Complexity Theory - Tutorial

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1. STRONGLY-CONNECTED := \{G = (V, E) : G is strongly connected directed graph\}. Prove that STRONGLY-CONNECTED is NL-complete.

2. Prove $NP \neq \text{SPACE}(n)$

3. Assume $P = NP$. Then there is a polynomial-time algorithm for solving SAT. Find in polynomial time an explicit algorithm that outputs a satisfying assignment to Boolean formulas whenever such an assignment exists.

4. Let A be an algorithm that’s supposed to solve SAT in polynomial time (that is, find a satisfying assignment whenever one exists), but that actually fails on some SAT instance of size $n$. Then if someone gives you the source code of A, you can, in time polynomial in $n$, find a specific SAT instance that actually witnesses A’s failure.

5. We say that a function is write-once computable if it can be computed by an $O(\log n)$-space Turing machine $M$ whose output-tape is ”write once”, meaning that $M$ can either keep its head in the same position on the tape or write to it a symbol and move to the right. The used space on the output tape is not counted against $M$’s space bound.
   On the other hand, the implicitly logspace computable function is defined as a function that is polynomially bounded and the languages $L_f = \{(x, i) : f(x)_i = 1 \}$ and $L'_f = \{(x, i) : i < |f(x)| \}$ are in $L$.
   Prove that $f$ is write-once computable if and only if $f$ is implicitly logspace computable.

6. $M = \{0^k1^k : k \geq 0\}$. Prove that $M \in L$.

NOTE: Sources of the problems and more problems of the same kind:
- [http://cse.iitkgp.ac.in/~abhij/course/theory(CC/Spring04/chap3.pdf](http://cse.iitkgp.ac.in/~abhij/course/theory(CC/Spring04/chap3.pdf)