Complexity Theory - Tutorial

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Part 1

1. Decide whether the following statements are true or false and justify your decision.

(a) For every class $C$ and $A \in C$; if $A$ is $C$-complete under Karp reductions, then $C \subset P^A$

**Solution True.** Any $B \in C$ can be transformed into instance of $A$. But this - by the definition - means that $B$ can be decided by polynomial computation (do the transformation into an instance of $A$) and then asking oracle $A$.

(b) If $L$ is NP-complete and $L'$ is coNP complete, then $L \cap L'$ is $NP \cap coNP$ complete.

**Solution False.** SAT is NP-complete. NOSAT is coNP-complete. Their intersection is empty (so that language would trivially always return false, but we know that there are non-trivial languages in $NP \cap coNP$, primality, for example.).

(c) If PH=PSPACE, then PH collapses to some level $\Sigma_k$.

**Solution True.** There is a complete problem for PSPACE (TQBF, for example). Since PH = PSPACE, this problem is in PH and therefore it is in $\Sigma_k$, for some k. But now all the problems in PH can be in polynomial time reduced to that one. Therefore, PH collapses to $\Sigma_k$.

(d) There is an undecidable language in $P/poly$

**Solution True.** All unary languages are in $P/poly$, unary variant of halting problem is in $P/poly$.

(e) If P=NP, then NP=coNP.

**Solution True.** Since NP = P and $P \subset coNP$ it is clear that $NP \subset coNP$. Take $L \in coNP$. This means that $\overline{L} \in NP = P$. Then there exists a poly-time Turing machine $M$ that decides $\overline{L}$. Create $M' = 1 - M$, that machine decides $L$ in polynomial time. Therefore, $coNP \subset P = NP$.

(f) If #L is #P-complete, then L is NP-complete (you may assume $P \neq NP$).

**Solution False.** #2-SAT is #P-complete, while 2-SAT is in P.
(g) Show that every circuit with only $\land$ and $\lor$ gates can be replaced by a circuit containing only majority gates.

2. DOUBLE-SAT = \{\phi: \phi is a Boolean formula with 2 satisfying assignments\}. Show that DOUBLE-SAT is NP-complete.

Solution: DOUBLE-SAT is clearly in NP (the two mentioned assignments could be given as a witness. Now we show how to reduce SAT to DOUBLE-SAT. Let $\phi$ be a 3-CNF formula. We define $\phi'(y, x) = \phi(y) \land (x \lor \bar{x})$.

If $\phi \not\in 3SAT$ then $\phi'$ is obviously not in SAT. On the other hand, if $\phi \in 3SAT$, it means there is a satisfying assignment for $\phi$. But from that one we can derive two satisfying assignments for $\phi'$.

NOTE: This is not the final version of the problem list, the new problems might be added throughout this week.