Complexity Theory - Tutorial

Ivan Gavran

January 31st, 2017

Part 1

1. Decide whether the following statements are true or false and justify your decision.

   (a) For every class C and $A \in C$; if A is C-complete under Karp reductions, then $C \subset P^A$
   Solution *True.* Any $B \in C$ can be transformed into instance of A. But this - by the definition - means that B can be decided by polynomial computation (do the transformation into an instance of A) and then asking oracle A.

   (b) If L is NP-complete and L' is coNP complete, then $L \cap L'$ is $NP \cap coNP$ complete.
   Solution *False.* SAT is NP-complete. NOSAT is coNP-complete. Their intersection is empty (so that language would trivially always return false, but we know that there are non-trivial languages in $NP \cap coNP$, primality, for example.).

   (c) If PH=PSPACE, then PH collapses to some level Σ_k
   Solution *True.* There is a complete problem for PSPACE (TQBF, for example). Since PH = PSPACE, this problem is in PH and therefore it is in Σ_k, for some k. But now all the problems in PH can be in polynomial time reduced to that one. Therefore, PH collapses to Σ_k.

   (d) There is an undecidable language in $\mathrm{P}/_{poly}$
   Solution *True* All unary languages are in P/poly, unary variant of halting problem is in P/poly.

   (e) If P=NP, then NP=coNP.
   Solution *True* Since NP = P and $P \subset coNP$ it is clear that $NP \subset coNP$. Take $L \in coNP$. This means that $\bar{L} \in NP = P$. Then there exists a poly-time Turing machine M that decides $\bar{L}$. Create $M' = 1 - M$, that machine decides L in polynomial time. Therefore, $coNP \subset P = NP$

   (f) If #L is #P-complete, then L is NP-complete (you may assume $P \neq$ NP).
   Solution *False.* #2-SAT is #P-complete, while 2-SAT is in P.

(g) Show that every circuit with only $\wedge$ and $\vee$ gates can be replaced by a circuit containing only *majority* gates.

2. DOUBLE-SAT $= \{\phi : \phi$is a Boolean formula with 2 satisfying assignments$\}$. Show that DOUBLE-SAT is NP-complete.
   Solution DOUBLE-SAT is clearly in NP (the two mentioned assignments could be given as a witness. Now we show how to reduce SAT to DOUBLE-SAT. Let ϕ be a 3-CNF formula. We define $\phi'(y,x) = \phi(y) \wedge (x \vee \bar{x})$. If $\phi \notin 3SAT$ then ϕ' is obviously not in SAT. On the other hand, if $\phi \in 3SAT$, it means there is a satisfying assignment for ϕ. But from that one we can derive two satisfying assignments for ϕ'.

NOTE: This is not the final version of the problem list, the new problems might be added throughout this week.