Reachability Problems in Multi-Queue Automata

Highlights on Games, Logic and Automata 2020, (not in) Aachen

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Multi-Queue Automata

\[ A_1: \]
\[ a \quad a \quad a \]
\[ \bar{a} \quad \bar{a} \]

\[ A_2: \]
\[ b \quad b \quad b, \bar{c}, c \]
\[ \bar{a} \quad \bar{a} \quad \bar{a} \]

\[ A_3: \]
\[ a \quad a \quad a, \bar{c} \quad \bar{c} \]
\[ \bar{a} \quad \bar{a} \quad \bar{a}, c \]

**Question**

Given some configuration, is it possible to clear each queue?
Multi-Queue Automata

\( \mathcal{A}_1: \)

- Create task \( a \)
- Finalize / execute task \( a \)

\( \mathcal{A}_2: \)

- \( a \rightarrow b \)
- \( b \rightarrow b, c \)
- \( b, b, c, \overline{c} \)

\( \mathcal{A}_3: \)

- \( \overline{a} \rightarrow a \)
- \( a \rightarrow a, \overline{c} \)
- \( a, \overline{c}, \overline{c} \)
- \( a, c \)

Question

Given some configuration, is it possible to clear each queue?
Multi-Queue Automata

$A_1$:  
$\overline{a} \xrightarrow{a} a \xleftarrow{\overline{a}}$  
$a \overline{a}$  

$A_2$:  
$b \xrightarrow{b} b, c$  
$c b$  

$A_3$:  
$a \xrightarrow{\overline{a}} a, c \xrightarrow{a, \overline{c}} \overline{a}, c$  
$a c a$  

Question

Given some configuration, is it possible to clear each queue?
Multi-Queue Automata

\( A_1: \)

\[ \begin{array}{c}
\bar{a} \\
\downarrow \quad \downarrow \\
\overline{a} \\
\end{array} \]

\[ \begin{array}{c}
a \\
\rightarrow \\
a \\
\end{array} \]

\[ \begin{array}{c}
\bar{a} \\
\downarrow \quad \downarrow \\
\overline{a} \\
\end{array} \]

\( A_2: \)

\[ \begin{array}{c}
b \\
\rightarrow \\
b, c \\
\end{array} \]

\( A_3: \)

\[ \begin{array}{c}
a \\
\rightarrow \\
a, c \\
\end{array} \]

\[ \begin{array}{c}
\bar{a} \\
\downarrow \quad \downarrow \\
\overline{a} \\
\end{array} \]

\[ \begin{array}{c}
a, \bar{c} \\
\rightarrow \\
\bar{a}, c \\
\end{array} \]

\[ \begin{array}{c}
\bar{a}, c \\
\rightarrow \\
\overline{a}, c \\
\end{array} \]

\( a\ a \\
\) \( b\ b\ b \)

\( a\ c\ a \)

Question

Given some configuration, is it possible to clear each queue?
Multi-Queue Automata

\( \mathcal{A}_1: \quad \overline{a} \xrightarrow{a} a \xrightarrow{a} \overline{a} \)

\( \mathcal{A}_2: \quad b \xrightarrow{b, c} b, c \)

\( \mathcal{A}_3: \quad a \xrightarrow{a} \overline{a} \xrightarrow{a, c} a, \overline{c} \xrightarrow{\overline{c}} \overline{a}, c \)

\[ \begin{array}{c}
\text{Signal} \quad \text{Signal} \\
\begin{array}{c}
a \\
a \\
a \\
a \\
a \\
a \\
\end{array} \\
\begin{array}{c}
a \\
a \\
c \\
c \\
c \\
\end{array} \\
\begin{array}{c}
a \\
c \\
\end{array} \\
\begin{array}{c}
\end{array}
\end{array} \]

Question

Given some configuration, is it possible to clear each queue?
Multi-Queue Automata

$\mathcal{A}_1$: \[ \bar{a} \quad a \quad a \]

$\mathcal{A}_2$: \[ \quad b \quad b, c \quad \bar{c} \]

$\mathcal{A}_3$: \[ \bar{a} \quad a, c \quad \bar{c} \quad \bar{a}, c \]

**Question**

Given some configuration, is it possible to clear each queue?
Reachability Problem

Inputs:
- $L \subseteq (A^*)^n$ a rational language of queue contents
- $T \subseteq \{a, \overline{a} \mid a \in A\}^*$ a rational language of transformation sequences

Compute:
- $\text{REACH}(L, T) \subseteq (A^*)^n$ the set of all queue contents after application of $T$ on $L$

Theorem (Brand, Zafiropulo 1983)

There are $L$ and $T$ such that $\text{REACH}(L, T)$ is undecidable.

$\Rightarrow$ Approximate $\text{REACH}(L, T)$ step-by-step!
A word $w$ is **connected** if its *sequence diagram* is a connected graph:

- $A_1$: 
  \[ \begin{array}{c}
    \text{a} \\
    \text{a} \\
    \text{a} \\
    \text{a} \\
  \end{array} \quad \text{is not connected,} \\
  \begin{array}{c}
    \text{b} \\
    \text{b} \\
    \text{b} \\
    \text{b} \\
  \end{array} \quad \text{is connected} \]

- $A_2$: 
  \[ \begin{array}{c}
    \text{a} \\
    \text{b} \\
    \text{c} \\
    \text{c} \\
  \end{array} \quad \text{is connected} \]

- $A_3$: 
  \[ \begin{array}{c}
    \text{a} \\
    \text{a} \\
    \text{a} \\
    \text{a} \\
  \end{array} \quad \text{is not connected,} \\
  \begin{array}{c}
    \text{b} \\
    \text{b} \\
    \text{b} \\
    \text{b} \\
  \end{array} \quad \text{is connected} \]

$\Rightarrow$ **$ab\overline{a}$** is not connected, **$abc$** is connected

A language $L$ is **connected** if each $w \in L$ is connected
A generalization of [Boigelot et al. 1997] and [K. 2019]:

**Theorem**

Let $L \subseteq (A^*)^n$ be recognizable, $W, R \subseteq A^*$ be recognizable such that $W$ is connected. Then $\text{REACH}(L, (W\overline{R})^*)$ is effectively recognizable. The construction is possible in polynomial time.

**Proof idea:** Simulate such multi-queue automaton by a 1-counter automaton.

Thank you!