TECHNISCHE UNIVERSITÄT ILMENAU

Verifying Multi-Pushdown Automata

Highlights on Games, Logic and Automata 2022, Paris

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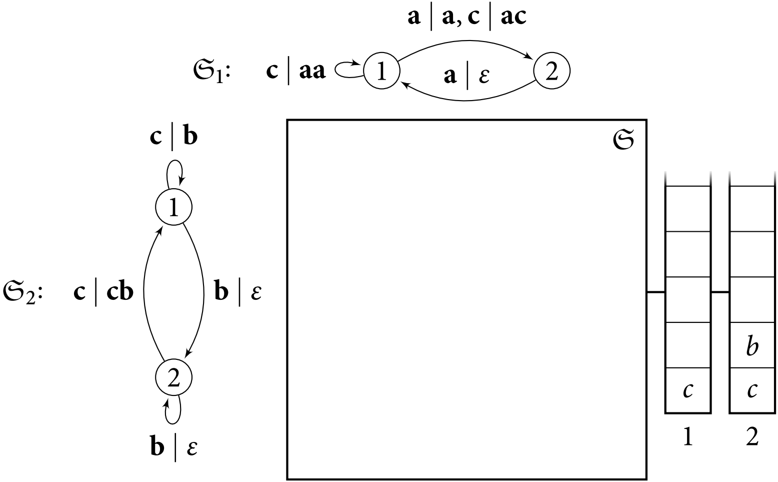
Motivation

- Consider automata with one or more pushdowns.
  - Model distributed systems with recursive programs.
- 2-pushdown automata are Turing-complete!
  - $\Rightarrow$ Verification problems are undecidable.
- Here: consider a special restriction to the automata.
  - $\rightsquigarrow$ cooperating multi-pushdown systems

- A cooperating multi-pushdown system (CMPDS) $\mathfrak{S}$ is a (special) subsystem of the synchronous product of 1-pushdown systems $\mathfrak{S}_1, \ldots, \mathfrak{S}_n$:



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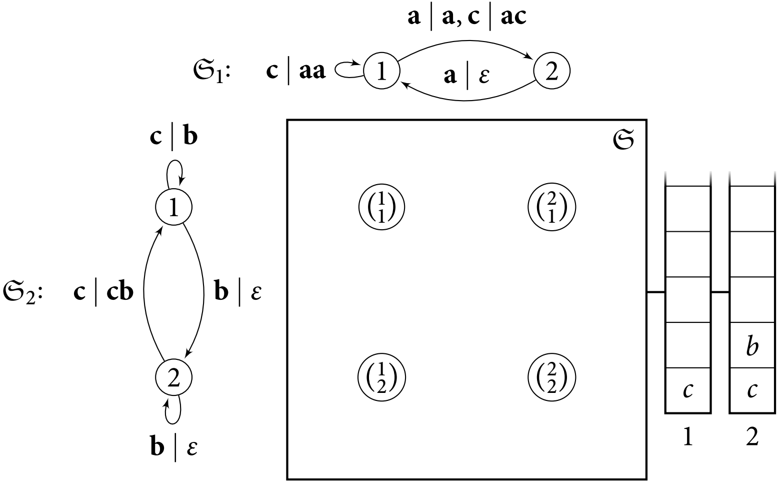

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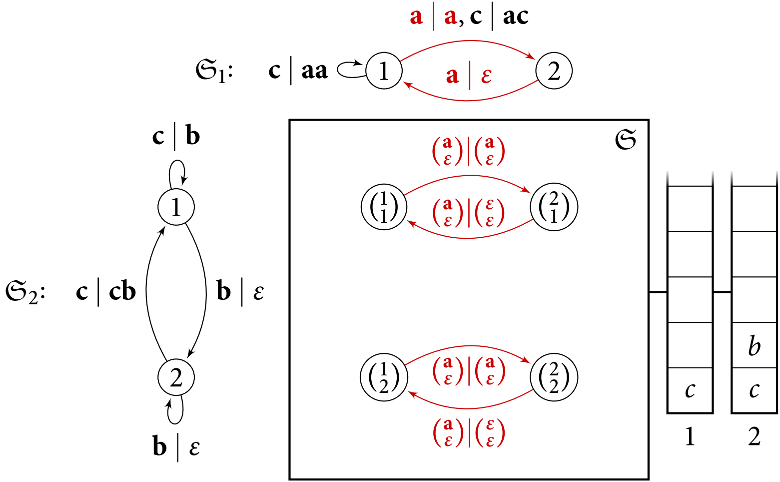

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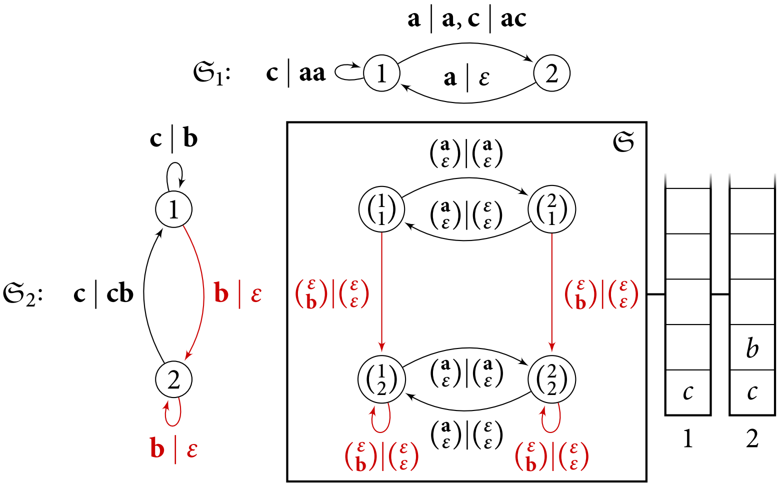

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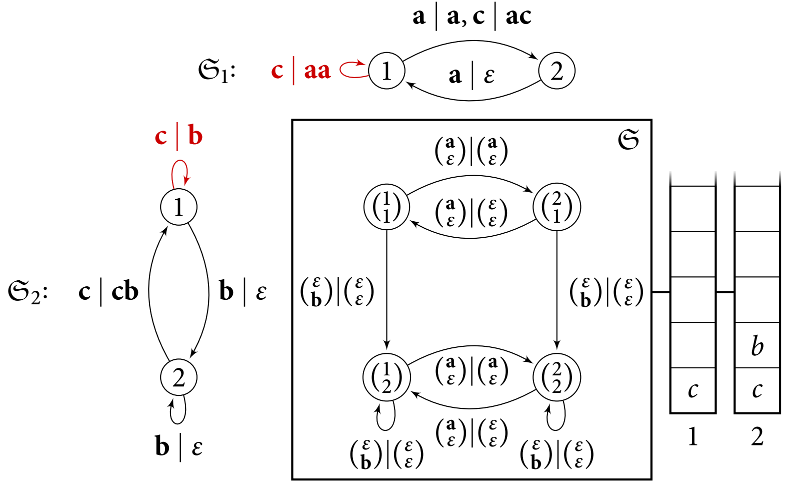

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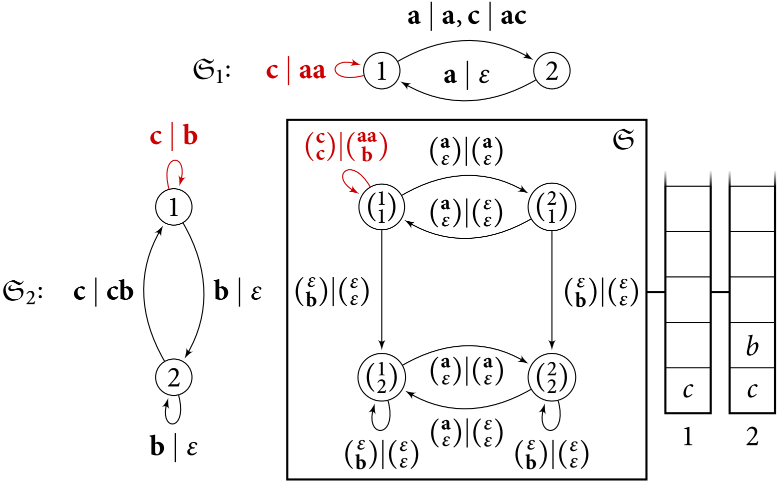

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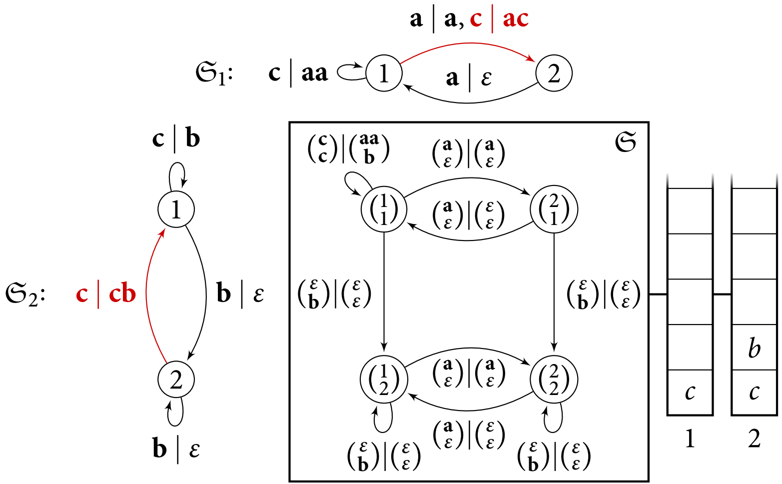

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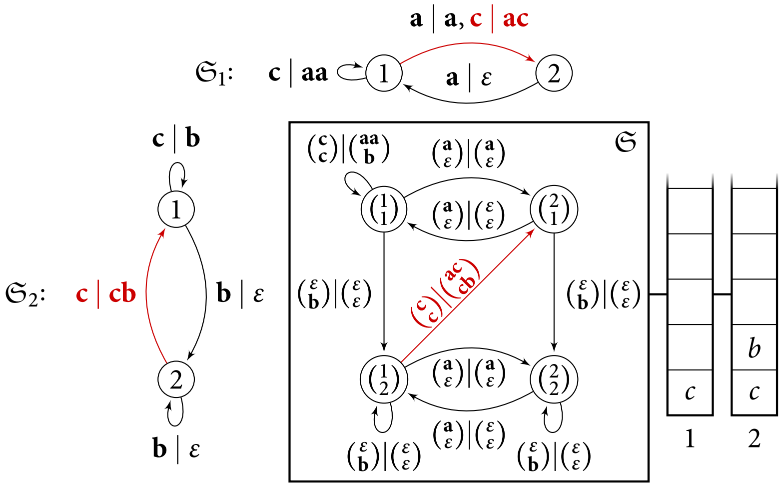

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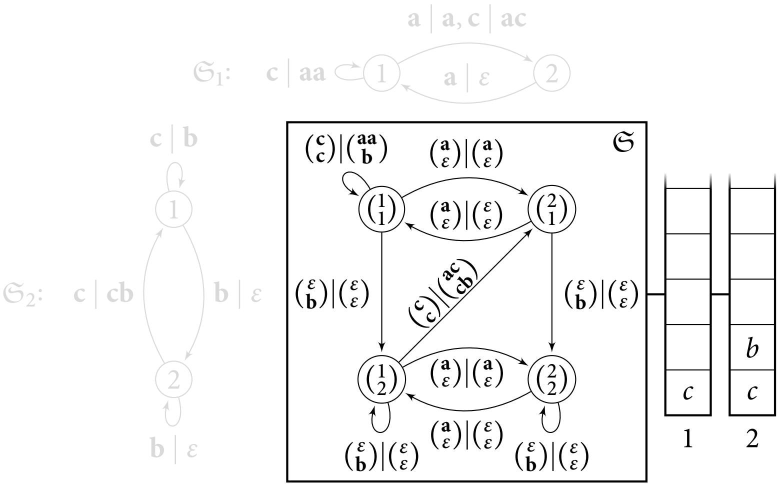

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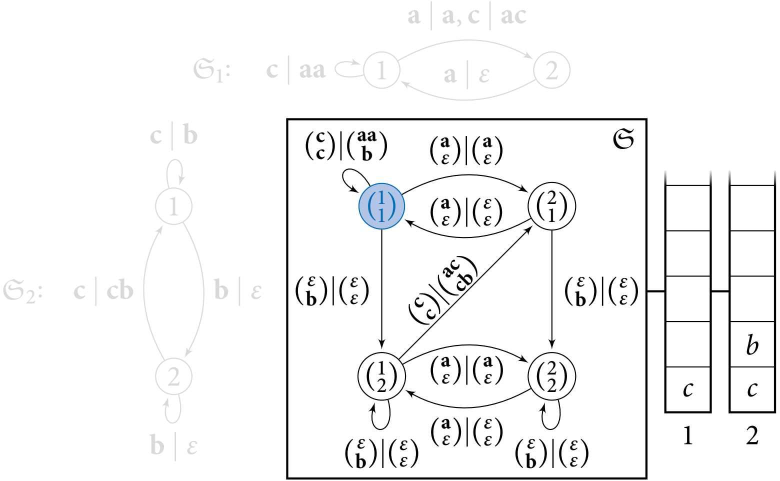

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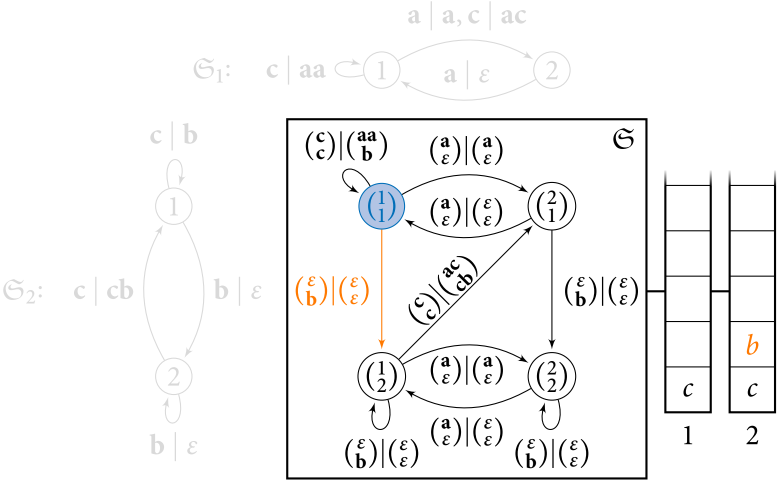

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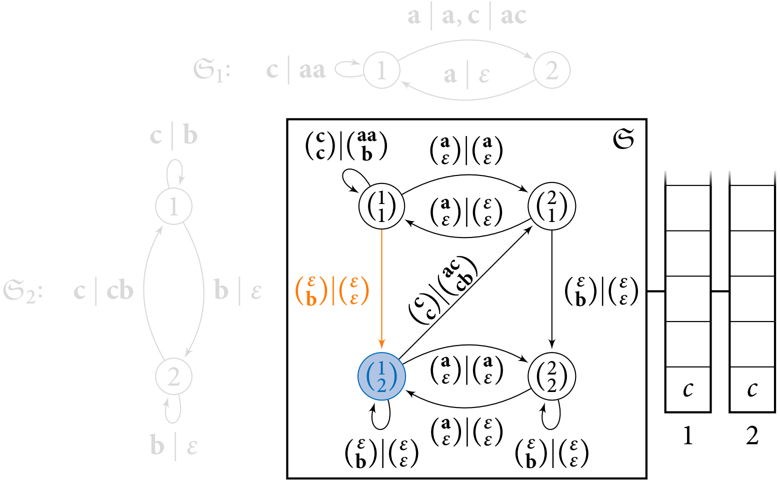

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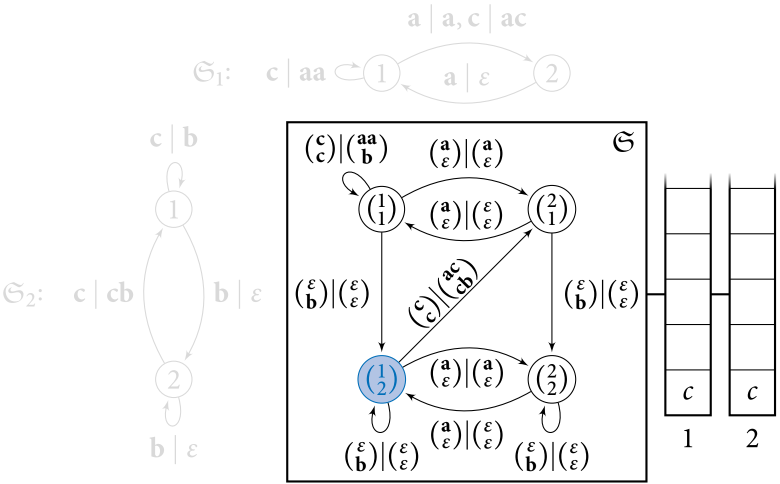

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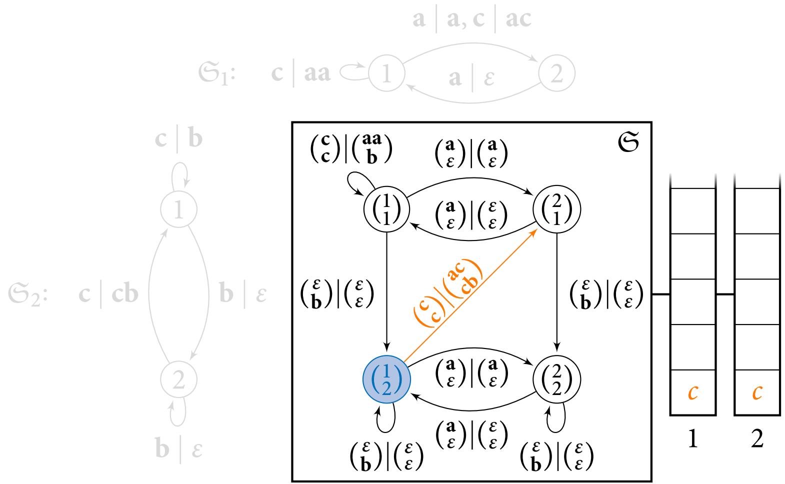

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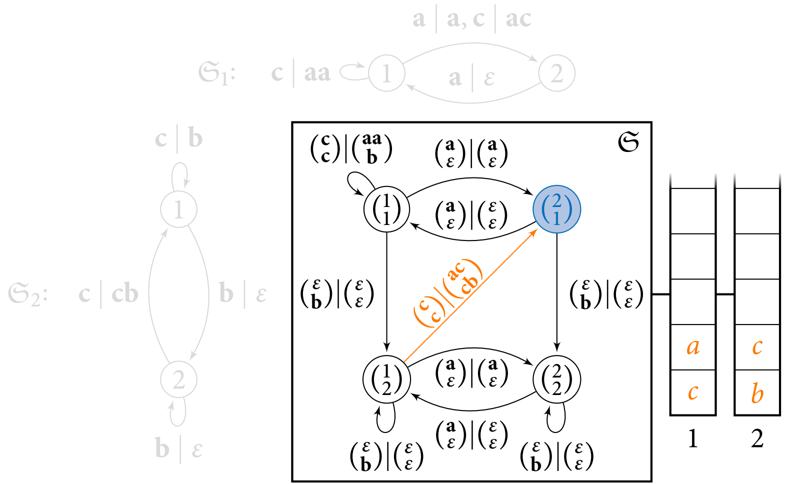

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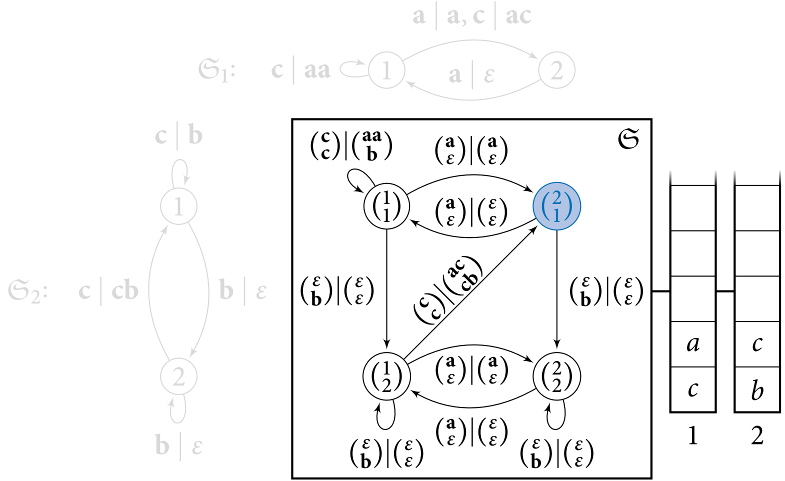

Backwards Reachability

- Let $\mathfrak{S}$ be a CMPDS and C a set of configurations.
- $\mathrm{pre}^*_{\mathfrak{S}}(C) := \{d \mid \exists c \in C \colon d \vdash^*_{\mathfrak{S}} c\}$

Theorem

1. *Let $\mathfrak{S}$ be a CMPDS and C a recognizable set of configurations (i.e., accepted by an asynchronous automaton). Then $\mathrm{pre}^*_{\mathfrak{S}}(C)$ is effectively recognizable (in polynomial time).*
2. *There are a CMPDS $\mathfrak{S}$ and a rational set C of configurations (i.e., accepted by an NFA) such that $\mathrm{pre}^*_{\mathfrak{S}}(C)$ is not rational.*

- Construction generalizes the one by Bouajjani, Esparza, and Maler.

Forwards Reachability

- Let $\mathfrak{S}$ be a CMPDS and C a set of configurations.
- $\mathrm{post}^*_{\mathfrak{S}}(C) := \{d \mid \exists c \in C\colon c \vdash^*_{\mathfrak{S}} d\}$

Theorem

1. *Let $\mathfrak{S}$ be a CMPDS and C a rational set of configurations. Then $\mathrm{post}^*_{\mathfrak{S}}(C)$ is effectively rational (in polynomial time).*
2. *There are a CMPDS $\mathfrak{S}$ and a recognizable set C of configurations such that $\mathrm{post}^*_{\mathfrak{S}}(C)$ is not recognizable.*

- Construction is inspired by the one by Finkel, Willems, and Wolper (but more involved).

Thank you!