CapablePtrs: Securely Compiling Partial Programs using the Pointers-as-Capabilities Principle

Abstract—Capability machines such as Cheri provide memory capabilities that can be used by compilers to provide security benefits for compiled code (e.g., memory safety). The existing C to Cheri compiler, for example, achieves memory safety by following a principle called “pointers as capabilities” (PAC). Informally, PAC says that a compiler should represent a source language pointer as a machine code capability. But the security properties of PAC compilers are not yet well understood. We show that memory safety is only one aspect, and that PAC compilers can provide significant additional security guarantees for partial programs: the compiler can provide security guarantees for a compilation unit, even if that compilation unit is later linked to attacker-controlled machine code.

As such, this paper is the first to study the security of PAC compilers for partial programs formally. We prove for a model of such a compiler that it is fully abstract. The proof uses a novel proof technique (dubbed TrICL, read trickle), which should be of broad interest because it reuses the whole-program compiler correctness relation for full abstraction, thus saving work. We also implement our scheme for C on Cheri, show that we can compile legacy C code with minimal changes, and show that the performance overhead of compiled code is roughly proportional to the number of cross-compilation-unit function calls.

Index Terms—

I. INTRODUCTION

In a conventional computer, memory is addressed using integers (pointers). In a capability machine (i.e., a capability-based computer), memory is addressed using capabilities [1, 2, 3, 4, 5, 6, 7], which carry more information than just a memory address—they also contain bounds information, indicating a range of memory that can be accessed using the capability, and possibly also other information such as access permissions. Load and store instructions take a capability (in a register), and the machine checks that the memory access is within the capability’s bounds, and the operation is compliant with the capability’s permissions. If not, the instruction fails with an exception. The hardware also ensures that integers and capabilities are not confused. One way of ensuring this is by tracking capabilities in memory and in registers by ensuring this is by tracking capabilities in memory and in that integers and capabilities are not confused. One way of implementing this is by tracking capabilities in memory and in that integers and capabilities are not confused.

A recent capability machine is Cheri. It has its own FreeBSD version and C compiler [8, 4, 9, 10, 11]. Many key design choices in Cheri were made to facilitate the use of memory protection in existing, large code bases. Specifically, Cheri supports the pointers-as-capabilities (PAC) principle, which intuitively dictates that a compiler should represent a source-level pointer as a target-level capability. To make this convenient, a Cheri capability contains (among other things): base and length addresses, and an offset relative to the base address [11]. Such a capability represents a pointer pointing to the address base+offset, and that is valid only if (base+offset) ∈ [base, base+length). Pointer arithmetic can be implemented by manipulating the offset. The following example illustrates how a compiler can map C pointers to such machine-level capabilities.

```c
extern void send_rcv(char* buffer);
static int secret;

void f() {
    char iobuffer[512];
    iobuffer[42] = 'X';
    send_rcv(iobuffer);
}
```

The C compilation unit (above) declares two module-scoped variables and defines a function f() using one of these variables. The assembly pseudocode (below) shows how a PAC compiler could translate the body of f(). The default data capability register $.ddc$ contains a capability for the global data section. The compiler knows that the variable $iobuffer$ occupies the first 512 bytes of that global data section. Hence, the first instruction ($.cs1$, set length of a capability) loads in register $.sc1$ a copy of $.ddc$ but with the length field reduced to 512. The next two instructions implement the assignment instruction of f(). Note that an out-of-bounds access would be trapped by the hardware. The final call instruction implements the function call in f() (assuming a calling convention where the argument is passed in register $.sc1$). All accesses to $iobuffer$ performed in $send_rcv$ will be bounds-checked, since the capability passed to $send_rcv$ carries the bounds information.

Which security benefits does such a PAC compiler provide? First, the compiler provides spatial memory safety. Since the bounds meta-data for a pointer is stored together with the pointer address in a single capability value, it is natural to implement a bounds-checking compiler [12, Section 4.3]. For

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1 As a typesetting convention, we use a blue, sans-serif font for source language elements and an orange, bold font for target language ones. Elements common to both languages are typeset in a black, italic font.
instance, an out-of-bounds access to iobuffer in our example will not access the secret variable, but just fail.\footnote{In principle, it is also possible to have the compiler provide temporal memory safety, but this is harder as it requires zeroing out all capabilities to a memory region when the region is freed. Efficient temporal memory safety for C is still an open problem and out of scope here. We refer the interested reader to prior work [13, 14, 15, 16, 17, 18].}

However, this is not the full story regarding security properties. Consider the example again under the assumption that the external send_rcv function is implemented directly in assembly. Now, we lose the guarantee that send_rcv cannot access the secret variable because an assembly level implementation can directly access $sdc$. Hence, upfront, memory safety is only guaranteed for complete programs: if all code in a program is compiled by the PAC compiler, then all out-of-bounds accesses will be trapped.

Nonetheless, a PAC compiler can provide a security guarantee even for partial programs by relying on capability unforgeability [19] and on a trusted control stack (that hides the capability of a program part from the context). The main contribution of this paper is to prove that a PAC compiler can, in fact, provide strong security guarantees for partial programs. For our example, the compiler we model in this paper provides the guarantee that secret is inaccessible, even if the function send_rcv is implemented in hand-crafted assembly.

To achieve non-trivial security guarantees for partial programs, the target capability machine needs to support a mechanism to define separate protection domains within a single process. For instance, CHERI provides support for so-called object capabilities [10, 9]. This makes it possible to put different program parts in separate protection domains. CHERI mainly uses object capabilities to compartmentalize programs: it offers an API to programmers to run parts of a program in a sandbox, a protection domain with reduced privileges. The current CHERI compiler, however, does not make direct use of the object capability mechanism: it can only be used through the provided API by the programmer who has to define and set up sandboxes. The PAC compiler we propose in this paper, on the other hand, will automatically set up a separate protection domain for every compilation unit. Doing so allows the compiler to provide strong security guarantees for partial programs.

Summary of our results: We study the security guarantees that a PAC compiler can provide for partial programs. Our setting is a PAC compiler from a simple imperative source language with pointers, to a capability machine with memory capabilities and a very basic form of protection domains/object capabilities. Our overarching contribution is a very strong security theorem for this compiler, namely, full abstraction (FA) [20, 21], which intuitively means that the compilation preserves and reflects observational equivalence of partial programs. FA implies the preservation of many security properties like confidentiality, even when target contexts are arbitrary target code (in our case, arbitrary assembly code) that may not respect the compiler’s conventions.

In proving FA for PAC, we make two additional contributions. Our first contribution is a new trace-based proof technique for FA that can simultaneously handle dynamic memory sharing between modules and, importantly, reuses whole-program compiler correctness as a black-box to simplify the FA proof. FA proofs with dynamic memory sharing are difficult and (whole-program) compiler correctness is usually proved anyhow, so reusing it for proving FA reduces work. Technically, our proof is structured as a new 3-way simulation called $\textit{TrICL}$ (read “trickle”). We expect $\textit{TrICL}$ to be of interest beyond our PAC setting.

Second, to prove FA, we find it essential to reflect some structure of capabilities at the source-code level, forcing the programmer to take into account some of the machine-code-level expressiveness when reasoning about the source program. Interestingly, not doing this may also lead to subtle security vulnerabilities as the following example illustrates.

```c
static bool secret = true;

int branch_on_secret(int* p, int* q) {
    if (((int) p) != (int) q) return 0;
    if (secret) return p[1]; else return q[1];
}
```

Function `branch_on_secret()` first tests whether its two argument pointers are equal addresses. This equality implies that the dereference operations $p[1]$ and $q[1]$ both evaluate to the same value (or both fail). Hence `branch_on_secret()` is actually intended to not really leak any information about secret. However, a machine-code level adversary can call `branch_on_secret()` with two capabilities that both point to the same address but that have different bounds information. In that case, accessing $p[1]$ could fail, while accessing $q[1]$ succeeds and returns a value. Thus, the behavior of `branch_on_secret()` in that case does leak information about secret. More dangerously, it leaks this information in a way to which the source-level programmer is oblivious. The source-level programmer does not have (through pointer operations) a way to access bounds information, while this bounds information actually opens up an information channel. Hence, to prove our FA results (which includes the preservation of source-level contextual equivalence), we extend the source language to make pointers carry bounds information. This essentially makes explicit exactly what aspects of the target language programmers need to take into account to reason about the security about partial source programs.

A second property we require for FA is that machine code does not have direct access to the program counter capability. This is easily checked at link time. It guarantees that the target context cannot confuse a partial program by providing it a code capability where it expects a data capability (a behavior that does not exist in our source language).

Finally, as our last contribution, we implement our compiler for C by adding a “compartmentalizing compiler-pass” on top of the existing C-to-CHERI compiler (the non-compartmentalizing PAC compiler) [10, 12], and we evaluate the performance cost as well as the compatibility of the compartmentalizing PAC compiler with existing code.
In summary, we make the following contributions:

- We state and prove the security properties of a pointers-as-capabilities (PAC) compiler for partial programs for the first time. In doing so, we make substantial technical contributions:
  - the definition of a sound and complete trace semantics for a C-like language (Section III-A) and for a language with capabilities (Section III-B). Both languages feature a memory model that allows fine-grained dynamic memory sharing;
  - the definition of a compiler between the aforementioned languages that embodies the PAC design (Section IV);
  - a proof that the said compiler is fully abstract, with a new trace-based technique, $\mathcal{TrICL}$, that handles dynamic memory sharing and allows reuse of whole-program compiler correctness for $\mathcal{FA}$ (Section V);
- An implementation of our compiler on top of the existing C-to-CHERI compiler and a measurement of its efficiency and compatibility with existing C code (Sections VI and VII).

We have simplified some aspects of the technical work for presentation. The full details as well as proofs are available as an anonymous technical report [22]. The implementation of our compiler and the related benchmarks will be made available publicly upon the publication of this paper.

II. FULL ABSTRACTION AND A NEW PROOF TECHNIQUE

We briefly recap compiler full abstraction ($\mathcal{FA}$), outline at a high-level how it is proved, explain why a new proof technique is needed and what our new technique (TrICL) does.

A. Preliminaries

Given execution states $s, s'$ of a language, and a small-step reduction relation $\rightarrow$ (with a reflexive transitive closure $\rightarrow^*$), we denote by $s \rightarrow s'$ the judgment that state $s$ executes and transitions into state $s'$. We call a state stuck if there is no $s'$ such that $s \rightarrow s'$. We treat exceptions and silent divergence the same way we do stuck states, so, with slight abuse of terminology, we use the term diverging for executions that get stuck, silently reduce forever or end in exceptions.

Programs, initial & terminal states: In our simple imperative setting, a (partial) program is a list of modules, and a module is a list of functions. A program is called whole if its functions refer only to other functions also defined within the program. Otherwise, it is called partial. Linking of a pair of programs is denoted $\times$. We define linking noncommutatively because this simplifies some technicalities (see Appendix A). For the sake of exposition, we distinguish two parts $\mathcal{C}$ and $p$ of a linked program $\mathcal{C} \times p$ as the context and the program, suggesting that the latter is the program part of interest because it is the program that is or has been translated by our compiler. Notice however that each of $\mathcal{C}$ and $p$ may themselves consist of more than one module (i.e., more than one compilation unit). As usual, only whole programs with a main entry-point function can execute. We also use the more conventional notation $\mathcal{C}[p]$ for $\mathcal{C} \times p$.

The initial state of a program $p$ is denoted $\text{init}(p)$. A state $s$ is called terminal when it satisfies a special judgment $\vdash_t s$. If the execution of a program of interest $p$ in a certain context $\mathcal{C}$ reaches a terminal state, then we say the execution converges (or instead say the program converges). We denote convergence by $\mathcal{C}[p] \Downarrow$, which is shorthand for $\exists s. \text{init}(\mathcal{C}[p]) \rightarrow^* s \land \vdash_t s$. Next, we define contextual equivalence of (partial) programs.

Definition 1 (Contextual equivalence).

\[ p_1 \simeq_{\mathcal{ctx}} p_2 \triangleq \forall \mathcal{C}. \mathcal{C}[p_1] \Downarrow \iff \mathcal{C}[p_2] \Downarrow \]

B. Compiler full abstraction

A compiler $[\cdot]$ is fully abstract when it preserves and reflects contextual equivalence. The use of full abstraction to establish compiler security is standard [20, 23, 24].

Definition 2 (Full abstraction). The compiler $[\cdot]$ is FA if for all $p_1, p_2$:

(i) (Reflection) $[p_1] \simeq_{\mathcal{ctx}} [p_2] \implies p_1 \simeq_{\mathcal{ctx}} p_2$

(ii) (Preservation) $[p_1] \simeq_{\mathcal{ctx}} [p_2] \iff p_1 \simeq_{\mathcal{ctx}} p_2$

Condition (i) ensures that the compiler is non-trivial (a trivial compiler might compile semantically different programs to the same output program, which is forbidden by reflection). Reflection usually follows immediately from backward simulation, the standard formalization of the compiler’s whole-program correctness [25, 26].

Condition (ii) is the “security-relevant” direction of $\mathcal{FA}$ as it ensures that no extra distinguishing power is gained by target contexts as compared to source contexts. It implies the preservation of any security property that can be formalized as program equivalence (a notable example being noninterference for confidentiality). It is usually proved in the contrapositive: Assume $[p_1] \not\simeq_{\mathcal{ctx}} [p_2]$, and show $p_1 \not\simeq_{\mathcal{ctx}} p_2$. From the assumption, there must be a target context that distinguishes $[p_1]$ and $[p_2]$ (causes one to diverge and the other to converge). From this, we need to construct a source context that distinguishes $p_1$ and $p_2$. This construction of the source context is called back-translation in literature on $\mathcal{FA}$. There are two broad approaches to back-translation in literature: syntax-directed and trace-directed [24]. Here, we follow the trace-directed approach as we find it technically more convenient (we discuss syntax-directed approaches in Section VIII).

The key idea of the trace-directed approach is to characterize the observable behavior of partial programs via a labeled transition system that produces (finite) traces describing how a partial program interacts with its environment.\(^3\) This is done separately for source and target languages. Denote the set of traces of a partial program $p$ as $\mathcal{T}r(p)$. Next, define trace equivalence $\equiv_T$ of partial programs in source and target languages separately: Two partial programs (both source or both target) $p_1, p_2$ are trace equivalent, $p_1 \equiv_T p_2$, if $\mathcal{T}r(p_1) = \mathcal{T}r(p_2)$. We then prove three lemmas.

\(^3\)For the purpose of proving $\mathcal{FA}$, finite traces suffice, so “trace” in this paper always refers to a finite trace.
Lemma 1 (Soundness of target trace equivalence).
\[ p_1 \not\approx_{\text{ctx}} p_2 \implies \not\exists \alpha \in \text{Tr}(p_1) \]

Lemma 2 (Compilation preserves trace equivalence).
\[ p_1 \not\approx p_2 \implies \not\exists \alpha \in \text{Tr}(p_1) \]

Lemma 3 (Completeness of source trace-equivalence).
\[ p_1 \not\approx p_2 \implies \not\exists \alpha \in \text{Tr}(p_1) \]

The composition of these three lemmas immediately yields our goal, the contrapositive of condition (ii)! The important point is that only Lemma 2 bridges the two languages. Lemma 2 follows immediately from the following two lemmas, which together say that the compiler preserves and reflects traces of partial programs.

Lemma 4 (No trace is omitted by compilation).
\[ \alpha \in \text{Tr}(p) \implies \alpha \in \text{Tr}(\text{comp}(p)) \]

Lemma 5 (No trace is added by compilation).
\[ \alpha \in \text{Tr}(p) \iff \alpha \in \text{Tr}(\text{comp}(p)) \]

Lemma 4 follows directly from compiler (forward) simulation, which is needed to prove the compiler correct anyhow. Hence, this lemma does not add additional proof effort. On the other hand, Lemma 5 is an additional (and the last remaining!) proof burden. The “difficulty” of this proof really depends on the complexity of the traces, i.e., the complexity of interaction between a program and its context.

C. The Why and What of TrICL

In prior work that uses trace-based back-translation, program modules cannot share memory or the shared memory is fixed upfront [27, 28, 24]. Traces are easy to define in this setting. In our PAC setting, however, the memory shared between modules can grow dynamically as the program shares more of its previously private memory with the context by passing it corresponding capabilities (or pointers in the source). The context can also pass capabilities back to the program but it should only pass back those capabilities that it started with or those that it received previously during the execution (since capabilities cannot be forged by design). Consequently, any trace on which the context passes back a capability that it didn’t receive earlier should be removed from consideration as invalid. However, this notion of trace (in)validity is not straightforward (in either language) because capabilities can be passed not just directly via function arguments, but also indirectly, as something that’s reachable in the heap at the point of a transition. This makes valid traces on partial programs rather hard to define. To avoid this problem, we use a different definition of traces.

A new definition of traces: We start from whole programs, not partial programs. The trace is defined by decorating the small-step semantics of the whole program with information about interaction between its modules. We then define the traces of a partial program as the traces it can produce when linked with some (existentially quantified) context. The presence of the context automatically takes care of ensuring that all traces are valid, so we don’t have to define validity separately. (Validity of a trace now follows implicitly from easier-to-define invariants on the whole execution state.)

Definition 3 (Traces of a partial program).
\[ \text{Tr}(p) \overset{\text{def}}{=} \{ \alpha \mid \exists \mathcal{C}, s, \varsigma. (\text{init}(\mathcal{C}[p]), \emptyset) \xrightarrow{\alpha} (s, \varsigma) \} \]

(The \emptyset and \varsigma in this definition are auxiliary state components, that we describe in Section V. The reader can ignore them for now.) To prove Lemma 5 with this definition of traces, we must show that, given a trace \( \alpha \) of \( p_1 \), there is an emulating source context \( \mathcal{C}_{\text{emu}} \) such that \( \mathcal{C}_{\text{emu}} |_{p_2} \) produces \( \alpha \). In other words, we must back-translate a trace \( \alpha \) into a source context \( \mathcal{C}_{\text{emu}} \) and show that \( \mathcal{C}_{\text{emu}} |_{p_2} \) can actually produce \( \alpha \). For this, we would like to set up a simulation between the target and source runs. However, this simulation can be very hard to set up because we constructed the emulating source context without considering the given target context (just from the trace prefix) and, hence, there can be differences between the internal behaviors of the emulating context and the given target context, e.g., in the specific order of updates to the shared memory, and in the internal function calls. These differences mean that the memory (and the call stack) do not remain in sync between the emulating source context and the given target context and our simulation relation needs to accommodate the big gap between the internal states of the target contexts and the emulating source contexts. We call this the “vertical gap”.

TrICL: This is where our new TrICL technique comes in. TrICL introduces a third “mediator” run to the simulation, namely, that induced by the compilation of the whole source program containing both \( p_2 \) and \( \mathcal{C}_{\text{emu}} \). Overall, the three runs are (i) the given (target) run of \( \mathcal{C} |_{p_1} \), where \( \mathcal{C} \) is the target context that induces the trace \( \alpha \), (ii) the emulating source run of \( \mathcal{C}_{\text{emu}} |_{p_2} \), and (iii) the mediating target run of \( \mathcal{C}_{\text{emu}} |_{p_2} \).

Introducing the mediator run simplifies the proof as follows. Because the mediator \( \mathcal{C}_{\text{emu}} |_{p_2} \) is obtained by a whole program compilation of the emulator \( \mathcal{C}_{\text{emu}} |_{p_2} \), we can reuse whole-program compiler correctness as a black box to immediately reduce the problem of showing that the emulating (source) run emulates the given (target) run to that of showing that the mediating (target) run emulates the given (target) run. This reduced problem is now about two runs in the same language (target), which enables us to write a big part of our simulation invariants as same-language (instead of cross-language) invariants. In other words, this simplifies the “vertical gap”.

Section V explains the TrICL simulation for our PAC compiler, but we note that the idea is general and should apply to other settings as well.

III. Source and Target Languages

Next, we introduce our source and target languages, ImpMod (Section III-A) and CHERIExp (Section III-B).
A. ImpMod: The source language

To keep our focus limited to the “pointers to capabilities” aspect of the translation, we design ImpMod and CHERIExp to be pretty close except in how they deal with memory. For example, ImpMod features only unstructured control flow in the form of a JumpIfZero instruction. But it still has functions and modules. In fact, modules are crucial. They are units of memory isolation, which makes programs interesting for security. Briefly, 1) every module gets its own module-global variables; every function inside the module can access these variables, whereas any function external to the module cannot access these variables directly by default, and 2) every module gets its own data stack on which it stores the frames of live (ongoing) calls to its functions.

For example, Listing III.1 shows a module (Main) with two module-global variables, ibuffer and secret (lines 4 & 5). All of the main function\(^4\) and the secret-handling functions read.secret, encrypt and decrypt are defined in the same module, and thus can each access the variables ibuffer and secret. The function send.rcv, in contrast, is external: it is defined in the Networking module which is presumably untrusted. On line 11, the Networking module gains access to ibuffer. The Networking module, however, does not gain access to secret through &ibuffer—so long as the other trusted functions read.secret and encrypt also make sure not to copy (pointers to) secret into the ibuffer. Any attempt to increment and access the pointer &ibuffer beyond the array bounds gets stuck. To understand how we model these bounds checks, we introduce the expression semantics and the memory model of ImpMod.

1) Expressions and memory model of ImpMod: Expressions are denoted with \(e\) and do not update memory.

\[
e ::= \mathbb{Z} | \text{VarID} | e \oplus e | e[e] | \&\text{VarID} | \&e[e] | \ast(e) | \text{start}(e) | \text{end}(e) | \text{offset}(e) | \text{capType}(e) | \text{limRange}(e, e, e)
\]

\[
V ::= \mathbb{Z} | \text{Cap}
\]

Base expressions are integers \(\mathbb{Z}\) and variable identifiers \text{VarID} (both local and global). Binary arithmetic expressions are generically denoted with \(e \oplus e\). Pointer and array expressions are: 1) the array-offset expression \(e[e]\), 2) the ampersand (address-of) operator of the forms \&\text{VarID} and \&e[e], and 3) the star (pointer dereference) operator \(\ast(e)\). The intuitive meanings of these expressions are as in C. Moreover, there are low-level expressions that are necessary for reflecting the target memory model in the source language (as mentioned in Section I): four getters: \text{start}(e), \text{end}(e), \text{offset}(e), and \text{capType}(e); and a setter: \text{limRange}(e, e, e). These low-level expressions operate on a capability-based representation of memory addresses that we explain next.

Addresses in ImpMod are represented as capabilities. Thus, built into the memory model is a type \text{Cap} for capabilities that is distinct from integers \(\mathbb{Z}\). Hence, run-time values \(V\) are integers \(\mathbb{Z}\) or capabilities \text{Cap}. A memory \(\text{Mem} : \mathbb{Z} \rightarrow \text{Cap}\) is a finite map from addresses to values. Note that the range of a memory may contain capability values. Concretely, we define the type \text{Cap} of capability values as \(\text{Cap} \stackrel{\text{def}}{=} \{\kappa, \delta\} \times \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}\). The first field of the capability value indicates its type: code (\(\kappa\)) or data (\(\delta\)). The getter of this field is the expression \text{capType}. The next three (integer) fields are respectively the \text{start}, \text{end}, and \text{offset} of a capability. The start and end identify the memory region on which this capability authorizes a code (\(\kappa\)) or data (\(\delta\)) access operation. The offset designates the address at which an access operation is performed. The offset should be within range (checked by Rule Eval-star below).

The semantics of expressions are defined in Figure 1 by the judgment \(e \vdash v\). Notice that expressions in ImpMod do not have side effects. The evaluation context for expressions consists of:

1) syntactic information about the program given by the function definitions \text{Fd}, the declarations of module-global variables \text{MVars}, and the layout and bounds \(\beta\) of all the program’s variables;

2) load-time information about the program given by the per-module data-segment-location \(\Delta\), and the per-module local-stack-location \(\Sigma\); and

3) execution-state information given by the memory \text{Mem}, the stack pointers \(\Phi\) of the module-local stacks and the program counter \(\text{pc}\).

We make two observations on the rules of Figure 1. First, Rules Eval-amp-local-var and Eval-amp-module-var are the only two rules (of expression evaluation) where a capability value originates. In fact, this capability value originates out of thin air. This behavior (the minting of a capability value during the course of the program execution) is precisely why this source semantics is not really executable as is on a capability architecture. On a capability architecture, a program does not have any means to forge a capability value. Instead, it can only refer to capability registers\(^6\). (Contrast these two rules to Rules evaldc and evalstc of Section III-B.) Replacing the

\(^4\)Notice in the main function on line 8 the use of variable \text{iobuffer}; the syntax for l-values is more explicit than in C.

\(^6\)It can also get capabilities by calling the memory allocator, which in turn will place in a register (or in the program’s memory) a new capability authorizing access to the allocated region.
mentions of program variables with expressions that mention capability registers is thus the key role of a PAC compiler.

Second, Rule Eval-star performs a bounds check before it loads a value from memory. Building-in this check is how we define ImpMod to be spatially safe by relying on the capability-based memory model. A similar bounds check is performed at store time as well (see rule Assign-to-var-or-arr of the Assign command in fig. 8).

2) Commands and execution state of ImpMod: Commands of ImpMod are denoted with Cmd. Unlike expressions, they can modify memory and other parts of the execution state.

\[
\text{Cmd ::= Assign } e_1 \ e_r \mid \text{Alloc } e_1 \ e_{\text{size}} \mid \text{Call } \text{fid } \varepsilon \mid \text{Return} \mid \text{JumpfZero } e_2 \ e_{\text{off}} \mid \text{Exit}
\]

The commands should be self-explanatory (fid is a function name). An execution state \( s \) defined \( \langle \text{Mem}, \Phi, \text{pc}, \text{stk}, \text{nalloc} \rangle \) of a program in ImpMod consists of the memory Mem, the stack pointers \( \Phi \) of the module-local stacks, the program counter pc, a trusted control stack stk—which is shared among all modules of a program—and the memory-allocation status represented by the next-free-address nalloc. The space for dynamic memory allocation (i.e., the heap) is also shared by all the program modules (like the trusted control stack). The purpose of modeling a control stack that is trusted and hence separate from the accessibile memory is that we want to rule out all ill-formed control sequences. No command in ImpMod can write directly to this control stack.

The semantics are small step and use the judgment \( \rightarrow \subseteq s \times s \) that is indexed with an evaluation context \( \text{Fd, MVar, } \beta, \Delta, \Sigma \) and an allocation limit \( \nabla \). The allocation limit (\( \nabla \)) is the maximum possible size of the shared heap.

B. CHERIExp: The target language

Our target language CHERIExp is, like ImpMod, an imperative language with modules. However, unlike ImpMod, it does not feature variables. Instead, it only features “capability registers” and integers as base expressions. The role of the compiler from ImpMod to CHERIExp is to implement operations on source pointers by using operations on capability registers. The memory model is like that of ImpMod (Section III-A1), i.e., it is capability based. Through this capability-based memory model and capability registers, CHERIExp models a slightly abstracted and simplified version of CHERI assembly [5].

1) Expressions and commands in CHERIExp:

\[
e ::= \varepsilon \mid \text{getddc} \mid \text{getstc} \mid e \oplus e \mid \text{inc}(e, e) \mid \text{deref}(e) \mid \text{start}(e) \mid \text{end}(e) \mid \text{offset}(e) \mid \text{capType}(e) \mid \text{limRange}(e, e, e)
\]

Expressions are denoted e. The language has three named capability registers: stc, ddc and pcc. The names of these registers hint at their recommended usage: ddc stands for default data capability while stc stands for stack capability. Our compiler uses ddc as a capability on the per-module data segment, and stc on the per-module stack. The third capability register, pcc, holds the program counter capability which points to the current command and allows the execution of commands. The getter expressions getddc and getstc immediately return the current value of the respective capability registers, ddc and stc. (See rule evalddc in Figure 2.) However, pcc cannot be read in CHERIExp. This is a simple way of enforcing the restriction (mentioned in Section I) on linking with contexts that mention the pcc register. No expression allows the fabrication of an arbitrary capability, thus enforcing capability unforgeability.

In CHERIExp, one can increment (decrement) the offset of a capability by an arbitrary integer value (see rule Eval-inc), resulting in a new capability that nevertheless has the same bounds as the original. The check that the offset lies within bounds is only performed at use time (e.g., use by means of a deref expression). Observe that rule Eval-deref performs the same bounds-check on the capability value that rule Eval-star (of ImpMod) performs. \( \text{limRange}(e_1, e_2, e_3) \) restricts the lower- and upper-bounds of the capability \( e_1 \) to the interval \( [e_2, e_3] \), returning a new capability.

Commands in CHERIExp are the same as commands in ImpMod (modulo expressions). Briefly, the small-step command reduction is denoted with \( \rightarrow \subseteq s \times s \) where a state \( s \) of a CHERIExp program, like in ImpMod, consists of a memory Mem, an allocation status nalloc and a trusted stack stk, but unlike ImpMod also consists of three capability registers: ddc, stc and pcc, and a map mstc holding a per-module capability authorizing access to the module’s local data stack (the local data stack is part of the memory Mem).

Note that we model both a trusted stack (and hence a secure calling convention), and a separate map for the per-module data-stack capability in CHERIExp. This built-in segregation may sound too abstract for a target language that already has low-level elements like capability registers. However, this modeling choice allows us to focus only the PAC principle for program variables and not worry about the integrity of the stack pointer. (Prior work has already shown how compilers can enforce well-bracketed control flow and stack encapsulation using capabilities [16].)

IV. OUR PAC COMPILER

Our ImpMod to CHERIExp compiler (\( \langle \rangle \)) translates pointers to capabilities (the PAC principle). In this section, we present its crucial bits, namely, the translation of ImpMod expressions to CHERIExp expressions. The translations of commands (denoted \( \langle \rangle \)) and of modules (\( \langle \rangle \)) are rather trivial due to the similarity of the syntax of the source and target commands and modules, so we elide those.

The translation of expressions \( \langle \rangle : e \rightarrow e \), whose excerpts are presented below, is indexed by the syntactic information \( \text{fid}, \text{modID} \) (function id and module id) providing the scope of the expression being translated, and \( \beta \), giving the layout and bounds of source variables.

\[
\begin{align*}
\langle z \rangle & \text{ def } \equiv z \\
\langle e_1 \oplus e_2 \rangle & \text{ def } \equiv \langle e_1 \rangle \text{ fid}, \text{modID}, \beta \oplus \langle e_2 \rangle \text{ fid}, \text{modID}, \beta
\end{align*}
\]
(Eval-amp-var)
\[(\text{fid}, _) = \text{pc} \land \text{vid} \in \text{localIDs}(\text{Fd}(\text{fid})) \cup \text{args}(\text{Fd}(\text{fid})) \land \text{mid} = \text{moduleID}(\text{Fd}(\text{fid}))\]
\[
\beta(\text{vid}, \text{fid}, \text{mid}) = [\text{st}, \text{end}] \\
\phi = \Sigma(\text{mid}).1 + \Phi(\text{mid})
\]

(\text{Eval-amp-module-var})
\[(\text{fid}, _) = \text{pc} \land \text{vid} \notin \text{localIDs}(\text{Fd}(\text{fid})) \cup \text{args}(\text{Fd}(\text{fid})) \land \text{mid} = \text{moduleID}(\text{Fd}(\text{fid}))
\]
\[
\text{vid} \in \text{MVar}(\text{mid}) \\
\beta(\text{vid}, 1, \text{mid}) = [\text{st}, \text{end}] \\
\]}

(\text{Eval-amp-arr})
\[
\&e_{\text{arr}} \downarrow (\delta, \text{st}, \text{end}, \text{off}) \land e_{\text{arr}} \downarrow (\text{vid}, \text{off} + \text{off}^{'}) \land \text{off}^{'}, e \in \mathbb{Z} \\
\&e_{\text{arr}}[e_{\text{arr}}] \downarrow (\delta, \text{st}, \text{end}, \text{off} + \text{off}) \\
\]}

(\text{Eval-star})
\[
\&e_{\text{arr}} \downarrow (\delta, \text{st}, \text{end}, \text{off}) \land e \downarrow (\delta, \text{st}, \text{end}, \text{off}) \\
\\ \\
\]}

\[
\mathbb{Z}\downarrow (\text{st} \leq \text{st} + \text{off} < \text{end})
\]

\[
*\downarrow (\text{Mem}(\text{st} + \text{off}))
\]}

Fig. 1: (Excerpt) Evaluation of expressions in \text{ImpMod}. The evaluation relation \downarrow is indexed with an evaluation context \text{Fd}, \text{MVar}, \beta, \Delta, \Sigma, \text{Mem}, \Phi, \text{pc}, which we elide for brevity.

(\text{Eval-dcc})
\[
\text{Mem}, \text{ddc}, \text{stc}, \text{pcc} \vdash \text{getddc} \downarrow \text{ddc}
\]

(\text{Eval-inc})
\[
\text{Mem}, \text{ddc}, \text{stc}, \text{pcc} \vdash e \downarrow (x, \text{st}, \text{end}, \text{off})
\]

(\text{Eval-deref})
\[
\text{Mem}, \text{ddc}, \text{stc}, \text{pcc} \vdash e \downarrow (\delta, \text{st}, \text{end}, \text{off}) \land \text{st} \leq \text{st} + \text{off} < \text{end}
\]

(\text{Eval-limitRange})
\[
\text{Mem}, \text{ddc}, \text{stc}, \text{pcc} \vdash \text{deref}(e) \downarrow \text{Mem}(\text{st} + \text{off})
\]

\[
\text{Mem}, \text{ddc}, \text{stc}, \text{pcc} \vdash \text{limitRange}(e_1, e_2, e_3) \downarrow (x, \text{st}', \text{end}', 0)
\]

Fig. 2: (Excerpt) Evaluation of expressions in \text{CHERIExp}.

\[
\{\&\text{vid}\}_{\text{modID}, \beta} \overset{\text{def}}{=} \text{limitRange(getddc,}
\]

\[
\text{start(getddc) + st,}
\]

\[
\text{start(getddc) + end)}
\]

\[
\text{when } \beta(\text{vid}, 1, \text{modID}) = (\text{st, end})
\]

\[
\{\&\text{e}_{\text{arr}}\}_{\text{modID}, \beta} \overset{\text{def}}{=} \text{let } s = \text{getstc in}
\]

\[
\text{let so = start}(s) + \text{offset}(s) \text{ in}
\]

\[
\text{limRange}(s, \text{st + so}, \text{end + so})
\]

\[
\text{when } \beta(\text{vid}, 1, \text{modID}) = (\text{st, end})
\]

\[
\{\text{vid}\}_{\text{modID}, \beta} \overset{\text{def}}{=} \text{deref(}\{\&\text{vid}\}_{\text{modID}, \beta})
\]

\[
\{\&\text{e}_{\text{arr}}\}_{\text{modID}, \beta} \overset{\text{def}}{=} \text{inc(}\{\&\text{e}_{\text{arr}}\}_{\text{modID}, \beta}, \{\&\text{e}_{\text{arr}}\}_{\text{modID}, \beta})
\]

\[
\{\text{e}_{\text{arr}}\}_{\text{modID}, \beta} \overset{\text{def}}{=} \text{deref(}\{\&\text{e}_{\text{arr}}\}_{\text{modID}, \beta})
\]

Translating expressions \text{start}, \text{end}, \text{offset}, \text{capType}, and \text{limRange} is straightforward and is similar to \{e_1 + e_2\}. As an example, we show the translation of Line 11 of Listing III.1.

\[
\langle \text{Call send_rcv(\&iobuffer)} \rangle
\]

\[
= \text{Call send_rcv (limitRange(}
\]

\[
\text{ddc, start(ddc) + 0, start(ddc) + 512)}
\]

Note that the compiler uses the bounds information that is given in the text of the program in the declaration of the array (Line 4) to introduce explicit curbing (using \text{limRange}) of the \text{ddc} capability so that the resulting capability is of the same size as the declared array size (512) and not bigger.

The curbing prevents the external function \text{send_rcv} from offsetting the capability beyond the span of \text{iobuffer} and from accessing other variables like \text{secret}. This curbing is, in fact, essential for full abstraction since a similar out-of-bounds access is prohibited in the source semantics.

V. PROVING THE COMPILER SECURE

We prove our compiler fully abstract following the steps in Section II. We fill in details of two of the steps here: The definition of traces and the proof of Lemma 5 using \text{TfHCL}.

A. Traces

As explained in Section II, we define traces by augmenting the small-step semantics of whole programs (Section III) with labels that capture information about the interaction between the program \text{p} of interest and its context. In our two languages, such an interaction happens only through shared memory or function arguments in only those steps that transfer control from the program to the context or vice-versa. Accordingly, a trace label arises only at such \text{border-crossing} control transfer steps, and the label records the shared memory, and the function arguments as in Laird [29]. In the following, we give a formal account of this development for the target language.
The labeled trace-step relation for CHERIExp is written \( \lambda_p \). The relation relates two trace states and a label \( \lambda \). A trace state \( (s, \varsigma) \) extends the normal execution state \( s \) with auxiliary information \( \varsigma \), which is the set of memory addresses shared so far (i.e., from the initial state and up until execution state \( s \)) between the program of interest \( p \) and the context. This auxiliary information is used to define an informative trace label \( \lambda \). A trace \( \alpha \) is a finite list \( \lambda \) of labels. Trace labels \( \lambda \) (of both our languages) have the following forms:

- A silent label \( \tau \) abstracts over any execution step that is internal to either the program \( p \) or the context.
- A termination label \( \checkmark \) indicates that a terminal execution state was reached. (Once a \( \checkmark \) appears, it re-appears in all subsequent trace labels.)
- An input call label \( \text{call}(fid) \) \( \tau \) ? \( Mem, \text{malloc} \) | \( \text{ret} \) ! \( Mem, \text{malloc} \) | \( \text{call}(fid) \) \( \tau \) ! \( Mem, \text{malloc} \)

\[ \lambda := \tau | \checkmark | \text{ret} ? \text{Mem, malloc} | \text{ret} ! \text{Mem, malloc} | \text{call}(fid) \tau ? \text{Mem, malloc} | \text{call}(fid) \tau ! \text{Mem, malloc} \]

A silent label \( \tau \) abstracts over any execution step that is internal to either the program \( p \) or the context.

- An input call label \( \text{call}(fid) \) \( \tau \) ? \( Mem, \text{malloc} \) indicates that at an execution state where the shared memory was \( Mem \), and the memory allocator state was \( \text{malloc} \), the context called the program’s function \( fid \) with the list of values \( \tau \) as arguments.
- An output call label \( \text{call}(fid) \) \( \tau \) ! \( Mem, \text{malloc} \) is similar to an input call label but the call goes in the opposite direction: the program called the context’s function \( fid \).
- An input return label \( \text{ret} ? \text{Mem, malloc} \) indicates that at an execution state where the shared memory was \( Mem \), and the allocator state was \( \text{malloc} \), the context returned to the program.
- An output return label \( \text{ret} ! \text{Mem, malloc} \) is similar except that the program returned control to the context.

Figure 3 shows the input return rule of \( \lambda_p \). Note that the third and fourth premises check that program counter capability \( pcc \) belong to \( p \)’s code memory after the transition but not before, implying that this is a border crossing from the context into the \( p \). Similar checks exist in other rules.

Next, given a reduction sequence of labeled steps, we drop all \( \tau \) labels from it, and concatenate the non-\( \tau \) labels into a trace \( \alpha \), writing \( (s, \varsigma) \xrightarrow{\lambda_p} (s’, \varsigma’) \) for the resulting steps.

Next, we define the traces of a partial program \( p \) as in Definition 3. We note that all traces are alternating in \( \text{"!"} \) and \( \text{"!"} \).

**Fact 1** (Traces are alternating). \( \alpha \in Tr(p) \implies \alpha \in Alt\checkmark^* \) where \( Alt \) \( \overset{\text{def}}{=} \{ (?|\epsilon) (|?)^* (|\epsilon) \} \text{ and } ? \) is the set of \( ? \)-decorated labels, and similarly for \( ! \).

**B. Proof of Lemma 5 using TrICL**

Lemma 5 assumes a trace \( \alpha \) for \( [[p_\alpha]] \) (in some target context, say, \( C \)) and requires constructing a source emulating context \( C_{emu} \) such that \( C_{emu}[[p_\alpha]] \) also has \( \alpha \). For this, we back-translate \( \alpha \) to a source context. We illustrate the back-translation through an example. Figure 4 shows one trace that is emitted by the compiled version of the module Main of Listing III.1.

The compilation of the first three commands generate \( \tau \) steps, which are dropped from traces. The next two non-\( \tau \) labels (shown in the example) are interesting:

1) The function call \( \text{send_rcv}(&\text{ioBuffer}) \) on Line 11 is border crossing, so its compilation emits an output call label which contains the callee function id (\( \text{send_rcv} \)), the argument to the call (the \( \delta \)-capability representing the translation of the pointer \( &\text{ioBuffer} \)), the direction of the call (\( ! \) denoting output, i.e., program-to-context), a snapshot of memory shared so far (namely, the contents of the array \( \text{ioBuffer} \)), and the value \(-1\) denoting the first heap address (the heap grows towards negative addresses in our semantics).

2) The target context (in which our compiled program executes) returns control to the compiled program, in this case, after zeroing out the contents of the shared memory. This emits an input return label.

Lemma 5 requires showing that exactly the same trace can be emitted by the source program in some source context. The proof of Lemma 5, therefore, requires us to construct such a source context, which we call the emulating context. The emulating context depends on the trace.\(^7\) Listing V.1 shows an emulating context for our example trace.

---

\(^7\) In principle, it could also depend on the target context, but this is usually not required. We also don’t use the target context.
This emulating source context consists of two modules, \textit{Networking}, which implements the API function \texttt{send\_rcv}, and \textit{HelperBackTranslation}, which implements helper functions and maintains metadata. We show just one example of such a helper function, namely \texttt{mimicMemory\_1()}. \texttt{mimicMemory\_1()} is called (on Line 11) by \texttt{send\_rcv()}. It zeroes out the IO buffer (to mimic the shared memory in the second action of the given target trace). The IO buffer is accessed by \texttt{mimicMemory\_1()} through the pointer stored in the global variable \texttt{arg\_store\_0\_send\_rcv\_0} (Line 18). This pointer is stored (not shown) by the function call \texttt{saveArgs\_send\_rcv\_1(iob\_ptr)} on Line 8.

We briefly explain what each helper function does. First, the context emulating a given trace defines a different set of helper functions for every position on the trace. The index of the corresponding trace label appears in the identifier of a helper function (for example, \texttt{mimicMemory\_1()}, \texttt{saveSnapshot\_0()}). To explain the helper functions, we follow the body of \texttt{send\_rcv(iob\_ptr)} line by line. In the beginning, the call to \texttt{readAndIncrementTraceIdx} keeps track of the current position in the trace. This knowledge of the current position in the trace is not used in our toy example, but it would be used if the API function (\texttt{send\_rcv} in this case) were called \textit{at more than one position} in the given trace; at each such position, we would use this knowledge to call the corresponding helper functions (e.g., \texttt{mimicMemory\_3()} instead of \texttt{mimicMemory\_1()}), which would copy to the shared memory the values that appear in trace position 3 instead of 1).

Next, on Line 8, we store the pointer \texttt{iob\_ptr} in a global variable by calling the \textit{HelperBackTranslation} module because we may need it to simulate a future trace position, not just the current call to \texttt{send\_rcv}. For the same reason, we save (on Line 9) a snapshot of the whole shared memory in global variables \texttt{snapshot\_0\_\sigma} to \texttt{snapshot\_0\_\sigma + 511}.

Next, on Lines 10 to 12, the actual emulation of the trace action at trace position 1 is done. In our example, \texttt{doAllocations\_1()} would do nothing because \texttt{nalloc} in Figure 4 does not change. Importantly, \texttt{mimicMemory\_1()} writes all the values (the zeros) to the shared memory before \texttt{send\_rcv} eventually returns, thus mimicking the given target trace.

\textit{The difficult simulation}: Returning back to the general picture of Lemma 5, after we have constructed the emulating source context from the given trace \(\alpha\), we must prove that the source program and this context emulate the given target trace. As explained in Section II, for this, we would like to set up a simulation between the target and source runs but this simulation can be very hard because there can be differences between the \textit{internal} behaviors of the emulating context and the given target context, e.g., in the specific order of updates to the shared memory, and in the internal function calls. Indeed, the target context likely does not use the kind of helper functions our emulating source context does! Our simulation needs to accommodate this “vertical gap”.

To add to this difficulty, we do want to simulate through (the \texttt{!}-decorated) execution steps of the program of interest \((p_3\text{ in Lemma 5})\). And since our compiler compiles it, this simulation would be very similar to what we would have to do anyhow just to prove our compiler correct, not fully abstract. So, we would like to “reuse” this part.

\textit{TrICL to the rescue}: This is where our new \textit{TrICL} simulation comes in. \textit{TrICL} introduces a third “mediator” run to the simulation, namely, that induced by the \textit{compilation of the whole source program} \([\mathcal{C}_{\text{emu}}[p_3]]\), containing both \(p_3\) and our emulating context, say, \(\mathcal{C}_{\text{emu}}\). As explained in Section II, this simplifies the proof because we can “reuse” whole-program compiler correctness as a black box to immediately reduce the problem of showing that the \textit{emulating} run emulates the given run to that of showing that the \textit{mediating} run emulates the given run. The emulating and mediating runs are in the same language, so this reduces the difficulty of the “vertical gap”.

Finally, to show that the mediating run emulates the given run in the target language, we rely on an alternating simulation in the target language that uses two different relations between the two runs – a \textit{strong} relation \(\approx_{[p_3]}\), which holds while the compilation of the program of interest executes, and a \textit{weak} relation \(\sim_{[p_3]}\), which holds while the contexts \(\mathcal{C}\) (which produced \(\alpha\)) and \([\mathcal{C}_{\text{emu}}]\) execute. The need for two relations will become clear shortly.

\textit{TrICL formally}: Formally, \textit{TrICL} is a relation between three trace states (a source emulating state \(s_{\text{emu}}\), a target mediating state \(s_{\text{med}}\) and a target given state \(s_{\text{given}}\) of the three runs explained above) that agree on the memory shared between the context and the program of interest. \textit{TrICL} is indexed by a trace \(\alpha\) that we are trying to emulate and a position \(i\) of that trace. The relation requires that (1) \(s_{\text{emu}}\) and \(s_{\text{med}}\) are related by the whole-program compiler correctness relation \(\approx_{[p_3]}\), (2) the source state satisfies an \textit{emulation invariant}, which basically captures that the construction of \(\mathcal{C}_{\text{emu}}\)
is indeed an emulation of the input steps of the trace $\alpha$, e.g., that the functions mimicMemory_1() and doAllocations_1(), etc. indeed emulate the input trace action $\alpha(1)$, and (3) the two target states $s_{\text{med}}$ and $s_{\text{given}}$ are related by the strong relation $\approx_{[p_1]}$ when execution is in $\lfloor p_3 \rfloor$ and by the weak relation $\sim_{[p_1]}$ when execution is in the contexts.

**Definition 4** (Trace-Indexed Cross-Language (TrICL) alternating simulation relation).

\[
\text{TrICL}(s_{\text{emu}}, s_{\text{med}}, s_{\text{given}}, \alpha, i, p) \overset{\text{def}}{=} s_{\text{emu}} \approx_{p_1} s_{\text{med}} \land \text{emulate_invariants}(s_{\text{emu}})_{\alpha, i, p} \\
\land (\alpha(i) \in \mathbb{I} \implies (s_{\text{med}}, \varsigma) \approx_{[p_1]} (s_{\text{given}}, \varsigma)) \\
\land (\alpha(i) \in \mathbb{O} \implies (s_{\text{med}}, \varsigma) \sim_{[p_1]} (s_{\text{given}}, \varsigma))
\]

Given this definition, we come to the key step of our proof, namely, that TrICL is an invariant.

**Lemma 6** (TrICL step-wise alternating backward-simulation).

\[
\alpha \in \mathbb{I} \land \text{TrICL}(s_{\text{emu}}, s_{\text{med}}, s_{\text{given}}, \alpha, i, p) \land (s_{\text{given}}, \varsigma) \overset{\alpha(i)}{\approx}_{[p_1]} (s_{\text{given}}, \varsigma') \\
\implies \exists s_{\text{emu}}, s_{\text{med}}, s_{\text{given}} \overset{\alpha(i)}{\approx}_{[p_1]} (s_{\text{med}}, \varsigma') \land (s_{\text{med}}, \varsigma) \overset{\alpha(i)}{\approx}_{[p_1]} (s_{\text{med}}, \varsigma') \land \\
\text{TrICL}((s_{\text{emu}}, s_{\text{med}}, s_{\text{given}}), \alpha, i, i+1, p)
\]

Before sketching the proof of this key lemma, we explain how the proof reuses the backward- and forward-simulation lemmas that are anyhow needed for compiler correctness:

- When control is in the contexts (case $\alpha(i) \in \mathbb{I}$ of Definition 4), we know by the emulation invariant that the source context $C_{\text{emu}}$ emulates the next action of the trace $\alpha$ (the emulation invariant holds of the construction shown in the example earlier), and we use forward simulation to argue that the mediating context, which is just the compilation of this source context, does the same.

- Dually, when control is in the program of interest (case $\alpha(i) \in \mathbb{O}$ of Definition 4), we know by the precondition of Lemma 6 that $\lfloor p_3 \rfloor$ produces the next action of the given trace $\alpha$, hence (by strong similarity that we explain below) $\lfloor p_2 \rfloor$ also produces the same action in the mediator trace. But now from knowing that an action of the mediator trace was produced, we use backward simulation to reason that the source program $p_1$ does the same action.

This “reuse” of the (forward- and backward-) simulations that are anyhow needed for compiler correctness is the simplification that TrICL affords. The proof of the two interesting cases of Lemma 6 is given in Appendix B.\(^8\)

\(^8\)The proof relies on forward- and backward-simulation, and also on key properties of the strong and weak similarities and of the emulate invariants. All of these properties (Lemmas 7 to 14) are given in Appendix B, and are proved in the TR [22].
runs, and in particular in the two target runs, which completes the strengthening proof.

Figure 5 depicts the alternating nature of the strong- and weak-similarity relations, and the strengthening and weakening at border crossings. The two traces are depicted as two horizontal sequences of states/transition pairs. The black lines that connect states from opposite traces show the nature of the simulation condition: option simulation (Lemma 11) is possible for weak similarity (the single black line), while for strong similarity (the double black line), only lock-step simulation (Lemma 9) is possible.

VI. IMPLEMENTATION IN A COMPILER FROM C TO CHERI

We have implemented the key ideas of Section IV in a compiler from C to CHERI. CHERI comes with a Clang/LLVM compiler [10] that already implements PAC, but no isolation for modules, which ImpMod has. Our compiler enforces this isolation via a C-to-C source pre-processing transform. The compiler relies on libcheri, CheriBSD's library for building, invoking and loading sandboxes—isolated units of computation with their own code and memory, which is private until explicitly shared. libcheri relies on CHERI's underlying support for object capabilities to provide sandboxes [31]. The important thing from our perspective is that libcheri requires the programmer to manually group functions into classes (the equivalent of modules), and to annotate functions to make them use object capabilities instead of the standard calling conventions (attribute cheri_ccall) and to specifically annotate functions that are exported from a class (attribute cheri_ccallee). Additionally, all initialization functions must be added to classes that call external functions.

Our source-to-source transform automates all this: It maps C modules (compilation units) to libcheri's sandboxes to isolate them from each other and automatically inserts all the required annotations. Examples of programs output by our transform are shown in the Appendix (Listings A.1 to A.5). To initialize sandboxes, we had to make some significant changes to libcheri as well. Briefly, libcheri requires a second phase of runtime linking to resolve cross-sandbox references. We implement this through a new recursive initialization function, called once before main, which loads and initializes all sandboxes that main depends on transitively, creates relevant object capabilities, and links the modules.

VII. EXPERIMENTAL EVALUATION

We evaluate the overheads of our proof-of-concept isolation scheme using four large open-source C libraries that we chose carefully for heterogeneity and compiled with the compiler: zlib [32], LibYAML [33], GNU-barcode [34], and libpng [35].

Code changes: We had to make some manual code changes to address incompatibilities between our scheme and the existing toolchain. These incompatibilities are: (a) Moving local variables whose addresses are shared with other modules to the heap; this is a fundamental limitation of CHERI, not specific to our scheme. (b) Explicitly sharing pointers to exported global variables of a module; this can be automated with further work on the CHERI linker. (c) In the existing toolchain, a function pointer always compiles to an execute capability. Our isolating scheme requires object capabilities for cross-module calls. To compensate for this incompatibility, we had to make manual changes to cross-module calls using function pointers.

Table 1 summarizes the magnitude of our changes on the various benchmarks. Overall, the changes are quite limited.

Performance overheads: The most significant performance overhead in our scheme comes from cross-module function calls, where we use object capabilities. The number of such calls is, of course, a characteristic of the application. To isolate this overhead, we compare the execution times of calls to library functions compiled with our scheme to those compiled with the vanilla CHERI compiler, which offers no module isolation and, hence, no security. We perform our experiments on a single-core CHERI VM implementing CHERI ISA version 5 [36] running our modified version of CheriBSD. This VM was hosted in another VirtualBox [37] VM with FreeBSD 9.1, which, in turn, runs on an arch-Linux host. The physical machine has a 4 core Intel Core i7-6700 CPU with 16 GB of RAM, of which 512 MB are allocated to the CHERI VM.

Figure 6 summarizes our results. The y-axes are execution times. The x-axes are log-scale. The lines “secure” and “vanilla” correspond to our scheme and vanilla CHERI, respectively. All times are averages of 10 runs. Standard deviations were all below 0.3s with the exception of the two longest LibYAML experiments, where they were 1.5s in each case. A small, constant delay of about 0.2s associated with the sandbox loading routines is included in both the baseline and the secure versions. We do not show the benchmark GNU-barcode as it takes very small inputs only.

Our overhead relative to vanilla CHERI is negligible for zlib as this benchmark has very few cross-module calls (43–441 as the payload size varies). In LibYAML, the number of cross-module calls grows linearly with input size, so the ratio of the two lines is nearly constant (our relative overhead is consistently between 35 and 40%). In libpng, the number of cross-module calls is constant but large (approx. 125,000), so the difference between the two lines is nearly constant and easily perceivable. Overall, these observations are in line with our expectation that the performance overhead of our isolation
A recent line of work proposes alternate criteria for compiler security, based on preservation of classes of properties and hyperproperties in the presence of adversarial contexts [45, 27, 47, 48]. Many of these criteria are incomparable to full abstraction, while some are stronger. Work on proof techniques for proving these criteria is still in early stages, but back-translation (both trace-directed and syntax-directed) features prominently. In particular, Abate et al. [27] describe a compiler and prove a strong robust safety property (not FA) for it using trace-directed back-translation. Their method also reuses whole-program compiler correctness to reduce a big part of cross-language reasoning to only target-level reasoning. However, their setting does not support sharing of memory across modules, which substantially simplifies their proof by eliminating the need for the strong and weak relations (Section V-B).

Another line of work [49, 50, 51, 52, 53] verifies that the compiler does not undo countermeasures that the programmer of cryptographic libraries implements in order to ensure protection against timing attacks or other secret-revealing attacks.

**Fully-abstract trace semantics:** Our trace labels are inspired by Laird’s work for a functional language with general references [29]. Laird relies on a bipartite LTS in which nodes are partitioned between program configurations and environment (context) configurations. We do not use this segregation explicitly, but our checks on pce in the trace semantics have the same effect.

**IX. LIMITATIONS AND FUTURE WORK**

Our work shows formally that PAC compilers can provide strong guarantees for partial programs. While we believe that this is a significant step forward in the understanding of the security properties of PAC compilers, we still make some simplifications and assumptions that would be interesting to remove in future work. First, our memory allocation model does not support de-allocation. This simplification allows us to represent the state of the memory allocator as just the next-free-address, and this is essential in keeping our model manageable. To the best of our knowledge, nobody has yet developed fully abstract trace semantics for languages with a realistic model of deallocation. Work in this direction would be interesting. Second, we do not yet model side channels. As such, our compiler is not guaranteed to preserve resistance against side-channel leaks [54]. There is recent related work that specifically investigates how to secure compilers such that they preserve side-channel resistance [49, 52, 53, 50, 51]. Combining PAC with these ideas would also be interesting.


APPENDIX

A. Why non-symmetric linking?

The fact that we chose to define linking as non-symmetric is just a side effect of trying to avoid some tedious proof [55], but linking being non-symmetric is not really essential for security.

We use non-symmetry to require that the program parts are first all linked together and used as the right operand of the linking operator. The left operand then represents the context in which this program runs. Having distinguished the program of interest from its context, we then define linking in such a way that the context’s data segment is placed in memory after the program’s data segment. There is no security motivation for this enforced order; it just makes the proof easier: the construction of the emulating context will occupy a data segment whose size is in principle larger (due to metadata) than the size of the data segment of the target context that we are emulating. This order of placing the data segments in memory ensures that this increase in size (due to metadata) does not impact the position of the program of interest’s variables in memory (in a simulating run compared to a given run).

However, lots of the metadata we store is redundant—we store this redundant data to make our life simpler. But in principle, we do believe one should be able to prove that the non-redundant metadata will at every execution state always fit within a data segment of the original size (i.e., the size from the given run). By proving this, there will be no need to define linking to be non-symmetric.

B. The proof of Lemma 6 and the (proved) assumptions of this lemma

We first show some excerpted definitions of key constituents of our TrICL relation, then in the end of Appendix B, we show the proof of Lemma 6, the step-wise backward simulation condition of TrICL, which is an interesting proof because it puts together the whole-program compiler correctness lemmas.

The goal is to prove that the TrICL relation satisfies the step-wise backward simulation condition (Lemma 6). A nice way to understand all the assumptions that are used in the proof of this lemma is to first group these assumptions (namely, to group Lemmas 7 to 14 that come later on) by the components of TrICL about which they talk. We recall that TrICL is defined in terms of four main relations/invvariants:

1) the vanilla (whole-program) compiler-correctness relation ($\approx_p$) between the source state and the mediator state satisfying lifted forward- and backward-simulations (Lemmas 7 and 8)
2) a strong-similarity relation ($\approx_p$) between the mediator state and the given state, a relation that satisfies lock-step simulation (Lemma 9)
3) a weak-similarity relation ($\sim_p$) also between the mediator state and the given state, a relation that satisfies option simulation (Lemma 11), and together with the strong similarity satisfying both weakening (Lemma 10) and strengthening (Lemma 12)
4) emulation invariants about the source state satisfying both adequacy (Lemma 13) and preservation by trace steps (Lemma 14)

Definition 5 (Weak-similarity relation (excerpt)).

$$(s_1, \varsigma) \sim_p (s_2, \varsigma) \overset{\text{def}}{=} s_1.\text{stk} \sim_p s_2.\text{stk} \land s_1.\text{mem} \sim_p s_2.\text{mem}$$

where weak stack similarity and weak memory similarity are defined as follows:

$s_1.\text{stk} \sim_p s_2.\text{stk}$ is defined to be the existence of a map
Fig. 7: Evaluation of expressions $e$ in ImpMod. The evaluation relation $\Downarrow \subseteq e \times V$ is defined on pairs of expressions $e$ and values $V$. The evaluation relation $\Downarrow$ is indexed with an evaluation context $Fd, MVar, \beta, \Delta, \Sigma, Mem, \Phi, pc$. The evaluation context is used in rules Eval-amp-local-var, Eval-amp-module-var and Eval-star. Instead of writing $Fd, MVar, \beta, \Delta, \Sigma, Mem, \Phi, pc \vdash e \Downarrow v$, we abbreviate it as $e \Downarrow v$.

\[
\begin{array}{llllllll}
\text{(Eval-binop)} & e_1 \Downarrow z_1 & z_1 \in \mathbb{Z} & e_2 \Downarrow z_2 & z_2 \in \mathbb{Z} & z_r = z_1 + z_2 & e_1 \oplus e_2 \Downarrow z_r \\
\text{(Eval-start)} & e \Downarrow (_\langle z_1, z_2 \rangle) & \text{start(e)} \Downarrow z \\
\text{(Eval-end)} & e \Downarrow (_\langle \ldots, z, \ldots \rangle) & \text{end(e)} \Downarrow z \\
\text{(Eval-offset)} & e \Downarrow (_\langle \ldots, z, \ldots \rangle) & \text{offset(e)} \Downarrow z \\
\text{(Eval-capType)} & e_c \Downarrow (x, st, end, \ldots) & e_s \Downarrow st' & st' \in \mathbb{Z} & e_e \Downarrow end' & end' \in \mathbb{Z} & \text{limRange(e_c, e_s, e_e)} \Downarrow (x, st', end', 0) \\
\text{(Eval-amp-local-var)} & (fid, \ldots) = pc & \text{vid} \in \text{localIDs}(Fd(fid)) \cup \text{args}(Fd(fid)) & mid = \text{moduleID}(Fd(fid)) & \beta(\text{vid}, fid, mid) = [st, end] & \phi = \Sigma(mid).1 + \Phi(mid) \\
\text{(Eval-amp-module-var)} & (fid, \ldots) = pc & \text{vid} \notin \text{localIDs}(Fd(fid)) \cup \text{args}(Fd(fid)) & mid = \text{moduleID}(Fd(fid)) & \beta(\text{vid}, \bot, mid) = [st, end] \\
\text{(Eval-amp-arr)} & \& e_{arr} \Downarrow (\delta, st, end, \ldots) & \& e_{arr} \Downarrow st' & \text{off'} \in \mathbb{Z} & e_{idx} \Downarrow \text{off'} & \text{limRange(e_c, e_s, e_e)} \Downarrow (x, st', end', 0) \\
\text{(Eval-star)} & e \Downarrow (\delta, st, end, \ldots) & \Phi(\text{vid}, \delta, \Delta(mid).1 + st, \Delta(mid).1 + end, 0) \\
\text{(Eval-end)} & e \Downarrow (\delta, st, end, \ldots) & \Phi(\text{vid}, \delta, \Delta(mid).1 + st, \Delta(mid).1 + end, 0) \\
\text{(Eval-offset)} & e \Downarrow (\delta, st, end, \ldots) & \Phi(\text{vid}, \delta, \Delta(mid).1 + st, \Delta(mid).1 + end, 0) \\
\end{array}
\]

between the indices of the entries of $s_1.stk$ and the indices of the entries of $s_2.stk$, such that this map is “successor preserving”, monotone, and every pair of indices that is in the map corresponds to two equal stack entries (one from $s_1.stk$ and the second from $s_2.stk$). (See section 6.3 of the technical report for formal definitions.)

$s_1.mem \approx_{p,<} s_2.mem$ is defined to be equality of the contents of memories $s_1.mem$ and $s_2.mem$ at all the currently-private addresses of $p$, where a currently-private address of $p$ is any address that is so far not shared (i.e., not in the set $\varsigma$) and that is reachable from $p$’s statically-allocated memory.

**Definition 6** (Strong-similarity relation (excerpt)).

\[
(s_1, \varsigma) \approx_{p} (s_2, \varsigma) \overset{\text{def}}{=} s_1.stk \approx_{p} s_2.stk \land s_1.mem \approx_{p} s_2.mem
\]

where strong stack similarity and strong memory similarity are defined as follows:

$s_1.stk \approx_{p} s_2.stk$ is defined to be the same as weak stack similarity, i.e., the existence of a map with the same properties mentioned in Definition 5, but with the extra condition that the top-most stack indices must be in the map. (This extra condition ensures that all function calls and returns happen in sync, hence, satisfying the lock-step simulation condition of Lemma 9.)

$s_1.mem \approx_{p} s_2.mem$ is defined to be equality of the contents of memories $s_1.mem$ and $s_2.mem$ at all the addresses reachable from $p$’s statically-allocated memory.

**Definition 7** (Emulation invariants (simplified excerpt)). The emulation invariants for a state $s$ and the prefix of a finite trace $\alpha$ up to position $i$ consists of invariants on the shape of the code that is currently executing (i.e., the code starting at address $s.pc$), in addition to invariants on memory $s.mem$ that ensure the metadata (that the back-translated context keeps) is compatible with the information from the trace prefix of $\alpha$ up to position $i$. The invariants on memory can be found in the technical report in definitions 116, 117, 118, and 126. The emulation invariants on the shape of the code are more interesting:

\[
\begin{array}{llllllll}
\text{emulate_invariants}(s)_{\alpha,i,p} \overset{\text{def}}{=} & \alpha(i) = ? \implies \exists j. j \leq i \land \text{upcoming_commands}(s) \land \text{emulate_responses_for_suffix}(\alpha, j, s.pc) & \\
\text{where:} & \text{upcoming_commands}(s, c) \text{ ensures that starting at address } s.pc \text{ of the code memory of } s, \text{ the commands } c \text{ are allocated, and} & \\
\text{emulate_responses_for_suffix}(\alpha, j, \ldots) \text{ is defined recursively on } j \text{ to be a nested switch statement:} & \text{switch(curent_trace_idx)} \{ & \\
\text{case } j : & \text{emulate_ith_action}(\alpha, j, \ldots); & \\
\end{array}
\]
Fig. 8: Small-step semantics of commands \texttt{Cmd} in \texttt{ImpMod}. The small-step relation $\Rightarrow \subseteq s \times s$ is defined on pairs of execution states $s$. The small-step relation $\Rightarrow$ is indexed with an evaluation context $Fd, MVar, \beta, \Delta, \Sigma$ and an allocation limit $\nabla$. The allocation limit $\nabla$ is used in rule \texttt{Allocate}.

\[
\begin{array}{l}
\text{(Allocate)}\\
\text{(Call)}\\
\text{(Jump-zero)}\\
\text{(Return)}\\
\text{(Jump-non-zero)}\\
\text{(Exit)}
\end{array}
\]

\begin{align*}
\text{emulate_responses_for_suffix}(\alpha, j + 2, \ldots) \\
\text{case } j + 2 : \\
\quad \text{emulate_i_th_action}(\alpha, j + 2, \ldots); \\
\quad \text{emulate_responses_for_suffix}(\alpha, j + 4, \ldots) \\
\ldots \\
\text{case } |\alpha| - 1 : \\
\quad \text{emulate_i_th_action}(\alpha, |\alpha| - 1, \ldots);
\end{align*}

(See definitions 120 and 121 in the technical report for the formal definitions.)

(Notice that the maximum depth of the nesting of the switch statement is $(|\alpha| - j)/2$, i.e., half the length of the suffix starting at position $j$—the half originates from the fact that we are only emulating every other trace label.)

(Notice also that, interestingly, most of the code generated by \texttt{emulate_responses_for_suffix} is actually unreachable; in fact for any finite trace $\alpha$, the size of the code generated is $O(|\alpha|^2)$, while the portion of this code that is reachable is always only linear in the length of the trace. But we found nevertheless that defining it this way makes some proofs
(evalconst)
\[ n \in \mathbb{Z} \]
\[ n, \text{Mem, ddc, stc, pcc} \downarrow n \]
\[ \text{(evaladd)} \]
\[ \text{ddc, Mem, ddc, stc, pcc} \downarrow \text{ddc} \]
\[ \text{(evalstc)} \]
\[ \text{stc, Mem, ddc, stc, pcc} \downarrow \text{stc} \]
\[ \text{(evalCapType)} \]
\[ e, \text{Mem, ddc, stc, pcc} \downarrow v \]
\[ v \in \{k\} \times \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \Rightarrow v' = 0 \]
\[ v \in \{\delta\} \times \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \Rightarrow v' = 1 \]
\[ \text{(evalCapStart)} \]
\[ e, \text{Mem, ddc, stc, pcc} \downarrow v \]
\[ v = (x, s, e, \text{off}) \in \text{Cap} \]
\[ v' = s \]
\[ \text{(evalCapEnd)} \]
\[ e, \text{Mem, ddc, stc, pcc} \downarrow v \]
\[ v = (x, s, e, \text{off}) \in \text{Cap} \]
\[ v' = e \]
\[ \text{(evalCapOff)} \]
\[ e, \text{Mem, ddc, stc, pcc} \downarrow v \]
\[ v = (x, s, e, \text{off}) \in \text{Cap} \]
\[ v' = \text{off} \]
\[ \text{(evalBinCp)} \]
\[ e_1, \text{Mem, ddc, stc, pcc} \downarrow v_1 \]
\[ v_1 \in \mathbb{Z} \]
\[ e_2, \text{Mem, ddc, stc, pcc} \downarrow v_2 \]
\[ v_2 \in \mathbb{Z} \]
\[ v' = v_1[\oplus]v_2 \]
\[ \text{(evalIncCp)} \]
\[ e, \text{Mem, ddc, stc, pcc} \downarrow v \]
\[ v_2 \in \mathbb{Z} \]
\[ e_2, \text{Mem, ddc, stc, pcc} \downarrow v_z \]
\[ v' = (x, s, e, \text{off} + v_z) \]
\[ \text{(evalDeref)} \]
\[ e, \text{Mem, ddc, stc, pcc} \downarrow v \]
\[ e_2, \text{Mem, ddc, stc, pcc} \downarrow s' \]
\[ e_2, \text{Mem, ddc, stc, pcc} \downarrow e' \]
\[ s' \in \mathbb{Z} \]
\[ e' \in \mathbb{Z} \]
\[ v = (x, s, e, \text{off}) \in \text{Cap} \]
\[ v' = \text{Mem}(s + \text{off}) \]
\[ \text{(evalLim)} \]
\[ \text{lim}(e, e_2, e_3, e_4, \text{Mem, ddc, stc, pcc}) \downarrow v' \]

considerably easier.)
(Observe that the meta-level function emulate_i th_action is the function that generates the sequence of function calls and the return command that are shown in the back-translation example of Listing V.l—lines 7 to 12)

Remember: We write \( s \overset{\alpha}{\rightarrow} p s' \) to denote that \( s \) is a state of the program-of-interest \( p \) (linked with some context), and that \( \alpha \) is a compressed trace prefix (i.e., with uninformative (\( \tau \)) labels dropped) that is emitted by the (multiple-step) execution of \( s \) until \( s' \). We write \( s \overset{\alpha}{\rightarrow} p s' \) to denote the same about the target language.

Lemma 7 (Compiler forward-simulation lifted to compressed trace steps).
\[ s_t \equiv_p (s_t, \zeta) \overset{\alpha}{\rightarrow} p (s'_t, \zeta') \]
\[ \exists s'_t, (s_t, \zeta) \overset{\alpha}{\rightarrow} p (s'_t, \zeta') \]

Lemma 8 (Compiler backward-simulation lifted to compressed trace steps).
\[ s_t \equiv_p (s_t, \zeta) \overset{\alpha}{\rightarrow} p (s'_t, \zeta') \]
\[ \exists s'_t, (s_t, \zeta) \overset{\alpha}{\rightarrow} p (s'_t, \zeta') \]

Lemma 9 (Lock-step simulation of strong similarity).
\[ (s_1, \zeta) \approx_p (s_1, \zeta) \]
\[ (s_2, \zeta) \overset{\tau}{\rightarrow} p (s_1, \zeta) \]
\[ (s_2', \zeta') \overset{\tau}{\rightarrow} p (s_1', \zeta') \]

Lemma 10 (Strong similarity is weakened by an output action).
\[ \lambda \in (s_1, \zeta) \approx_p (s_2, \zeta) \]
\[ (s_1, \zeta) \overset{\lambda}{\rightarrow} p (s_1', \zeta') \]
\[ (s_2, \zeta) \overset{\lambda}{\rightarrow} p (s_2', \zeta') \]

Lemma 11 (Option simulation of weak similarity).
\[ (s_1, \zeta) \approx_p (s_2, \zeta) \]
\[ (s_1, \zeta) \overset{\tau}{\rightarrow} p (s_1', \zeta') \]

Lemma 12 (Weak similarity is strengthened by aligned input actions).
Lemma 13 (Adequacy of the emulation invariants).
\[ \text{emulate_invariants}(s)_{\alpha,i,p} \land \alpha(i) \in \{? \cup \{\checkmark}\} \implies \exists s'. (s,\checkmark) \overset{\alpha(i)}{\longrightarrow}_p (s',\checkmark) \]

Lemma 14 (Preservation of the emulation invariants).
\[ \text{emulate_invariants}(s)_{\alpha,i,p} \land (s,\checkmark) \overset{\alpha(i)}{\longrightarrow}_p (s',\checkmark) \implies \text{emulate_invariants}(s')_{\alpha,i+1,p} \]

By relying on the lemmas above we were able to prove that TR-ICL satisfies step-wise backward-simulation (Lemma 6) as shown next:

**Proof.** (Simplified)

**Hypotheses (unfolding Definition 4):**

1. \( \alpha \in \text{Alt} \)
2. \( s_{\text{emu}} \overset{\alpha}{\rightarrow}_p s_{\text{med}} \)
3. \( \text{emulate_invariants}(s_{\text{emu}})_{\alpha,i,p} \)
4. \( \alpha(i) \in \{? \cup \{\checkmark\} \} \)
5. \( \alpha(i) \in \{? \} \)
6. \( (s_{\text{given}},\checkmark) \overset{\alpha(i)}{\longrightarrow}_p (s'_{\text{given}},\checkmark) \)

We consider the following two cases for the trace step \( \alpha(i) \). We obtain them by unfolding the definition of Alt in hypothesis 1 (we ignore in this proof sketch the case of \( \alpha(i) = \checkmark \)):

- Case \( \alpha(i) \in {? nil \}: 
  - (Return-to-program)
    \[ s \rightarrow s' \quad s.M_c(s.pcc) = \text{Return} \]
    \[ s.pcc \not\subseteq \text{dom}(p.M_c) \quad s'.pcc \not\subseteq \text{dom}(p.M_c) \]
    \[ \varsigma' = \text{reachable_addresses_closure}(\varsigma,s'.\text{Mem}) \]
    \[ \text{Mem}_{\text{shr}} = s'.\text{Mem}|_{\varsigma'} \]
  - (Return-to-context)
    \[ s \rightarrow s' \quad s.M_c(s.pcc) = \text{Return} \]
    \[ s.pcc \subseteq \text{dom}(p.M_c) \quad s'.pcc \subseteq \text{dom}(p.M_c) \]
    \[ \varsigma' = \text{reachable_addresses_closure}(\varsigma,s'.\text{Mem}) \]
    \[ \text{Mem}_{\text{shr}} = s'.\text{Mem}|_{\varsigma'} \]
- (Call-internal-context-silent)
  \[ s \rightarrow s' \quad s.M_c(s.pcc) = \text{Call fid} \varnothing \]
  \[ s.pcc \not\subseteq \text{dom}(p.M_c) \quad s'.pcc \not\subseteq \text{dom}(p.M_c) \]
  \[ (s,\varsigma) \overset{\tau}{\longrightarrow}_p (s',\varsigma') \]
- (Call-internal-program-silent)
  \[ s \rightarrow s' \quad s.M_c(s.pcc) = \text{Call fid} \varnothing \]
  \[ s.pcc \subseteq \text{dom}(p.M_c) \quad s'.pcc \subseteq \text{dom}(p.M_c) \]
  \[ (s,\varsigma) \overset{\tau}{\longrightarrow}_p (s',\varsigma') \]

Fig. 10: (Simplified Excerpt) Trace semantics of CHERIExp. The trace-step relation is indexed with a program \( p \). Other rules are in the supplementary material (figure 9 on page 151). Also, in the supplementary material, the definition of memory reachability and lemmas about it can be found (on page 19).
Thus, instantiate Lemma 12 (strengthening) to obtain 
\( (s'_{\text{med}}, s) \approx_{[p]} (s'_{\text{given}}, s) \).

(d) Obtain \texttt{emulate_invariants}(s_{\text{emu}})^{\alpha,i+1,p}, as follows:

Instantiate Lemma 14 (preservation of the emulation invariants) with hypothesis 3 and statement (a) that we proved above.

The four statements obtained above conclude this case.

- Case \( \alpha(i) \in \cdot \):

(a) Obtain \( s'_{\text{med}} \) where \( (s_{\text{med}}, s) \xrightarrow{\alpha(i)}_{[p]} (s'_{\text{med}}, s') \) and \( (s'_{\text{med}}, s') \approx_{[p]} (s'_{\text{given}}, s) \) as follows:

First, unfold the definition of the given step of hypothesis 6 to obtain star-many \( \tau \) steps leading to some state \( s'_{\text{given}} \) where also \( (s'_{\text{given}}, s) \approx_{[p]} (s'_{\text{med}}, s) \).

Now instantiate (the star version of) Lemma 9 (lock-step simulation) with both hypothesis 4 (after applying symmetry of strong similarity) and state \( s''_{\text{given}} \) (the \( \tau \) steps) to obtain some \( s''_{\text{med}} \), where \( (s''_{\text{med}}, s') \approx_{[p]} (s''_{\text{given}}, s') \).

Now use this together with the step \( (s''_{\text{given}}, s) \xrightarrow{\alpha(i)}_{[p]} (s''_{\text{given}}, s') \), which we obtained above, to instantiate Lemma 10 (weakening of strong similarity).

Now apply symmetry of weak similarity to obtain the desired subgoal.

(b) Obtain \( s'_{\text{emu}} \) where \( (s_{\text{emu}}, s) \xrightarrow{\alpha(i)}_{[p]} (s'_{\text{emu}}, s') \) and \( s'_{\text{emu}} \approx_{p} s'_{\text{med}} \) as follows:

Instantiate Lemma 8 (backward simulation) using both hypothesis 2 and the step of \( s_{\text{med}} \) that we obtained in the previous subgoal.

(c) Obtain \texttt{emulate_invariants}(s_{\text{emu}})^{\alpha,i+1,p}, as follows:

Instantiate Lemma 14 (preservation of the emulation invariants) with both hypothesis 3 and the step of \( s_{\text{emu}} \) that we obtained in the previous subgoal.

The three statements obtained above conclude this case.

\[ \Box \]

C. Output of the source-to-source transformation

Listing A.1: Source-to-source compilation output. Source is Listing III.1.

Listing A.2: Source-to-source compilation output. Initialization module init.c

Listing A.3: Source-to-source compilation output. Transformed main.c

Listing A.4: Source-to-source compilation output. Transformed lib1.c
Listing A.5: Source-to-source compilation output. Transformed lib2.c

```c
extern struct cheri_object lib2;
__attribute__((cheri_ccallee))
__attribute__((cheri_method_class(lib2)))
int f2(void);

int f2(void)
{
    [..]
}
```