Complexity Theory - Tutorial

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Part 1

1. Decide whether the following statements are true or false and justify your decision.
   (a) For every class C and $A \in C$; if A is C-complete under Karp reductions, then $C \subset P^A$
   (b) If L is NP-complete and L' is coNP complete, then $L \cap L'$ is $NP \cap coNP$ complete.
   (c) If PH=PSPACE, then PH collapses to some level Σ_k
   (d) There is an undecidable language in P$/_{poly}$
   (e) If P=NP, then NP=coNP.
   (f) If #L is #P-complete, then L is NP-complete (you may assume $P \neq$ NP).
   (g) Show that every circuit with only $\wedge$ and $\vee$ gates can be replaced by a circuit containing only *majority* gates.
2. DOUBLE-SAT $= \{\phi : \phi$is a Boolean formula with 2 satisfying assignments$\}$. Show that DOUBLE-SAT is NP-complete.
3. Show that $UPATH \in RL$. You may use the following fact: if graph G has a path from s to t, then a random walk of length $8 \cdot |V(G)| \cdot |E(G)|$ visits t with probability $\geq \frac{1}{2}$
4. Show that $SPACE(f(n)) \subset RSPACE(f(n)) \subset NSPACE(f(n))$

Part 2

5. We showed that there is an undecidable language in P/poly. Show that there are decidable languages in P/poly that are not in P. Use the following waypoints:
   (a) show that there is a decidable language L that is not in EXP

(b) define $L' := \{1^m : m \in L\}$ and show that L' is decidable and member of P/poly

(c) show that L' is not in P

6. Suppose that we have a poly-time algorithm A such that for $\phi \in$ USAT, $A(\phi) = 1$ and for $\phi \notin SAT$, $A(\phi) = 0$. Show that then NP = RP. Afterwards, show that AM[2] = BPP.

7. Show that regular languages are in NC^1. (Hint: note that the final state can be reached from the initial one in n steps if there is some intermediate state q that can be reached from the initial one in $\frac{n}{2}$ steps and the final state can be reached from q also in $\frac{n}{2}$ steps. Repeat the same idea to the leaves.)

NOTE: Some of the problems and solutions are taken from

- `http://soltys.cs.csuci.edu/blog/wp-content/oldpage/cu-f07/chp5.pdf`
- `http://cse.iitkgp.ac.in/~abhij/course/theory/CC/Spring04/soln5.pdf`