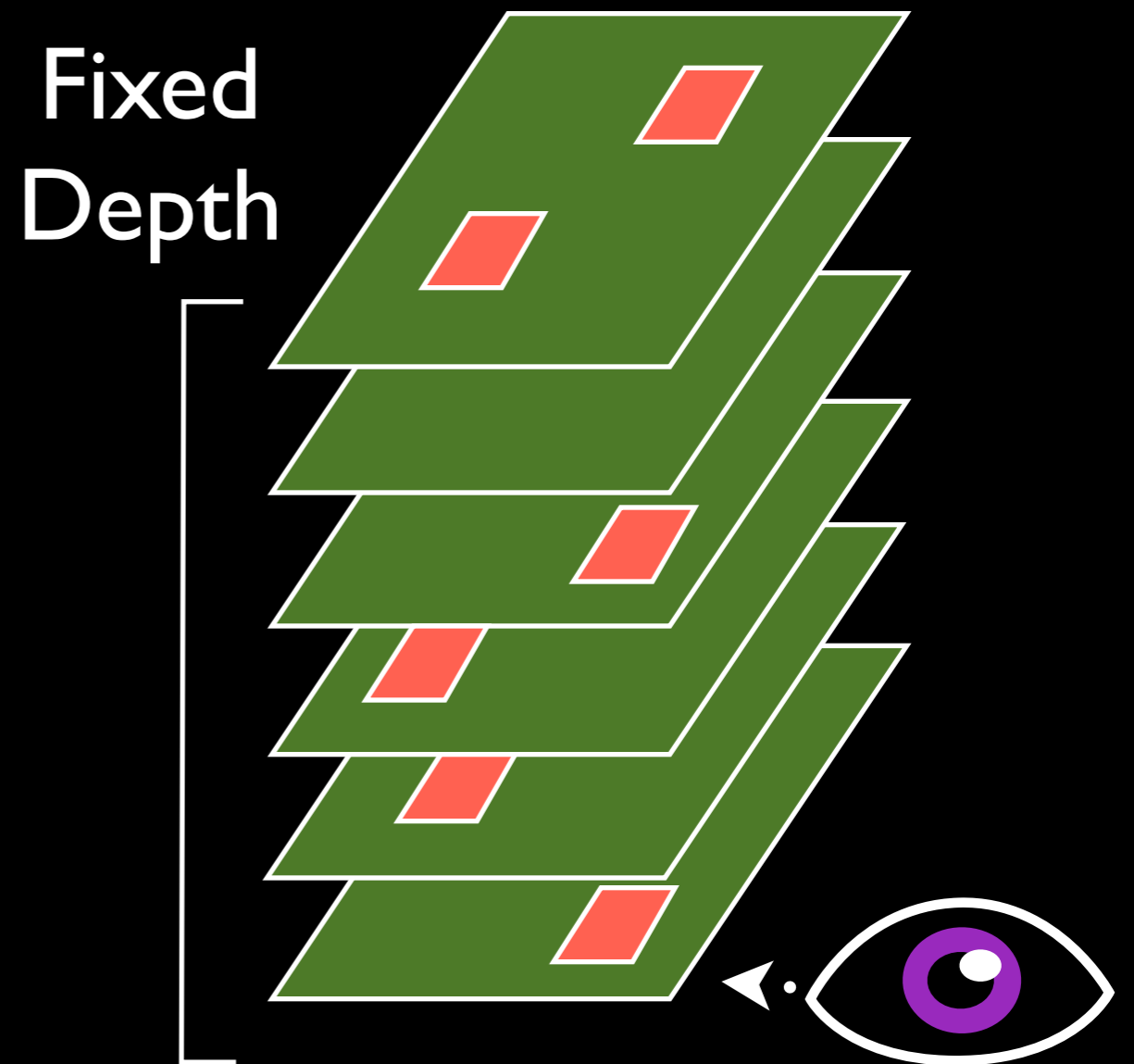


I. Random
Mutations

Spreadsheet Experiments



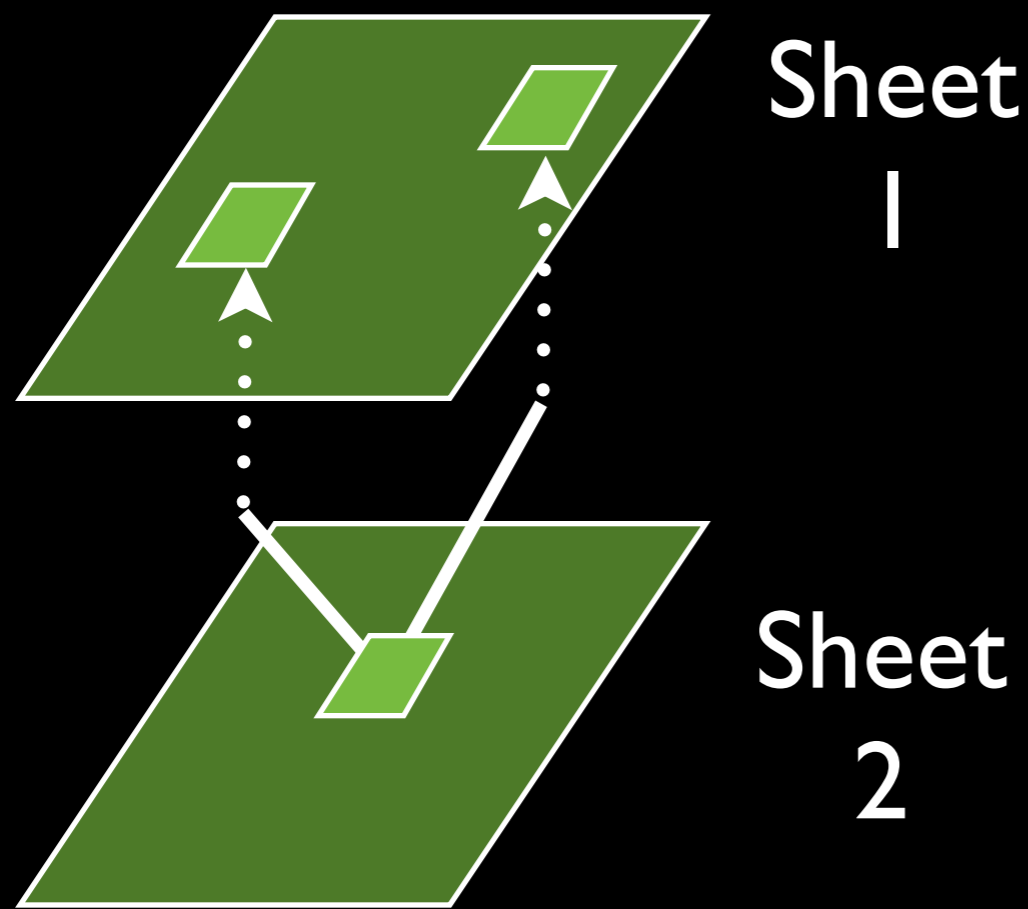
Random binary
formula



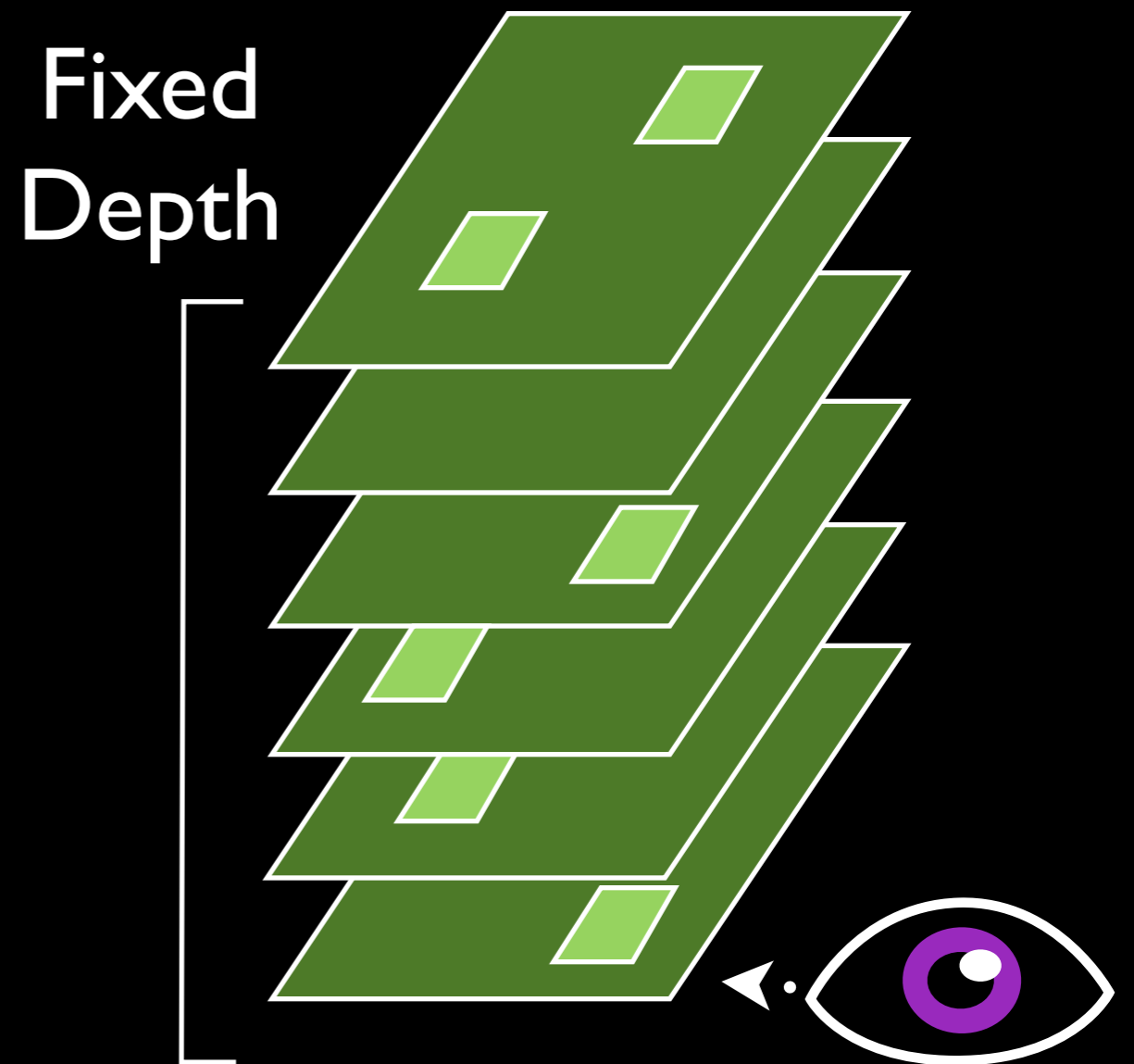
1. Random
Mutations

2. Observe
last sheet

Spreadsheet Experiments



Random binary
formula

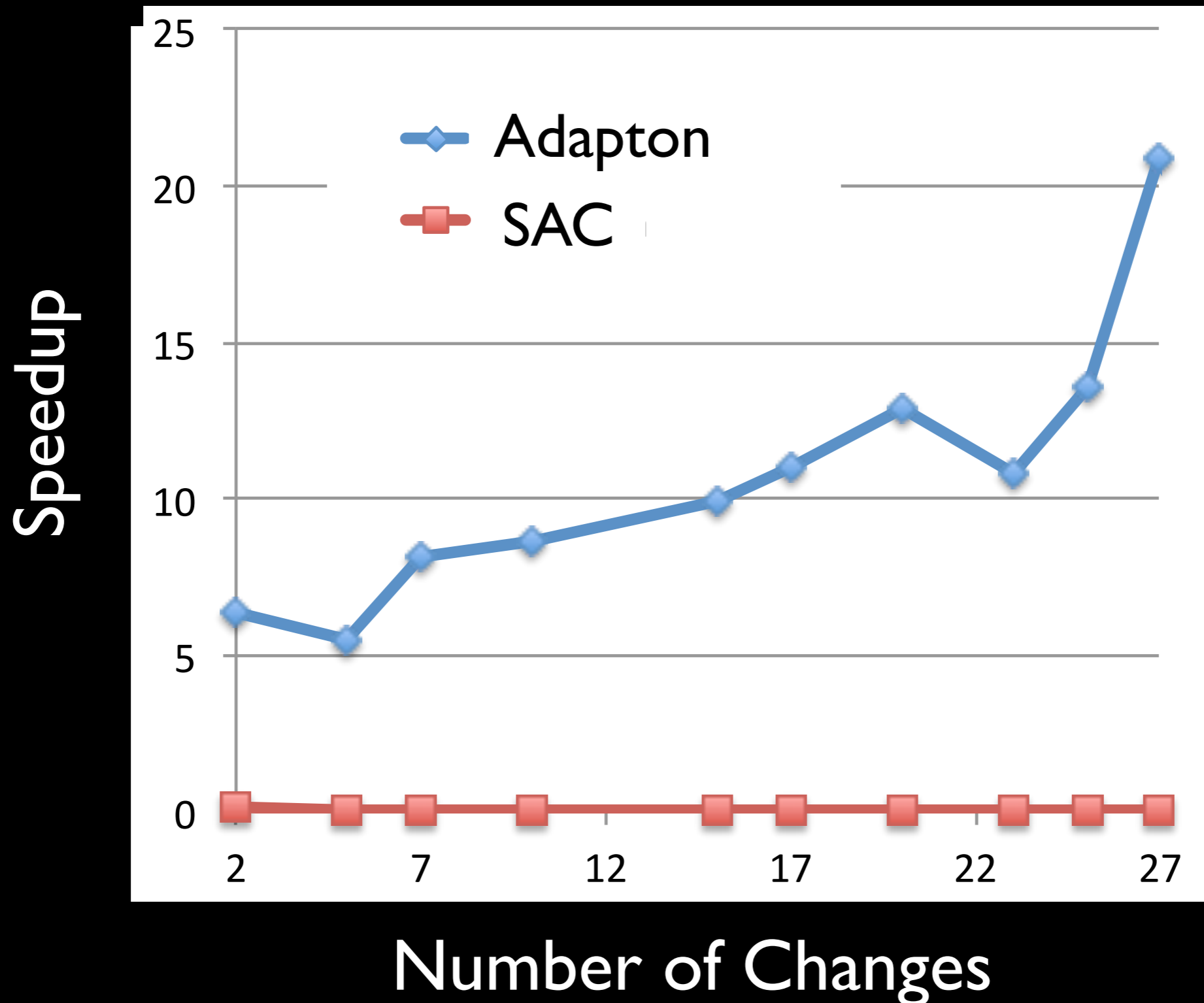


1. Random
Mutations

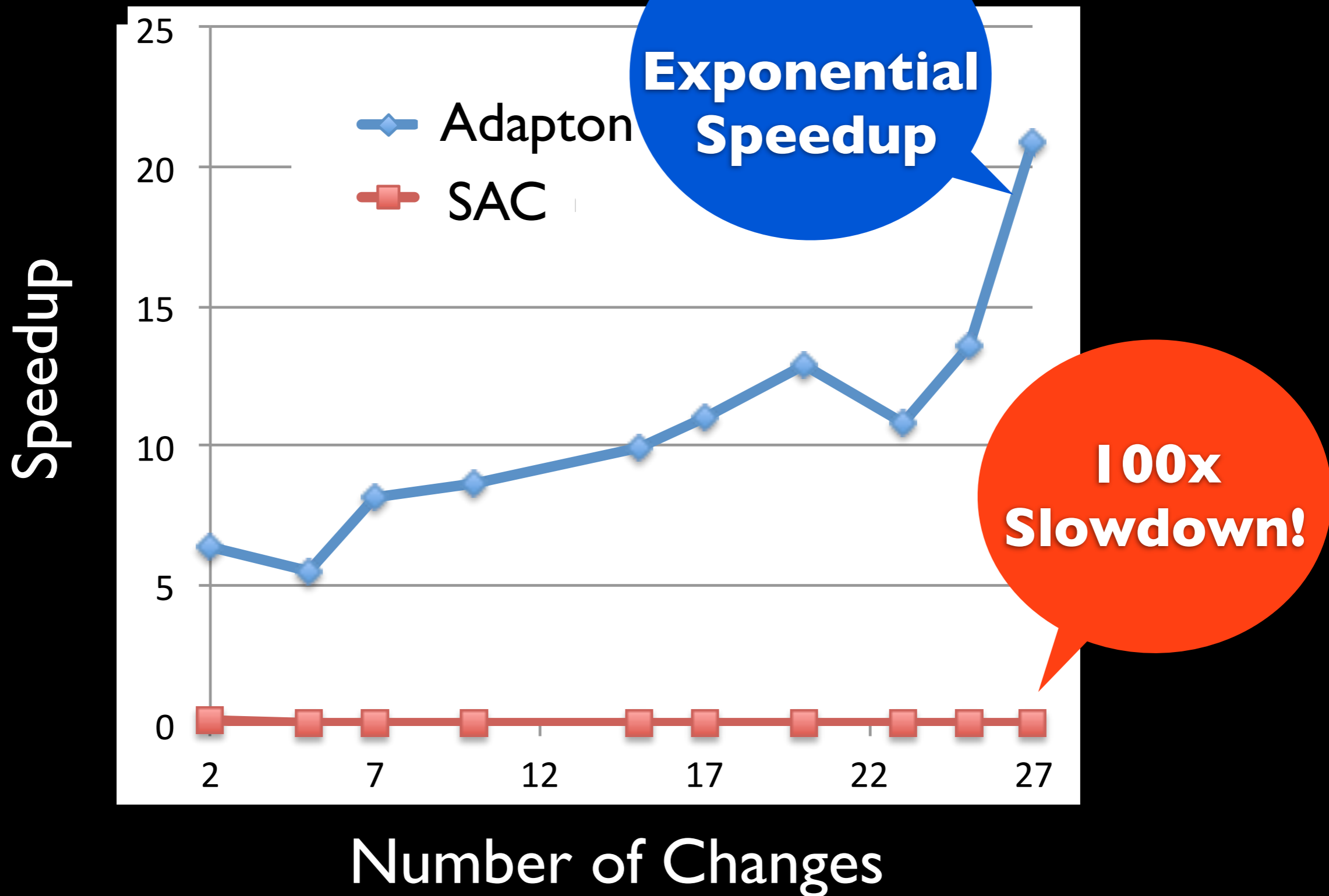
2. Observe
last sheet

Speedup vs # Changes

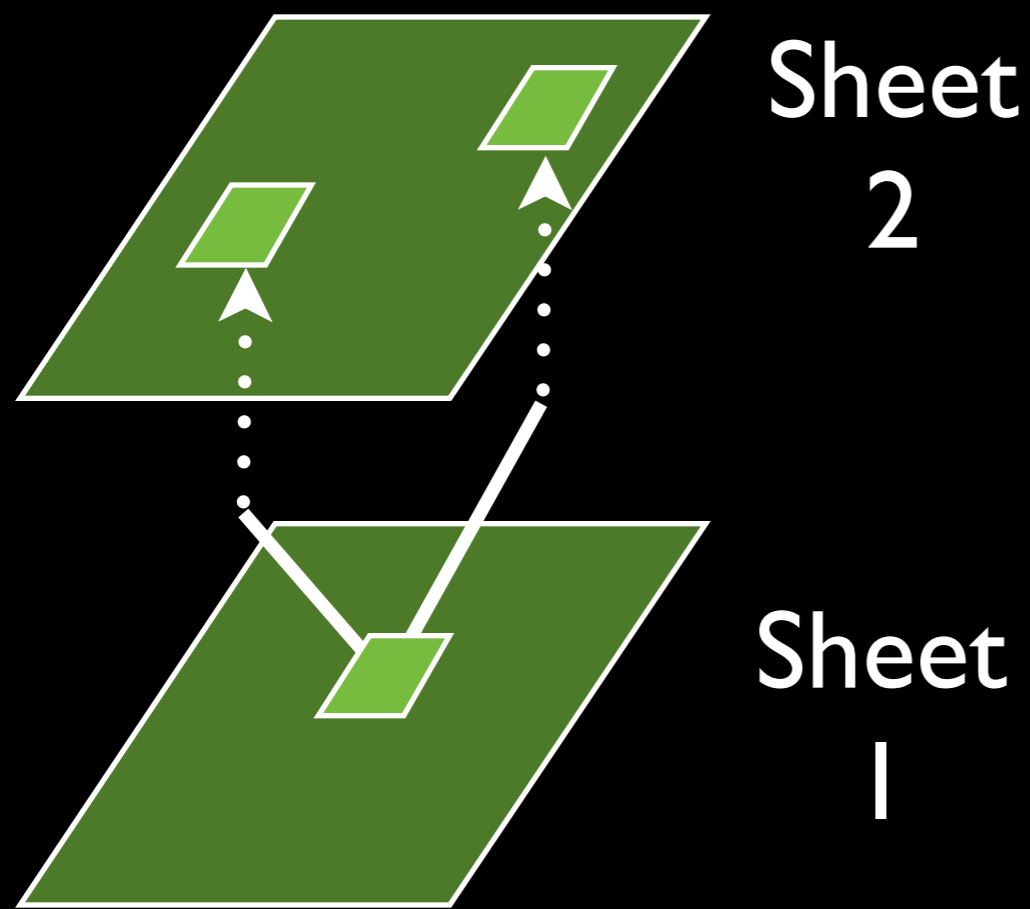
(15 sheets deep)



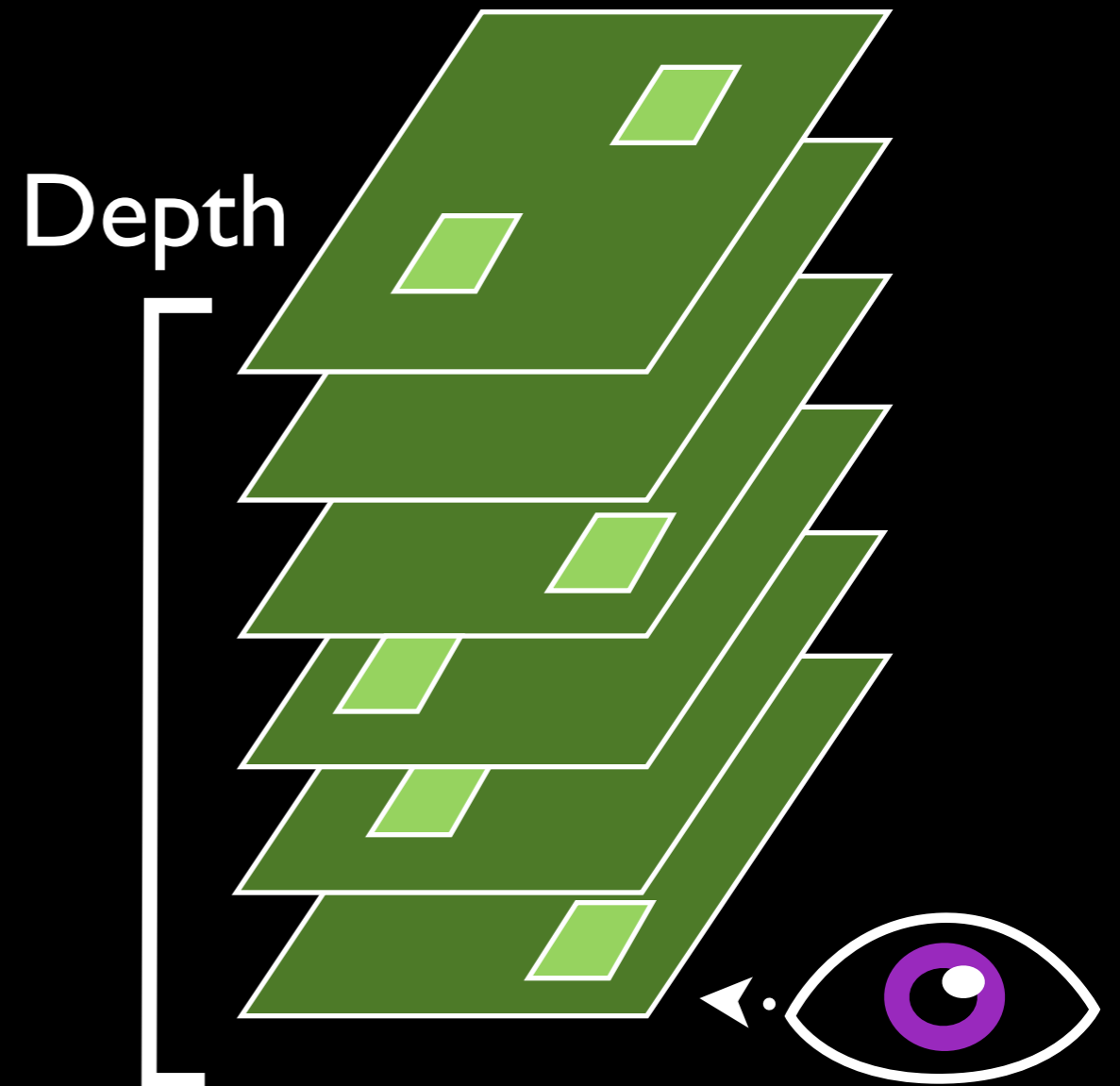
Speedup vs # Changes



Spreadsheet Experiments



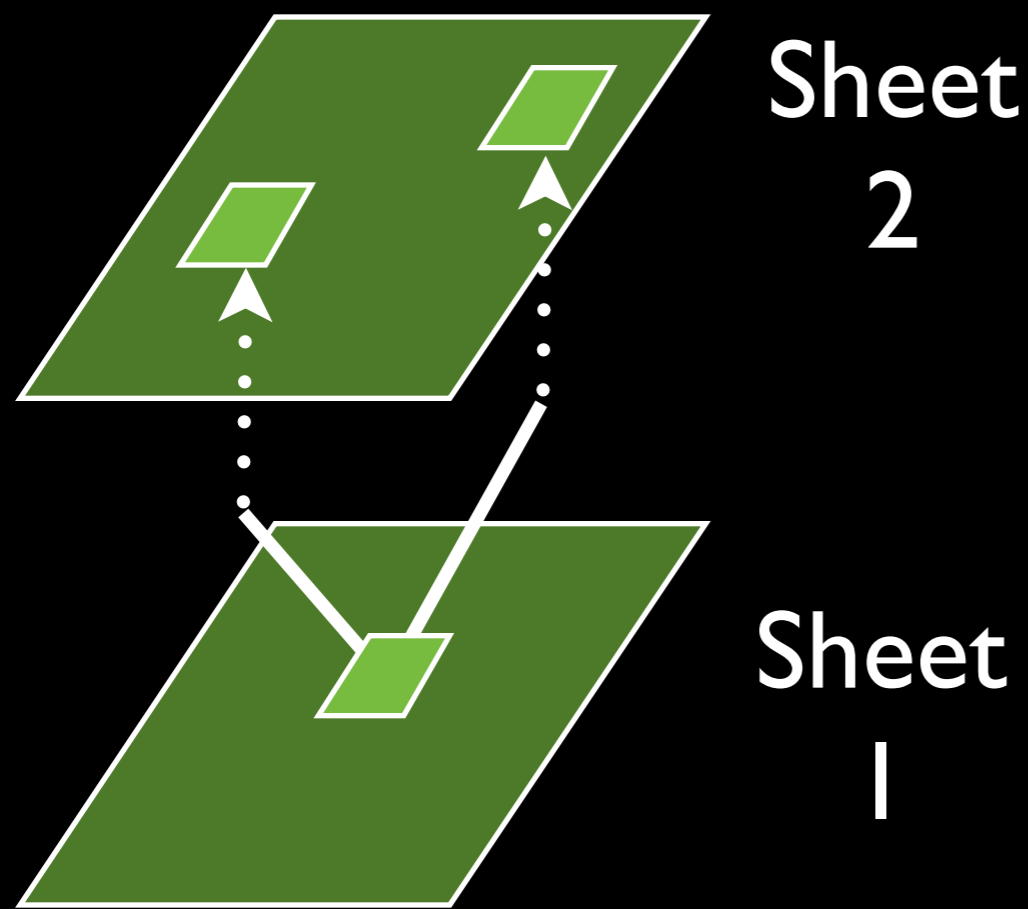
Random binary
formula



1. Random
Mutations

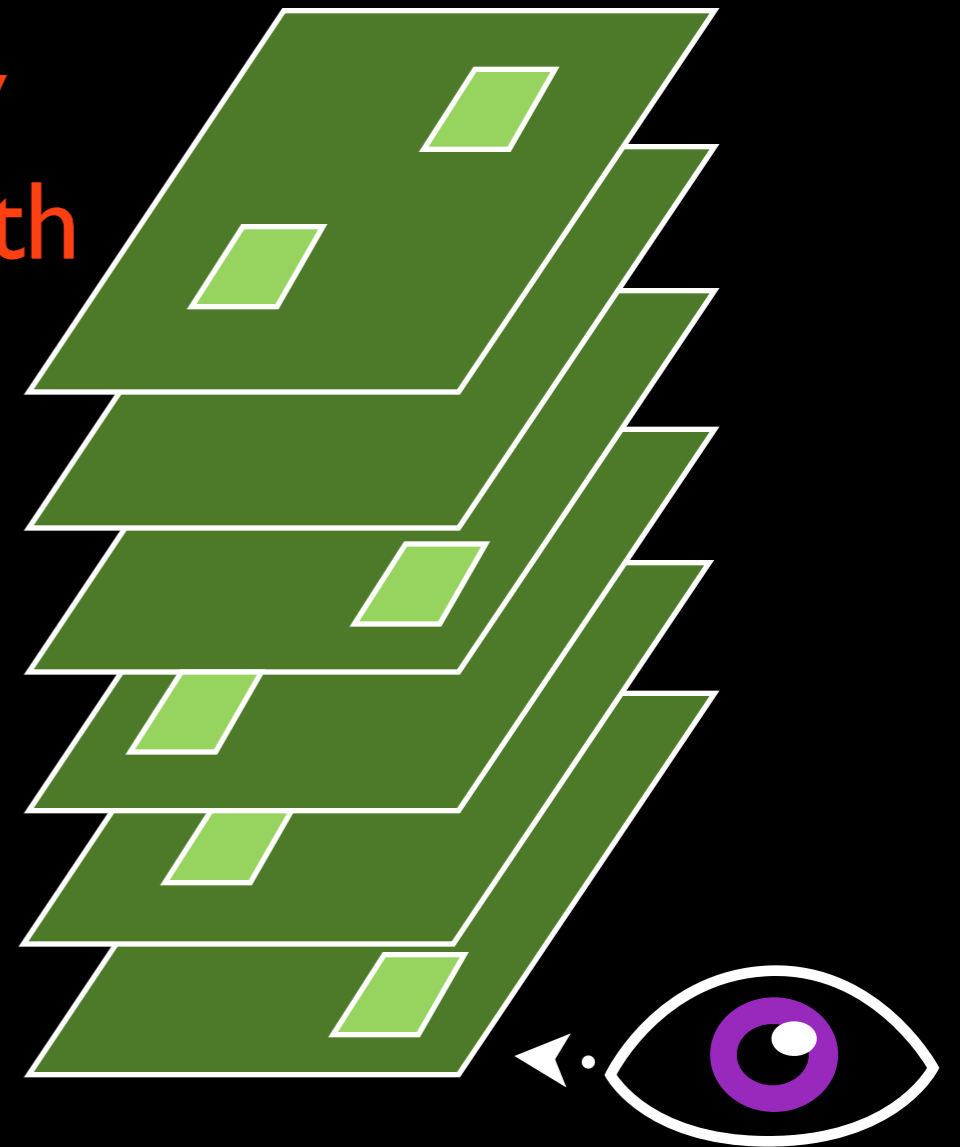
2. Observe
last sheet

Spreadsheet Experiments



Random binary
formula

Vary
Depth

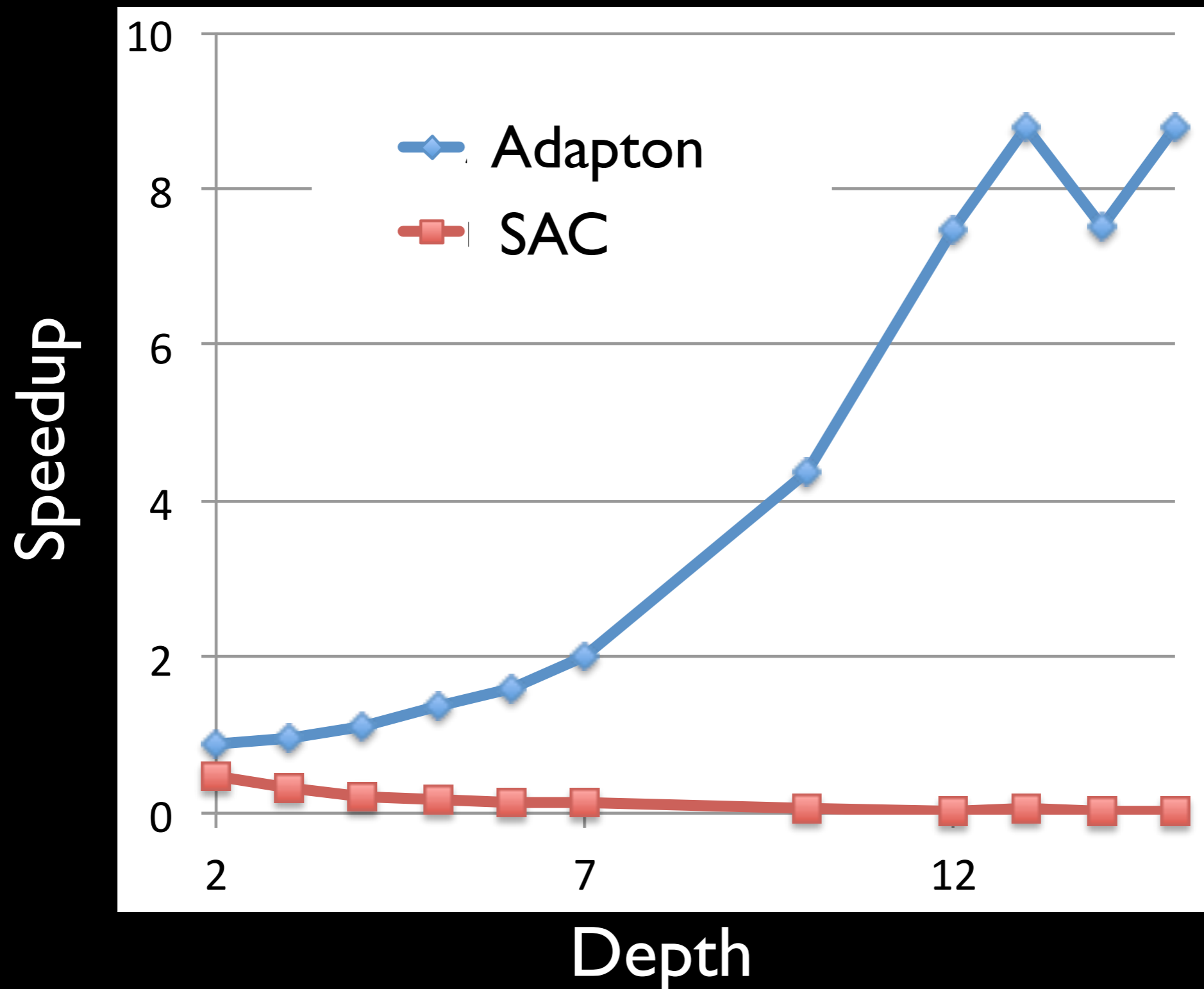


1. Random
Mutations

2. Observe
last sheet

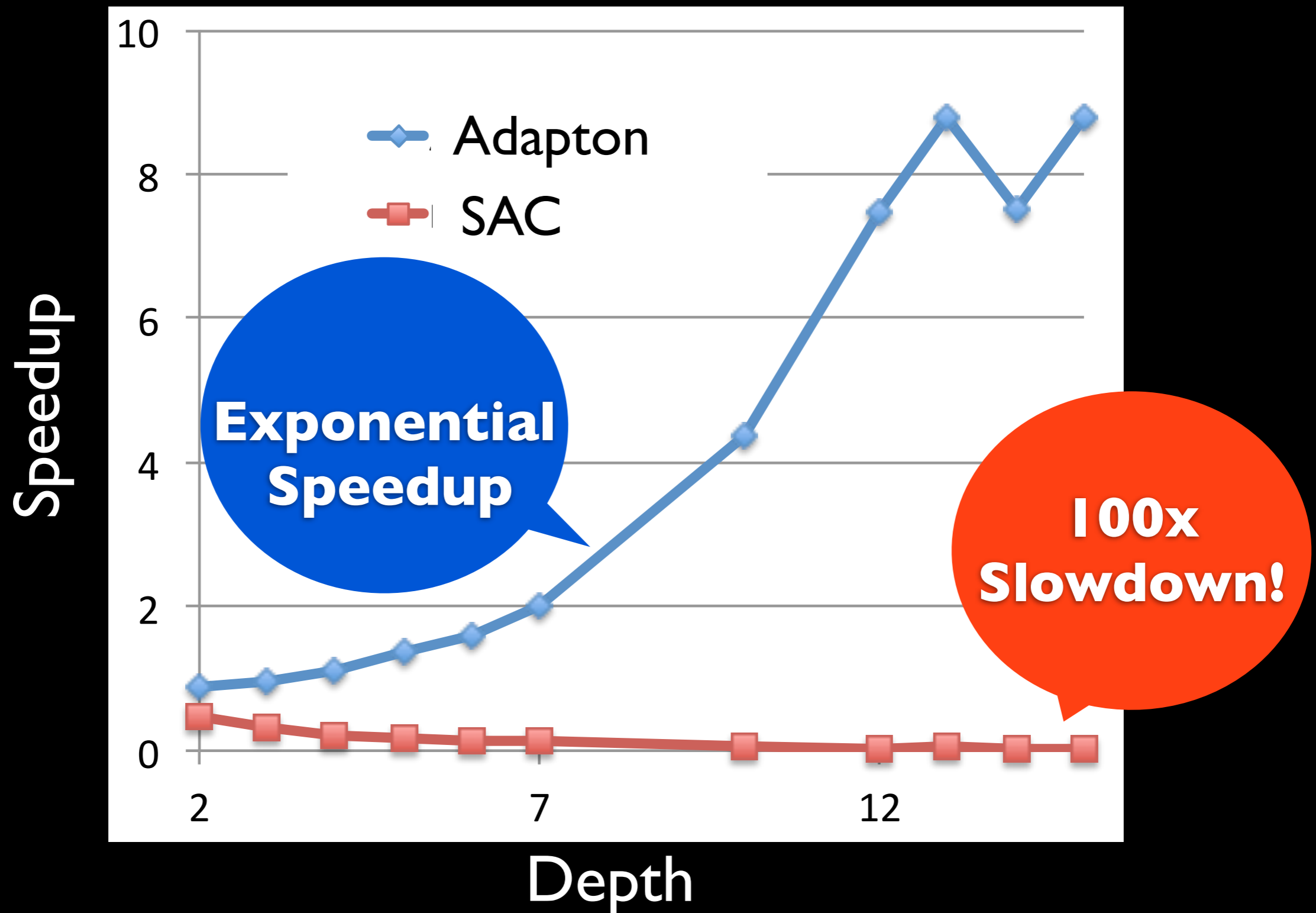
Speedup vs Sheet Depth

(10 changes between observations)



Speedup vs Sheet Depth

(10 changes between observations)



Paper and Technical Report

- ***Formal semantics*** of Adapton
- ***Algorithms*** to implement Adapton
- More empirical data and analysis

Aside: Formal Semantics

- ▶ **CBPV + Refs + Layers** (*outer versus inner*)
- ▶ Syntax for **traces** and **knowledge**
formally represents **DCG** structure
- ▶ Formal specification of change propagation
- ▶ **Theorems:**
 - **Type soundness**
 - **Incremental soundness**
(“*from-scratch consistency*”)

Summary

- ▶ **Adapton**: Composable, Demand-Driven IC
 - *Demand-driven* change propagation
 - Reuse patterns:
Sharing, swapping and switching
- ▶ Formal specification (see paper)
- ▶ Implemented in OCaml (and Python)
- ▶ Empirical evaluation shows **speedups**

<http://ter.ps/adapton>

			LazyNonInc		ADAPTON		EagerTotalOrder	
	pattern	input #	baseline	time	time	mem	time	mem
			(s)	(MB)	spdup	ovrhd	spdup	ovrhd
filter	lazy	1e6	1.16e-5	96.7	12.8	2.7	2.24	8.0
map		1e6	6.85e-6	96.7	7.80	2.7	1.53	8.0
quicksort		1e5	0.0741	18.6	2020	8.7	22.9	144.1
mergesort		1e5	0.346	50.8	336	7.8	0.148	96.5
filter	swap	1e6	0.502	157	1.99	10.1	0.143	17.3
map		1e6	0.894	232	2.36	6.9	0.248	12.5
fold(min)		1e6	1.04	179	472	9.1	0.123	33.9
fold(sum)		1e6	1.11	180	501	9.1	0.128	33.8
exptree		1e6	0.307	152	667	11.7	10.1	11.9
updown1	switch	4e4	0.0328	8.63	22.4	14.0	0.00247	429.9
updown2		4e4	0.0326	8.63	24.7	13.8	4.28	245.7
filter	batch	1e6	0.629	157	2.04	10.1	4.11	9.0
map		1e6	1.20	232	2.21	6.9	3.32	6.6
fold(min)		1e6	1.43	179	4350	9.0	3090	8.0
fold(sum)		1e6	1.48	180	1640	9.1	4220	8.0
exptree		1e6	0.308	152	497	11.7	1490	9.7