WHAT REALLY IS pWCET?

A RIGOROUS AXIOMATIC PROPOSAL

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Sergey Bozhko, Filip Marković, Georg von der Brüggen, and Björn Brandenburg

MAX PLANCK INSTITUTE FOR SOFTWARE SYSTEMS

technische universität dortmund

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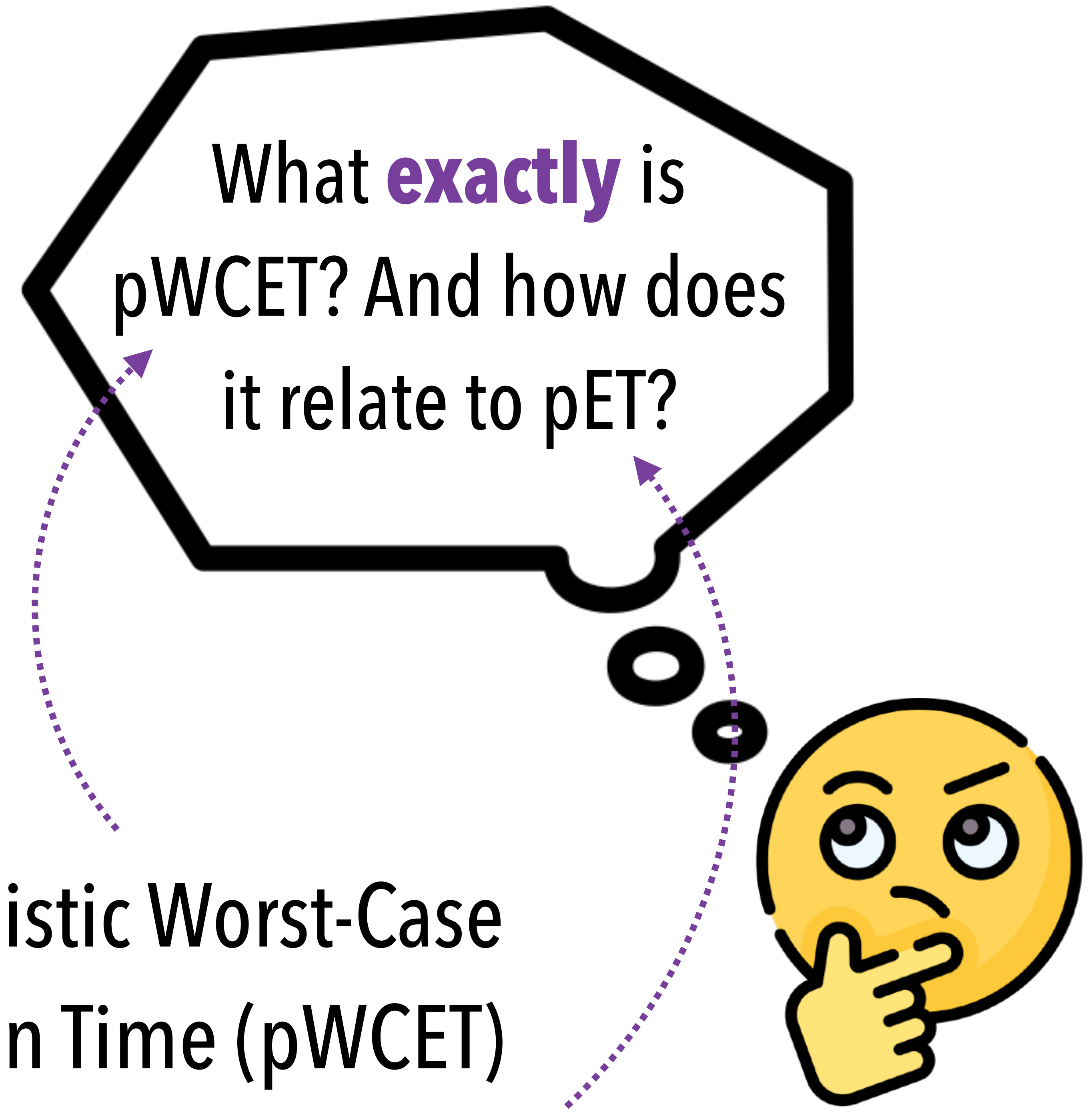
THIS PAPER IN A NUTSHELL



Probabilistic Worst-Case Execution Time (pWCET)

Probabilistic Execution Time (pET)

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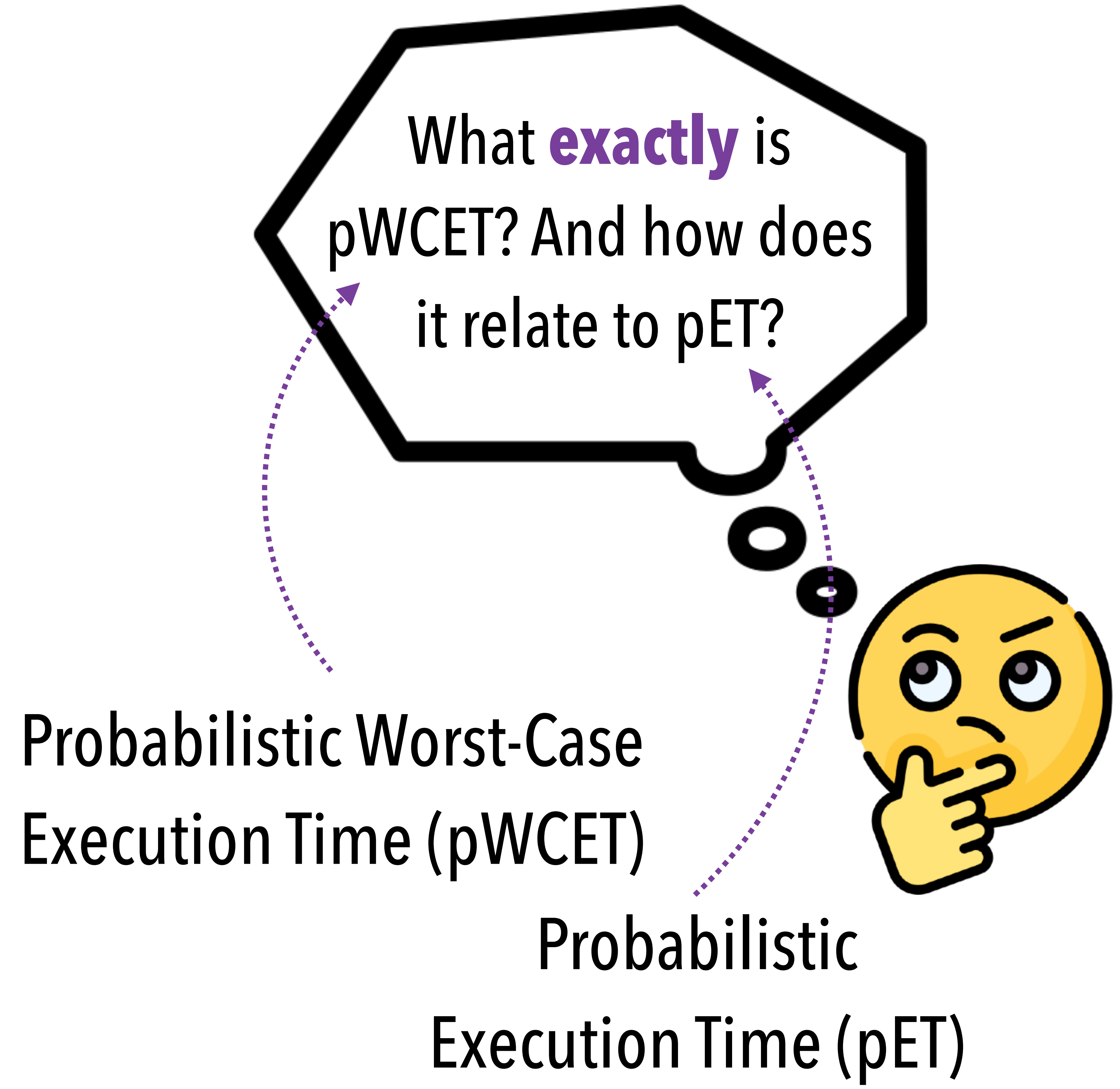
Independent and identically distributed

We propose

➔ First **fully formal** definitions of pET and pWCET

➔ **Adequacy property** capturing the notion of "IID upper bound on pET"

➔ **Prove** that our proposal of pWCET is adequate in this sense



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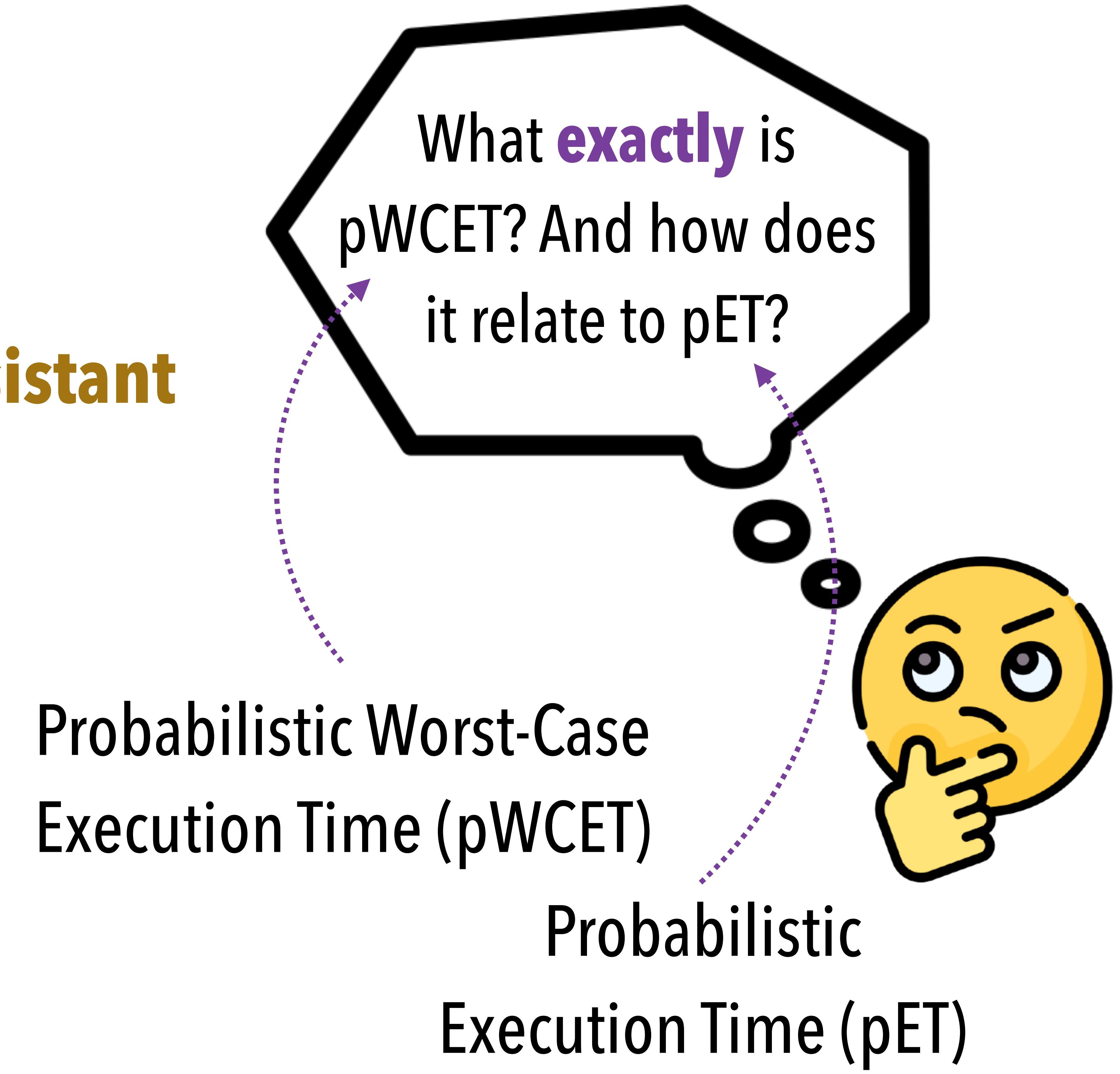

We formalized our proposal with the Coq proof assistant

➔ Semantics of stochastic real-time systems

➔ Definitions of pET, pWCET, and the adequacy property

➔ **Machine-checked** proof that pWCET is adequate

The Coq Proof Assistant

coq.inria.fr

ANALYSIS OF REAL-TIME SYSTEMS: THE BIG PICTURE

THE BIG PICTURE

Predictions

THE BIG PICTURE

Worst-Case Execution Time (WCET)

To get the predictions right, we need:

- ➔ Model with the right **specification**
  - ➔ *E.g.*, model must include WCETs to allow (classical) response-time analysis

Model of the system

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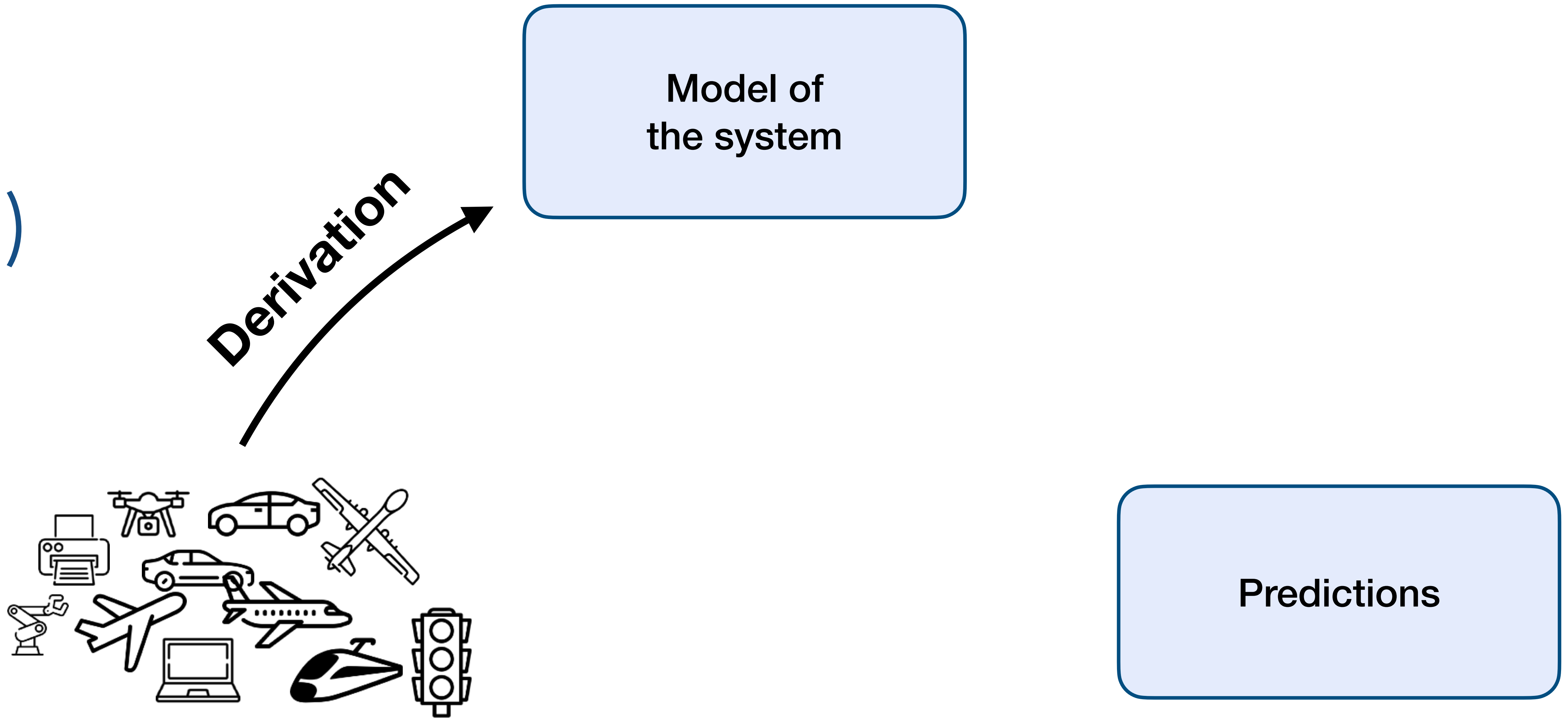

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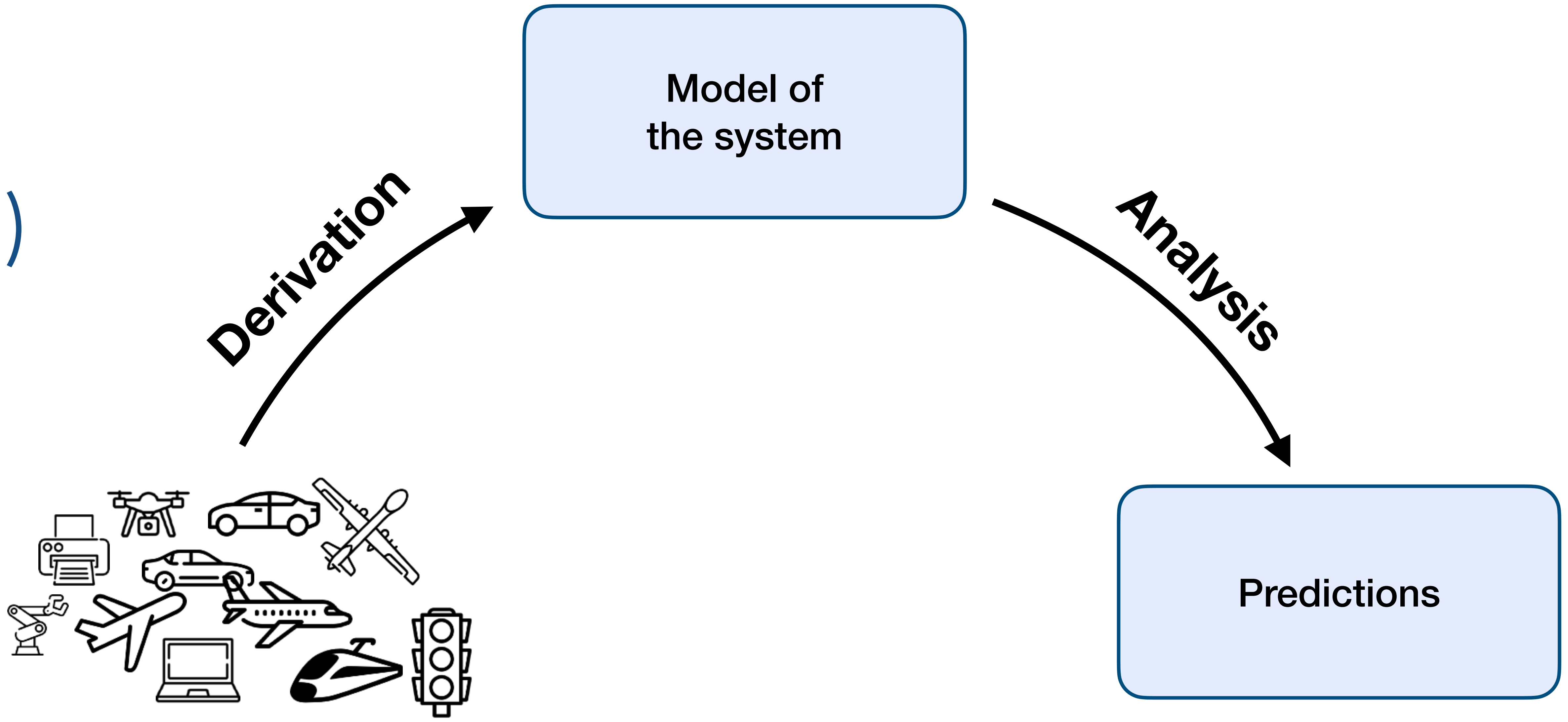

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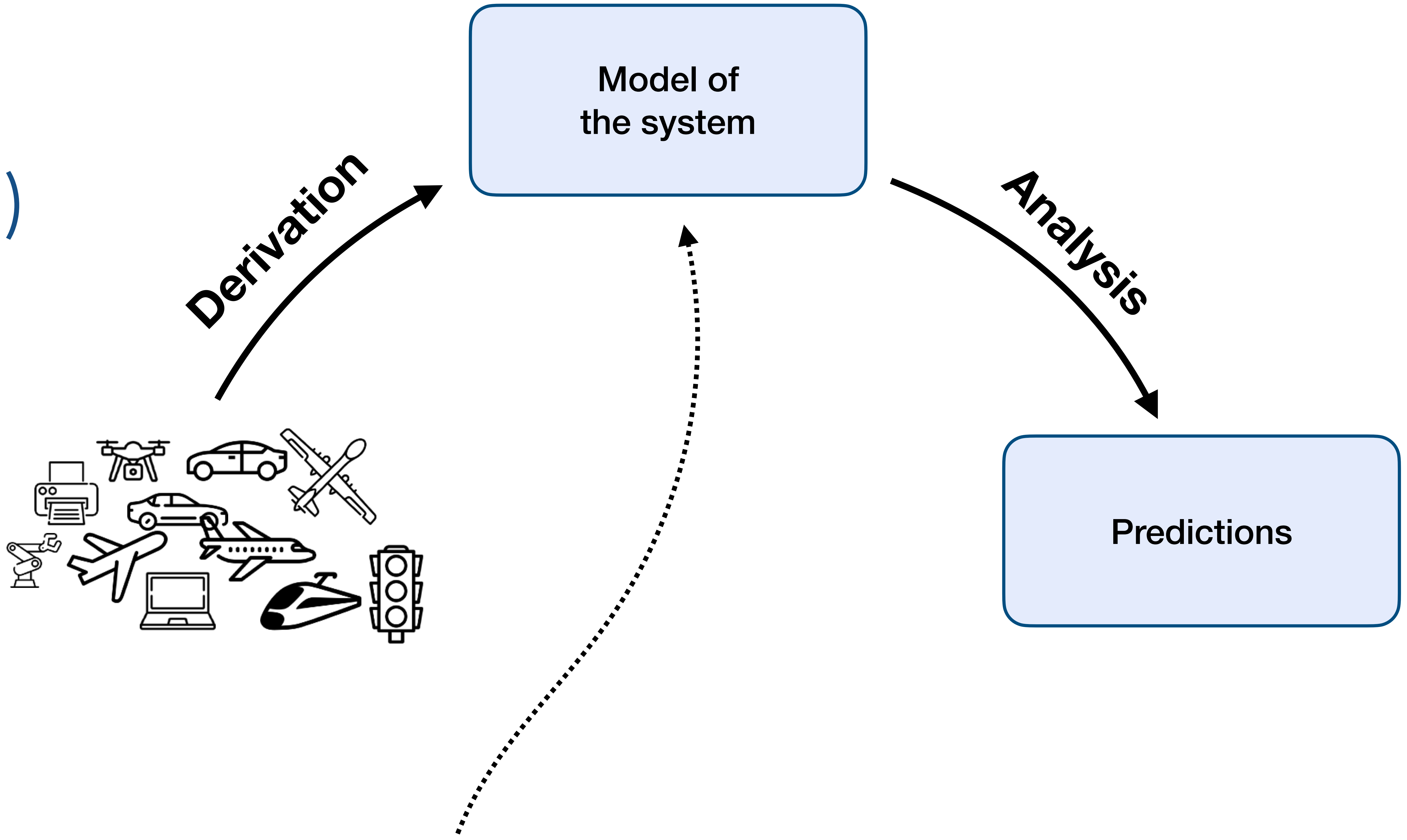

Interpretation of common models is pretty straightforward in the deterministic case

SPECIFICATIONS ARE LESS OBVIOUS IN THE STOCHASTIC CASE

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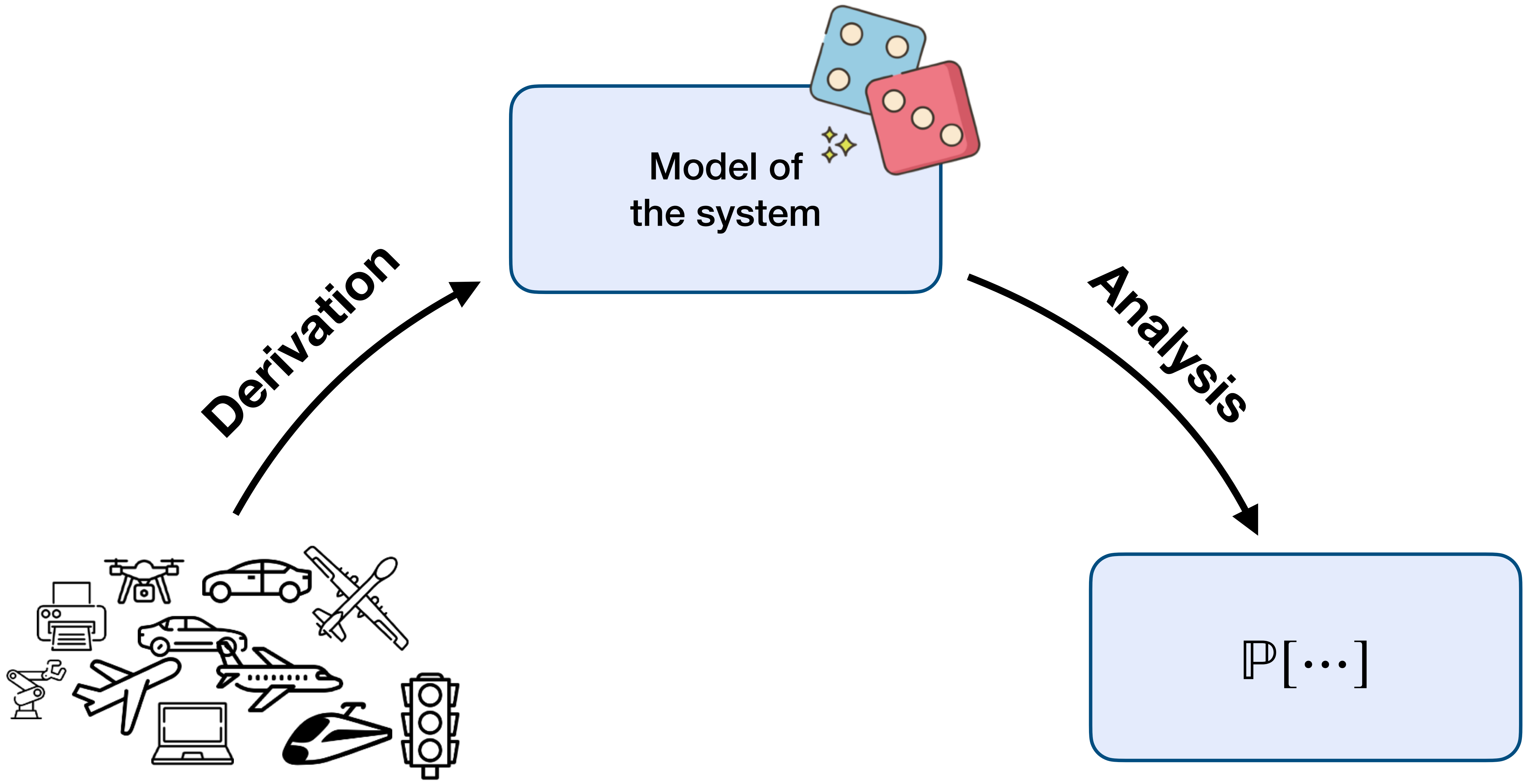

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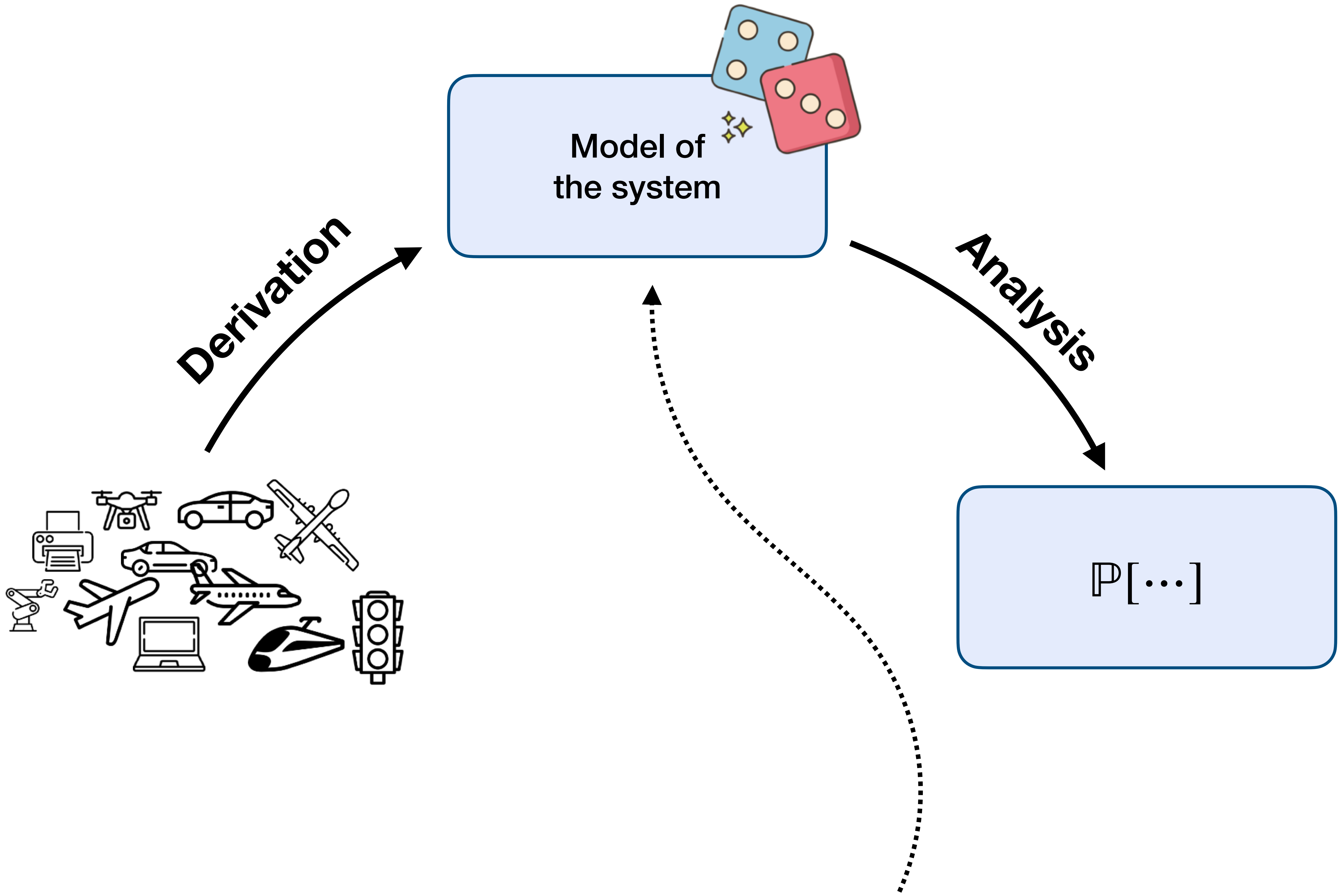

Specifications of stochastic RTSs are much less straightforward

THE CASE FOR STOCHASTIC RTS

Stochastic real-time systems

➔ Model of RTSs, where workload parameters are modelled *stochastically*

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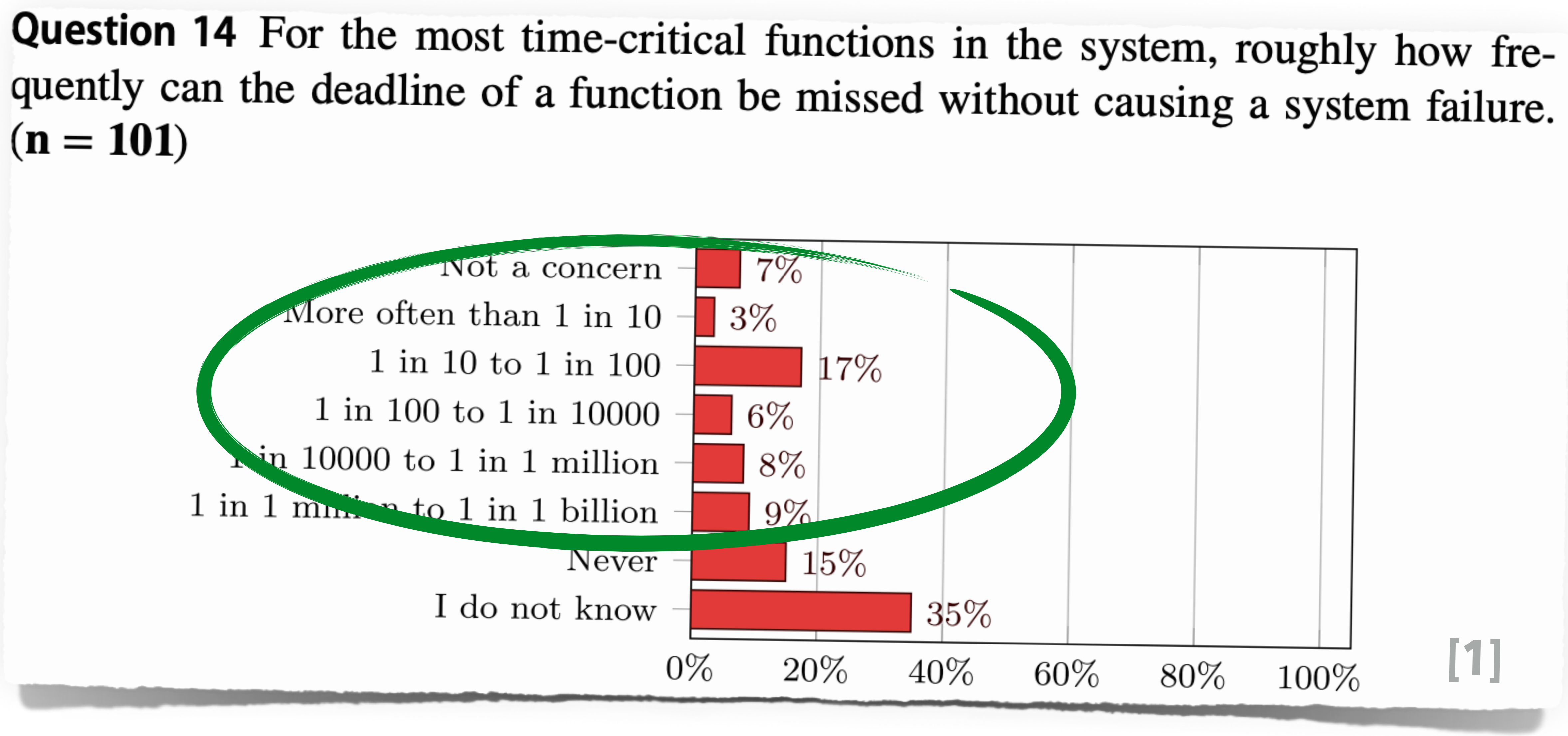

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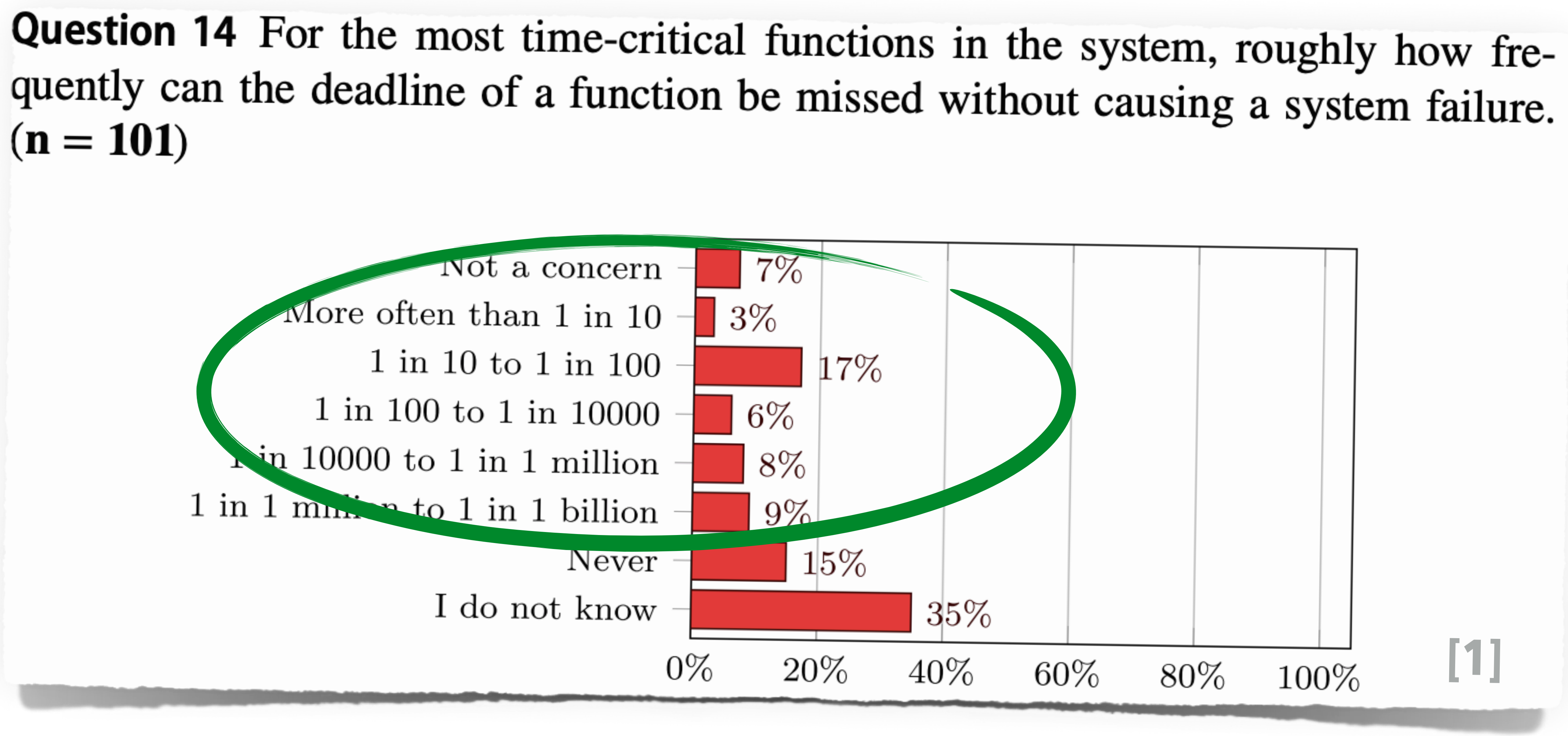

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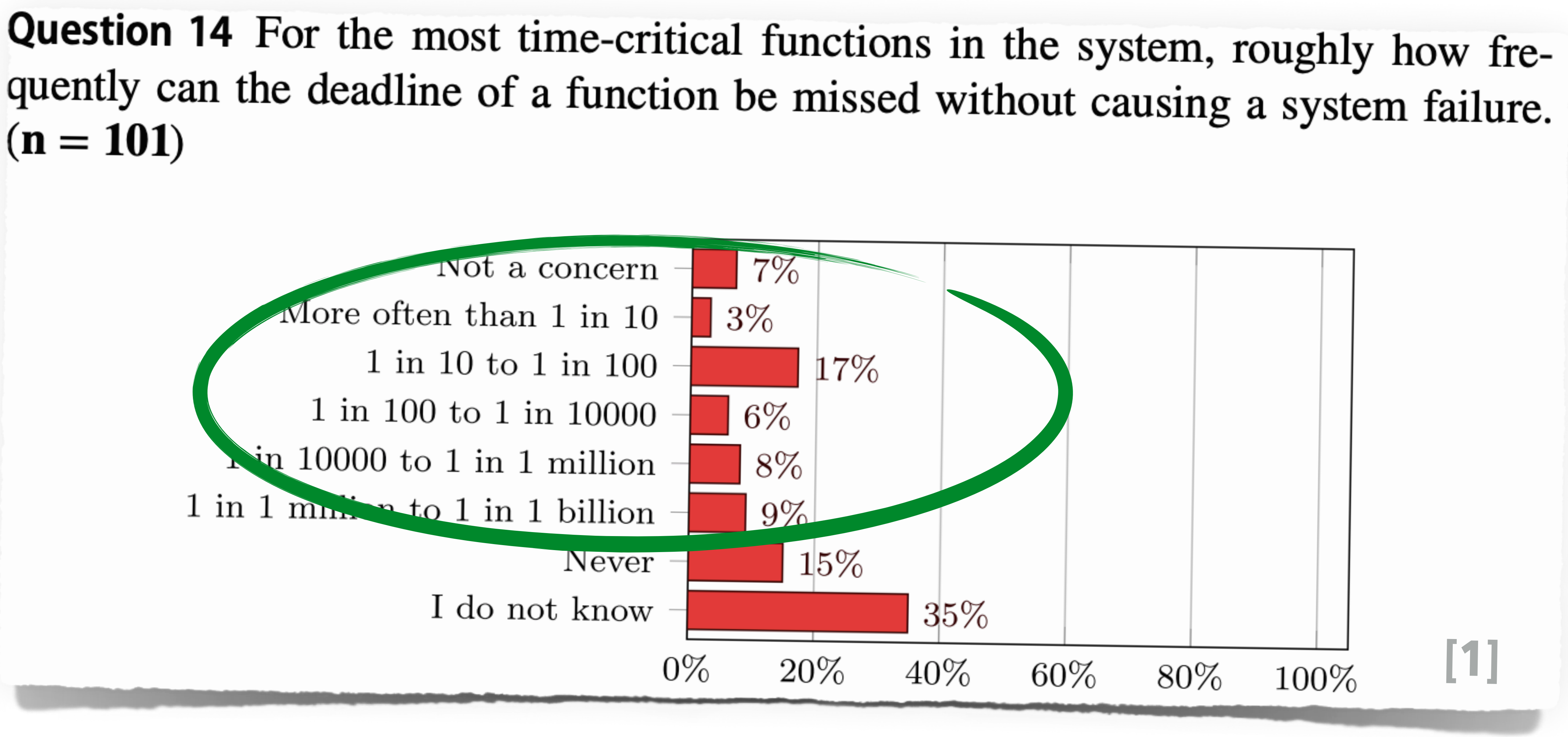
→ Enables analysis of transiently **overloaded systems**
  → Ubiquitous in practice
  → *E.g., FMTV Challenge 2016*



The total utilization of that system goes above 100%. Using response time analysis in such situation automatically yields unbounded (infinite) worst-case response times. [2]

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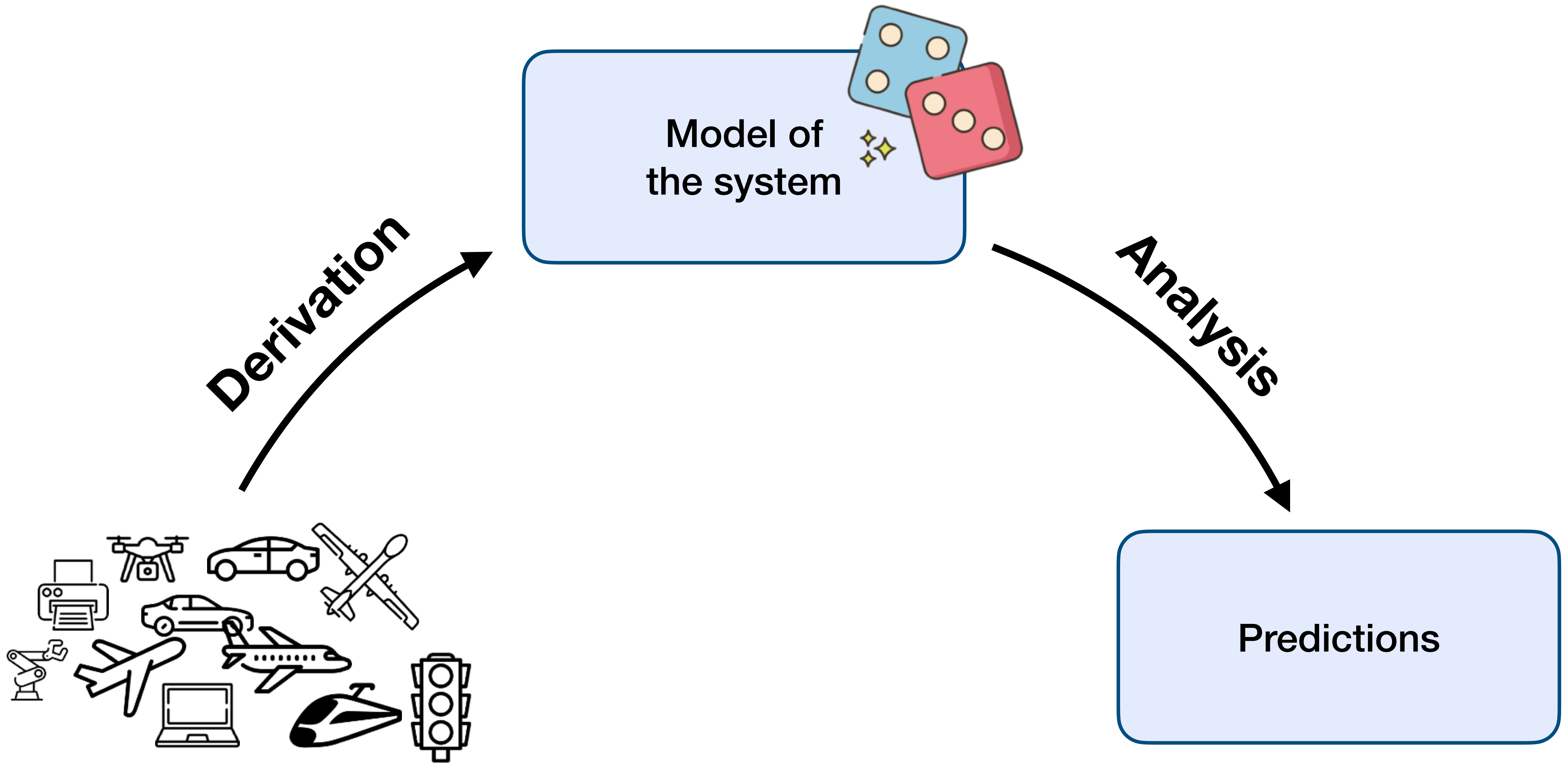
DEPENDENCY IN STOCHASTIC RTS

THE PROBLEM OF STOCHASTIC RTS: DEPENDENCY



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Probabilistic Execution Times (pETs)

Ground-truth behavior
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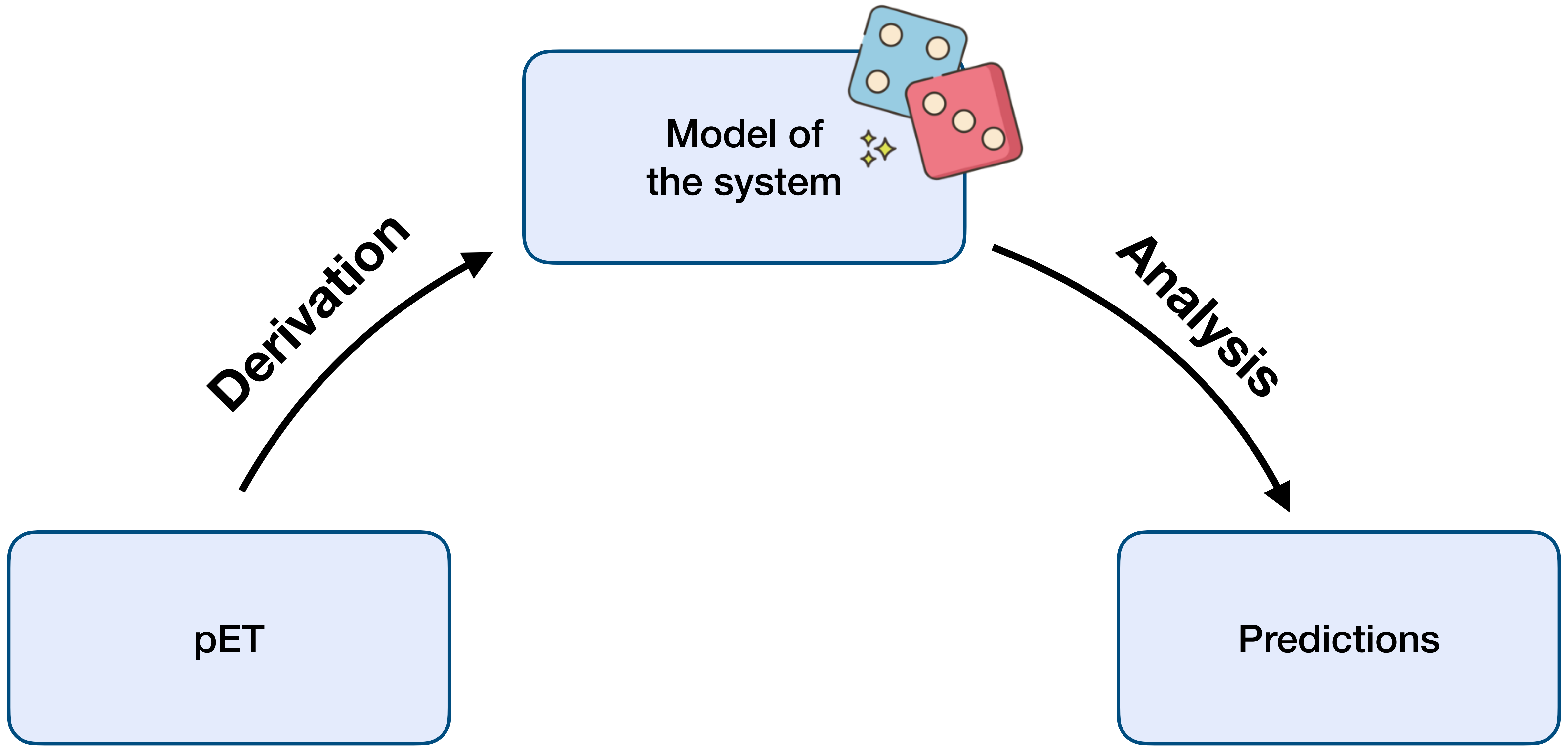

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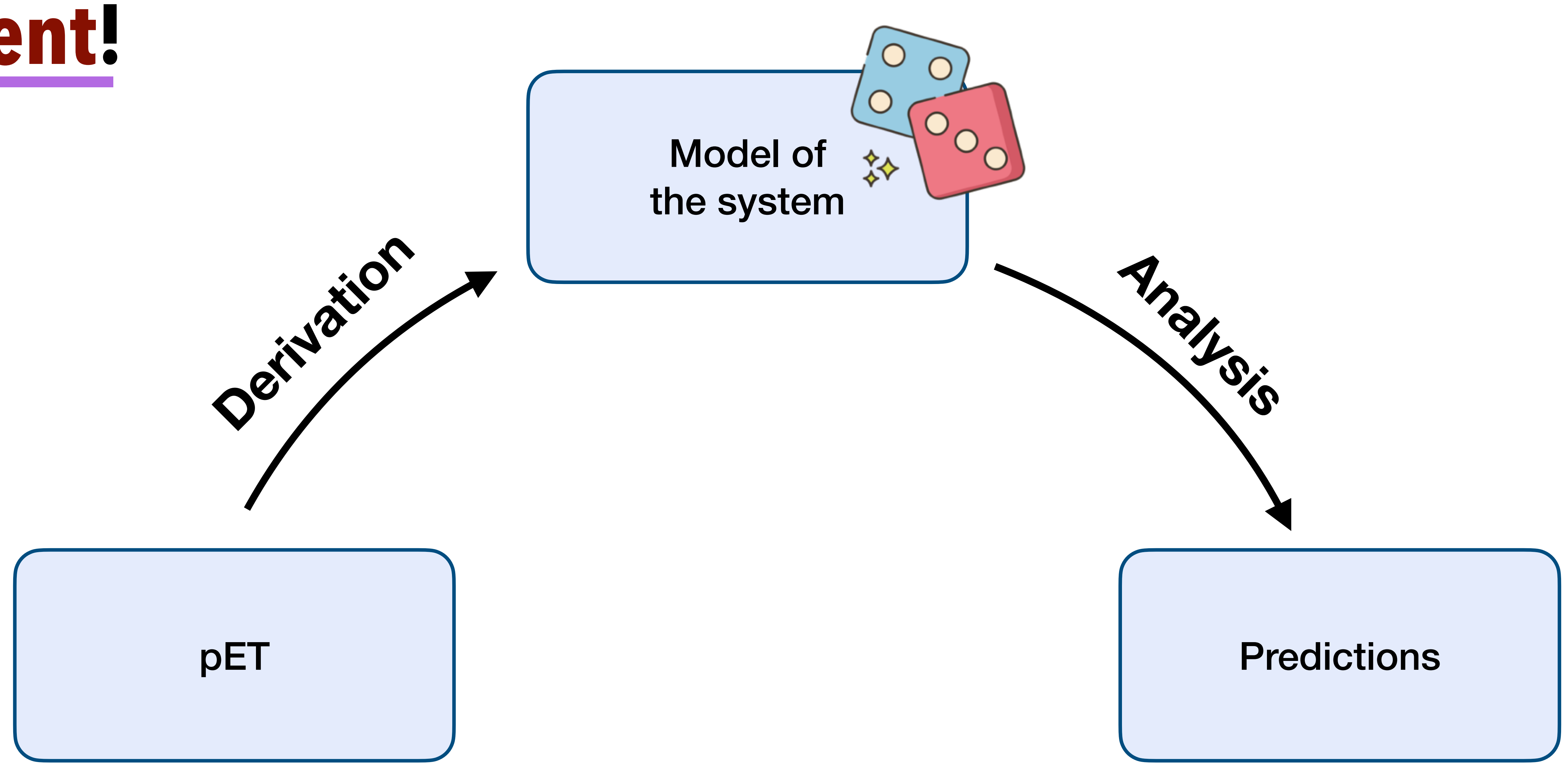

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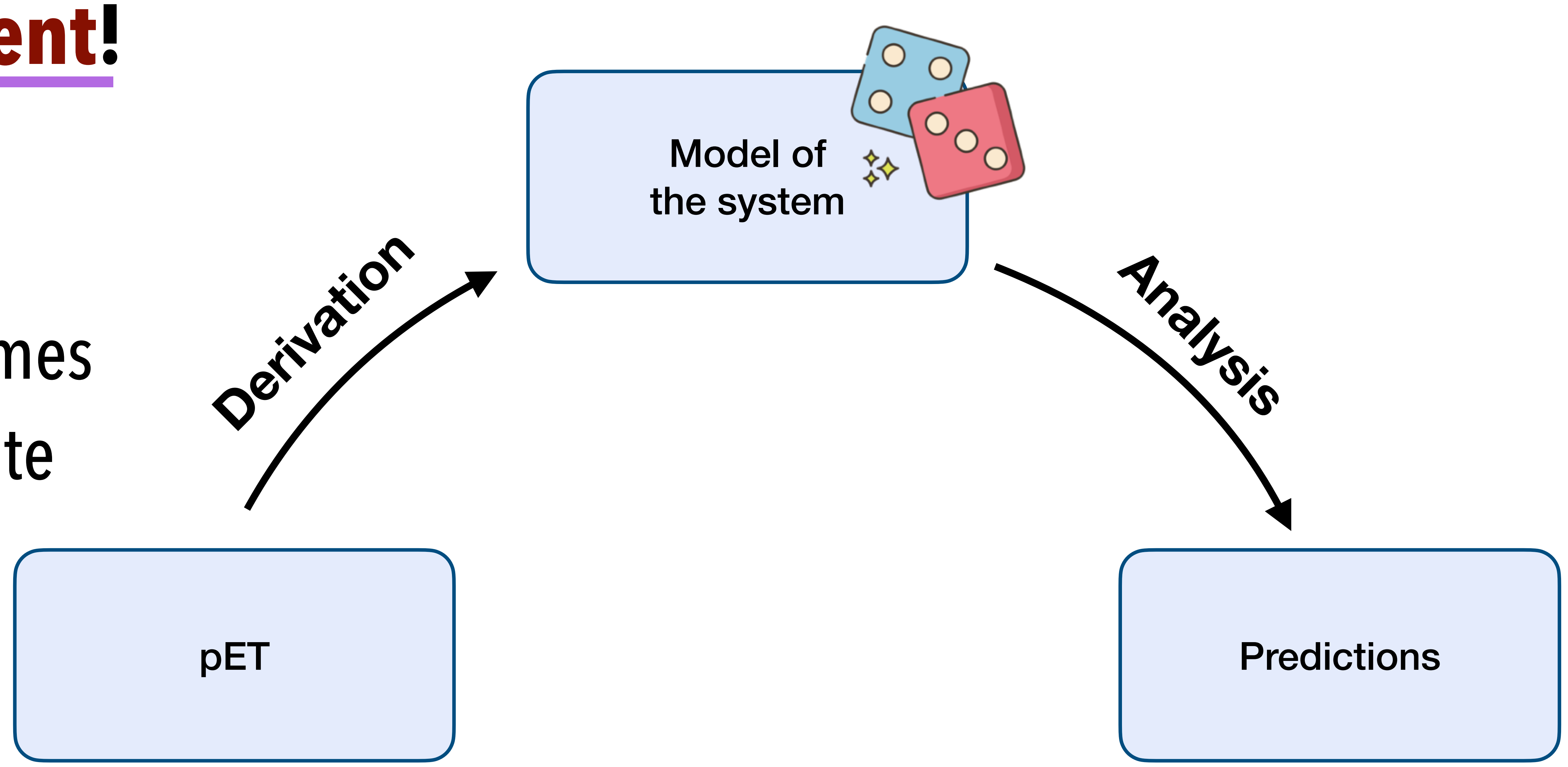

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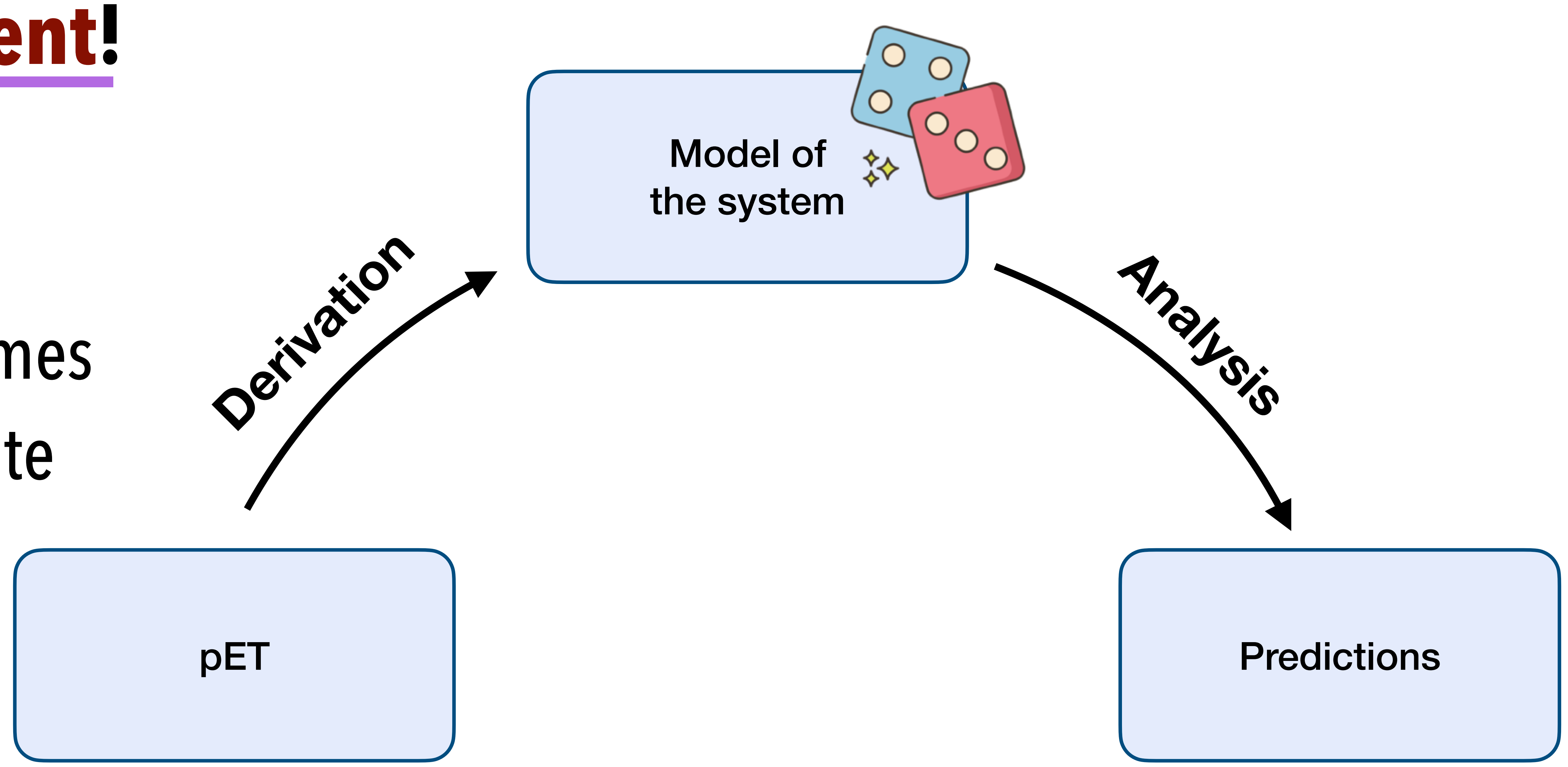

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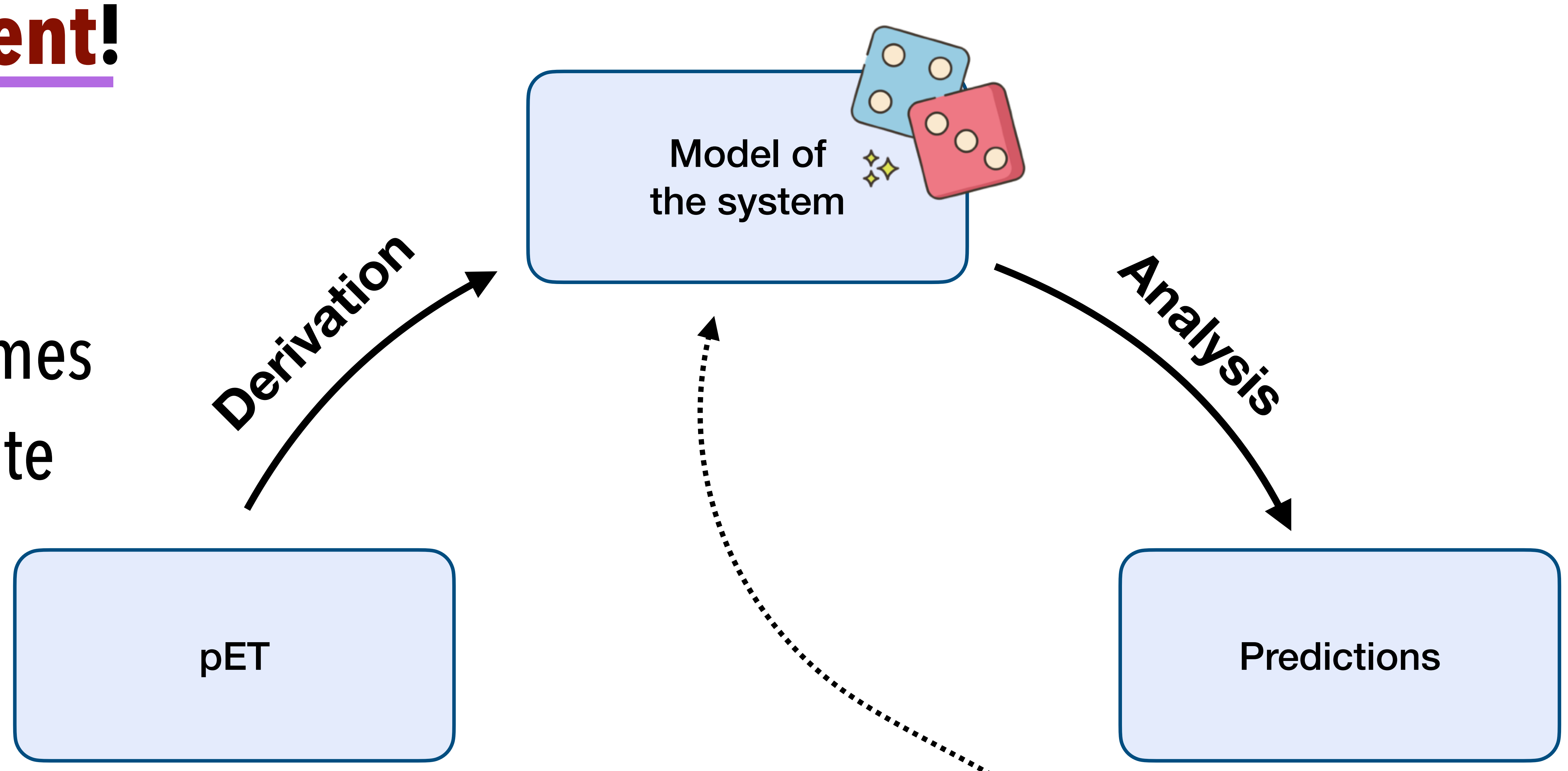

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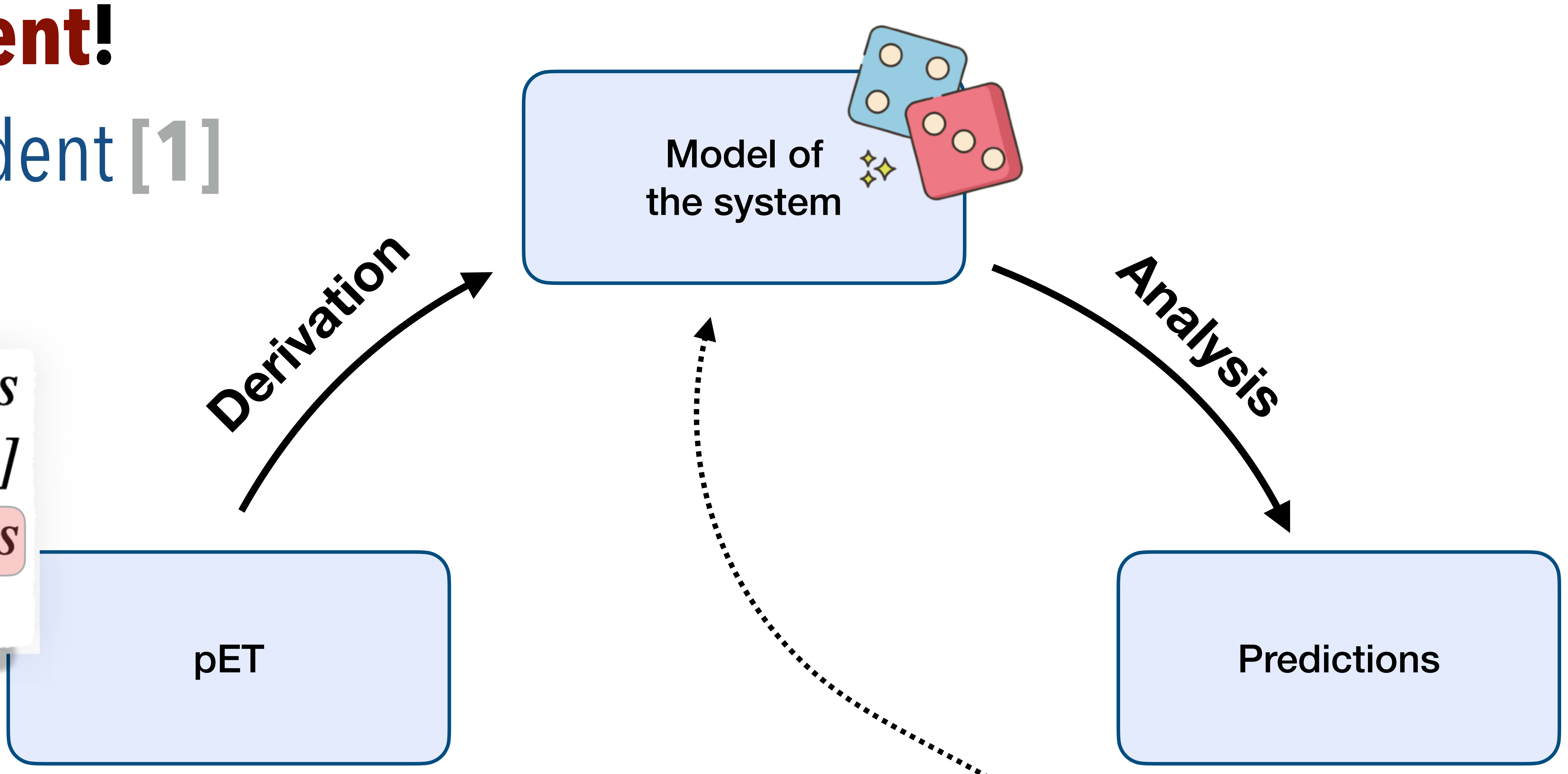
THE PROBLEM OF STOCHASTIC RTS: DEPENDENCY

Probabilistic Execution Times (pETs) are dependent!

- ➔ Tia *et al. 1995*: computation times are **not** independent [1]
  - ➔ Ignoring this fact may lead to incorrect bounds

> *"Unfortunately, the computation times of individual requests are not statistically independent. [. . .] As a consequence, the probability of meeting deadlines thus computed may be overly optimistic."*



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A: No, the results can be optimistic

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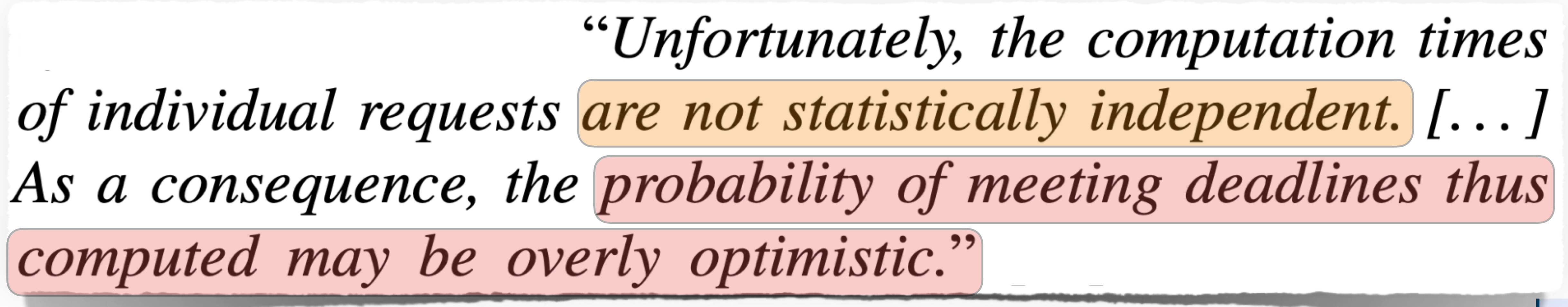
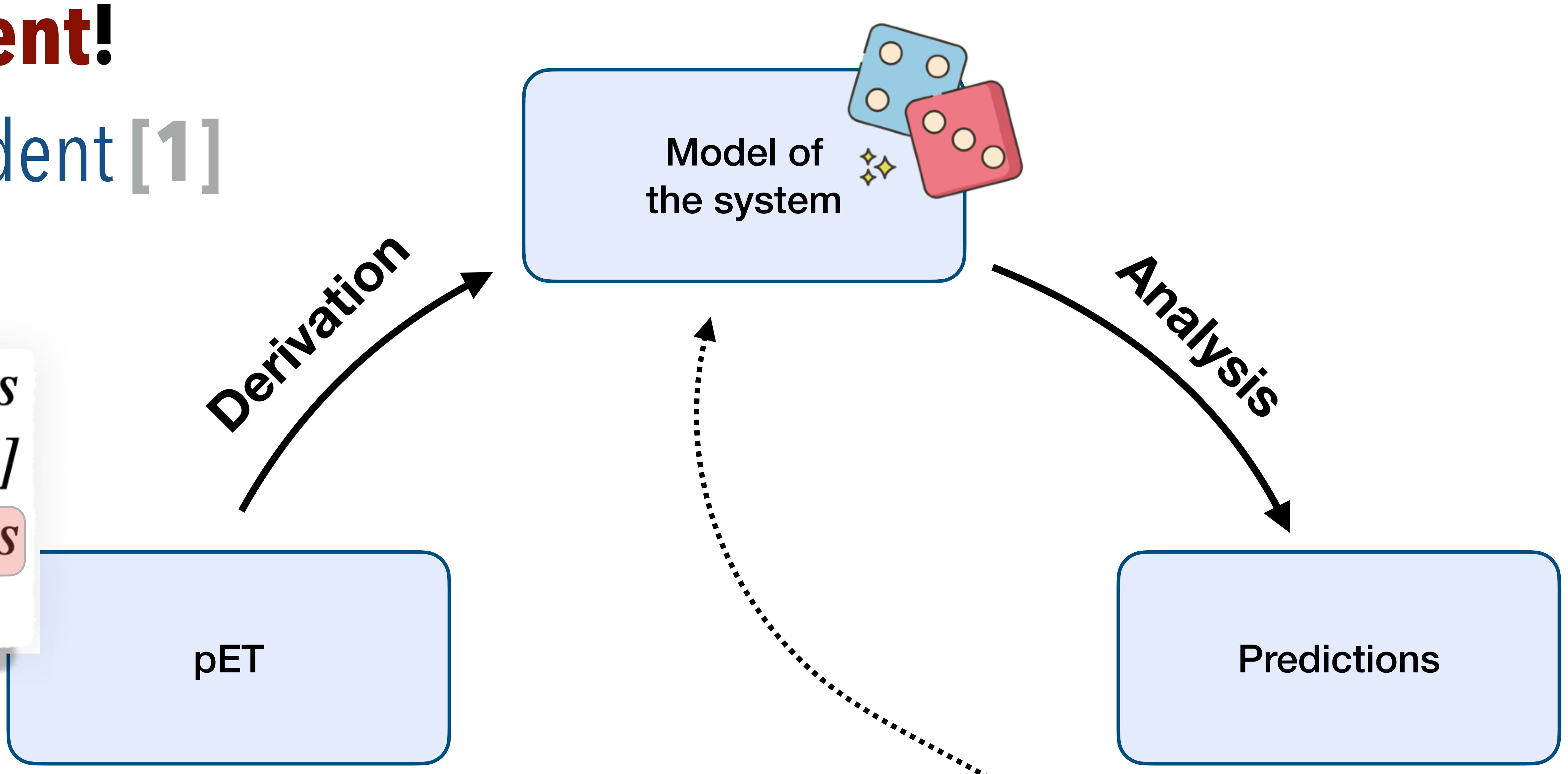


"Unfortunately, the computation times of individual requests are not statistically independent. [...] As a consequence, the probability of meeting deadlines thus computed may be overly optimistic."

- ➔ Limits the application of probability theory tools
  - ➔ *E.g.*, convolution is not applicable



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PRIOR WORK: pWCET

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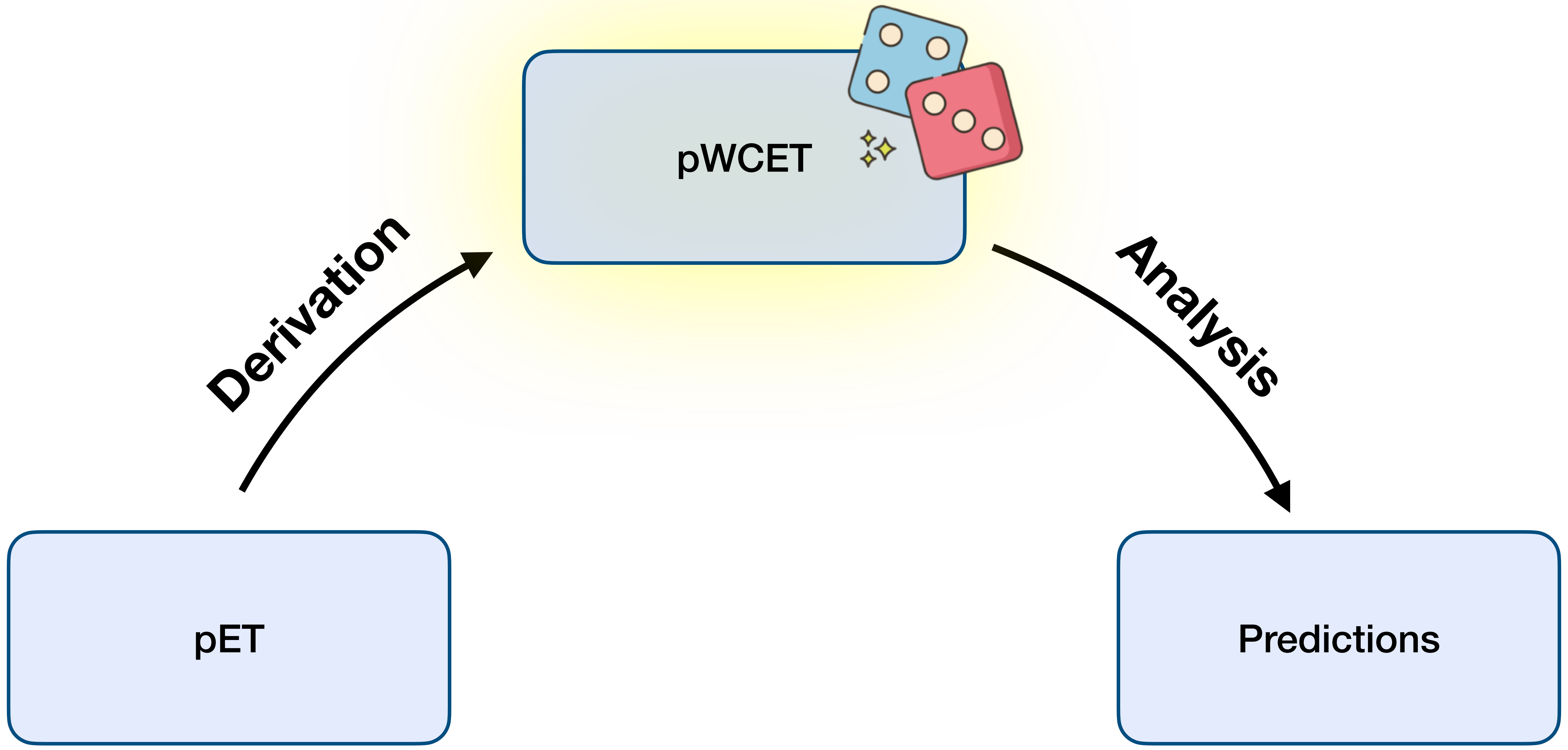

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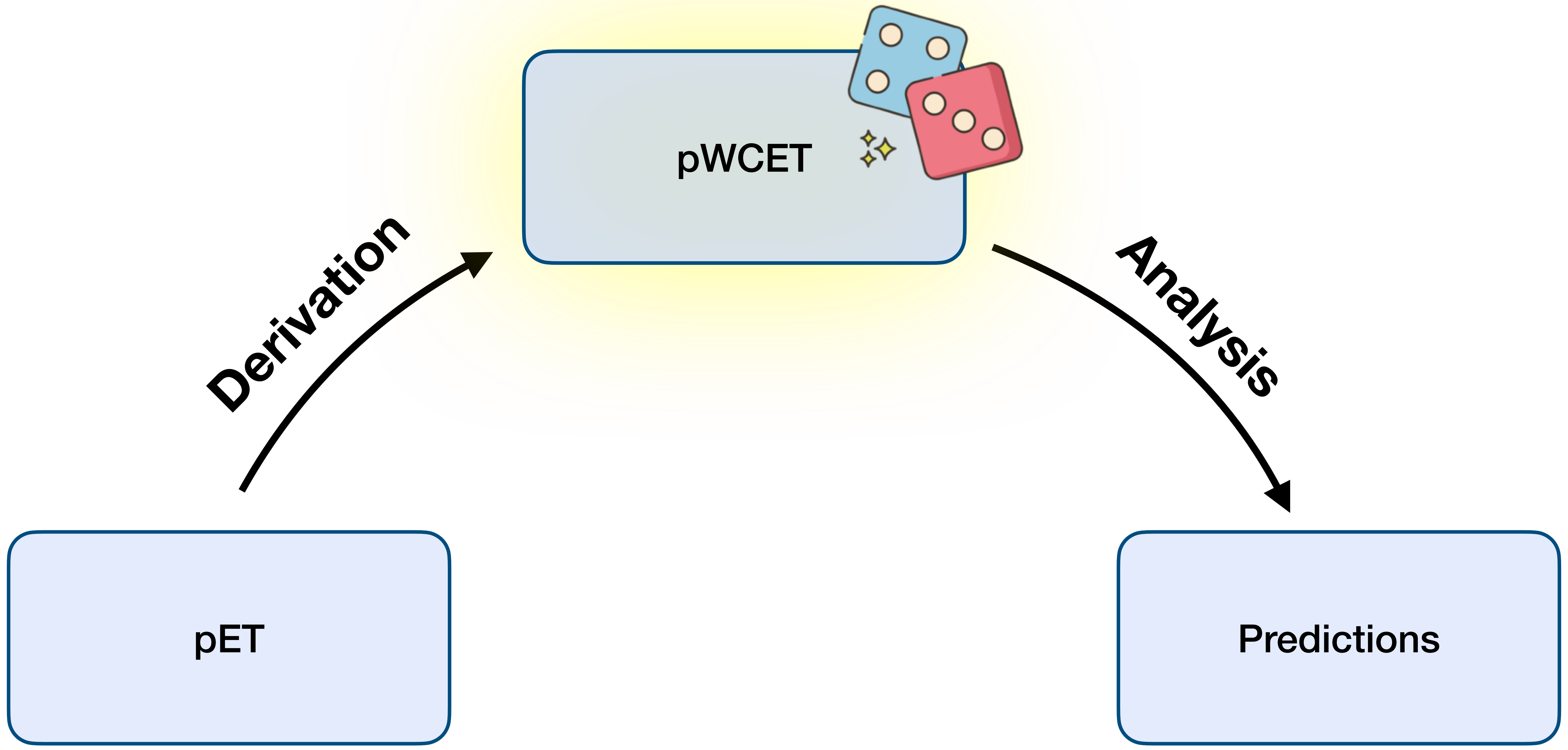

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We note that the actual execution times for a sequence of jobs of a task, which exercise the same or different paths, may well show strong correlations and dependences. It is the *modelling* of the execution times via an appropriate pWCET distribution which enables probabilistic independence to be assumed. (This is similar to the conventional case of a single WCET which can similarly be used in this way, even though the actual execution times of different jobs have strong dependences).

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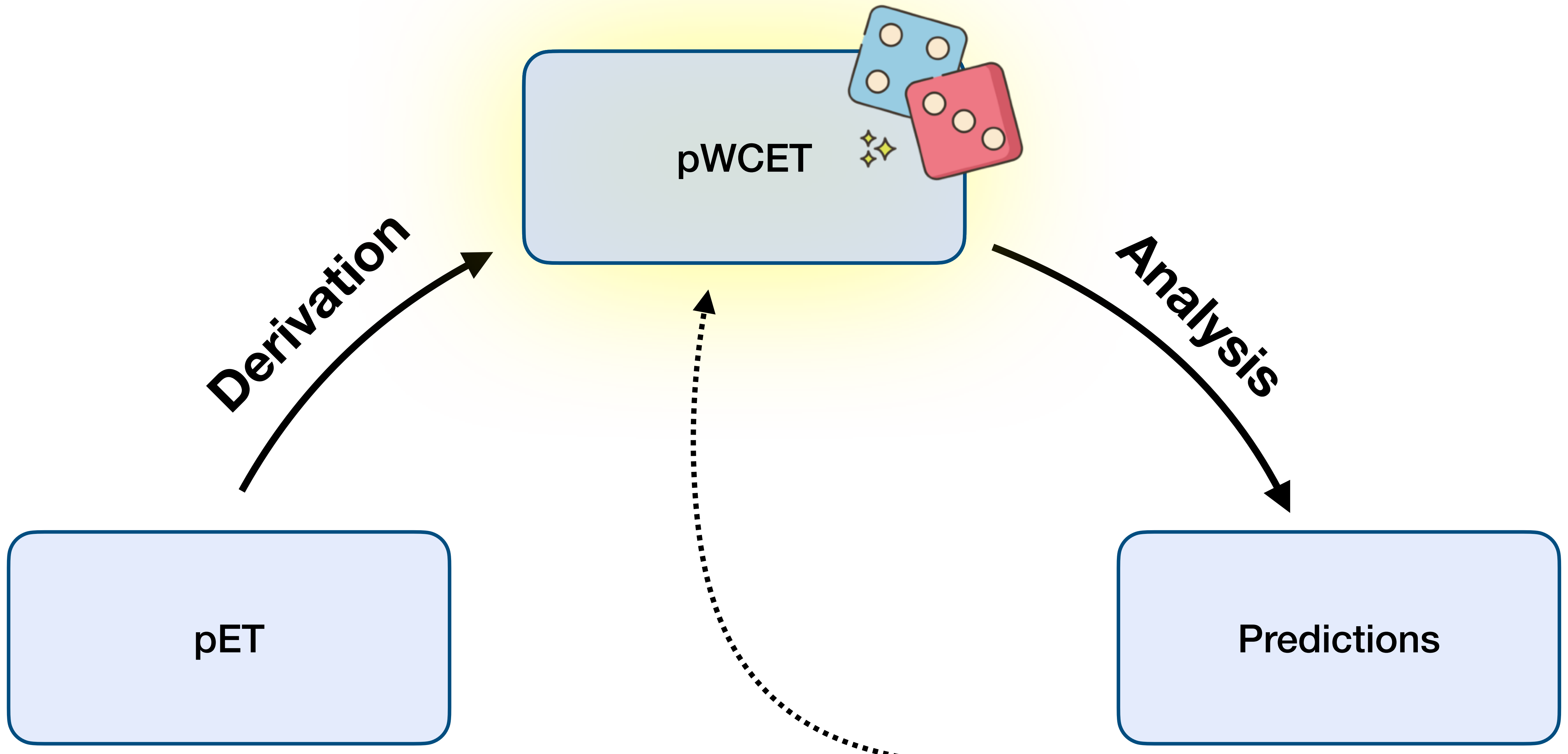

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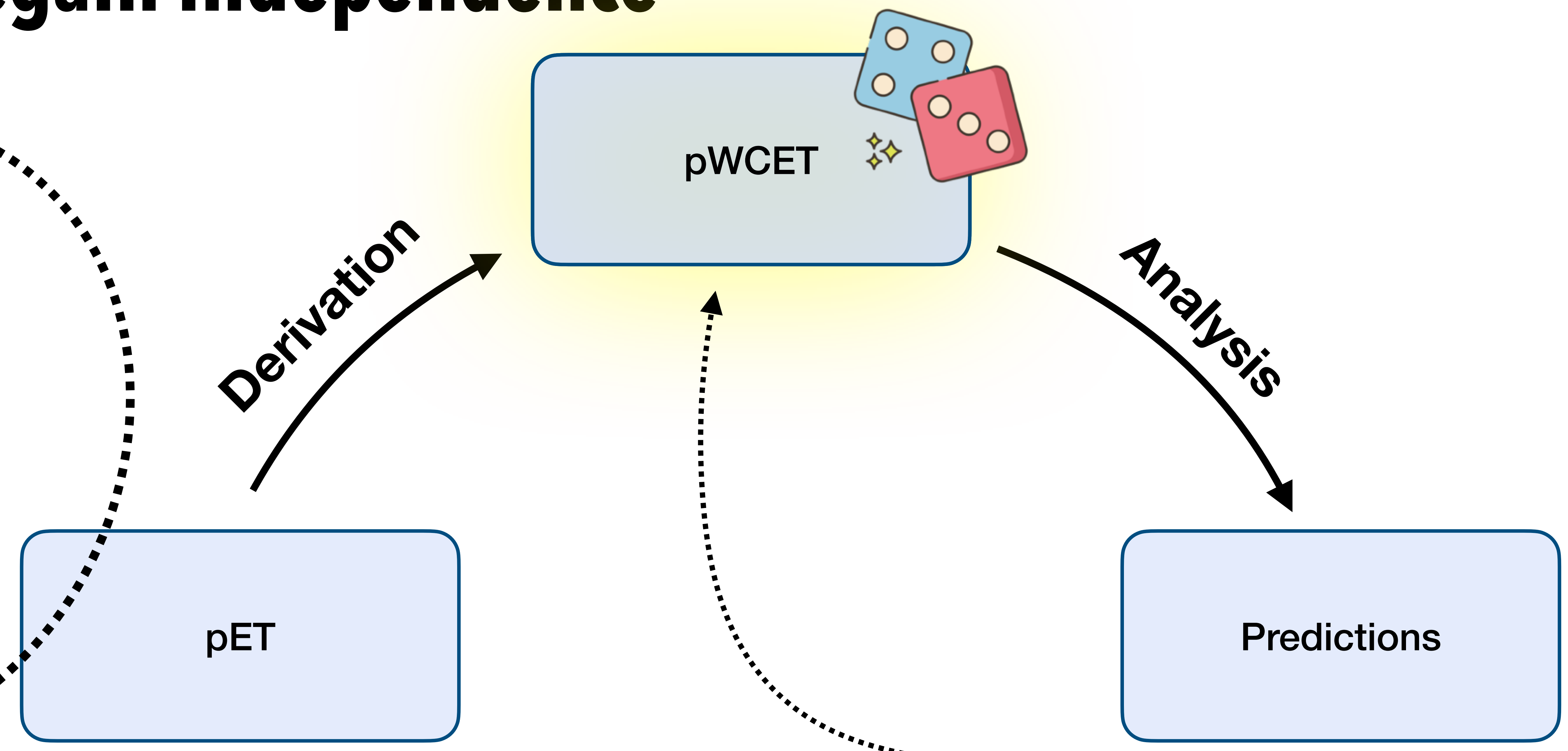
➔ *Goal*: enable "IID reasoning"



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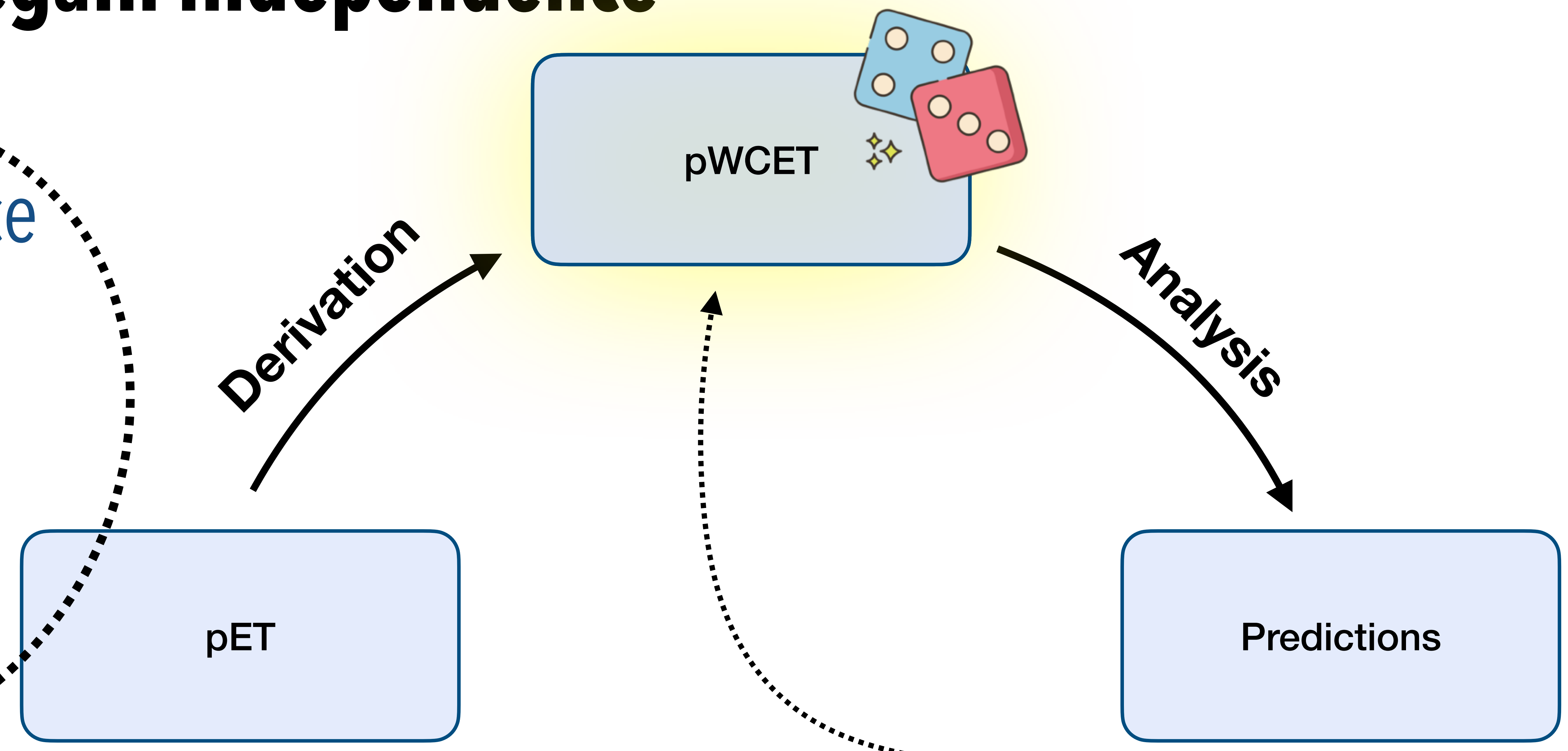

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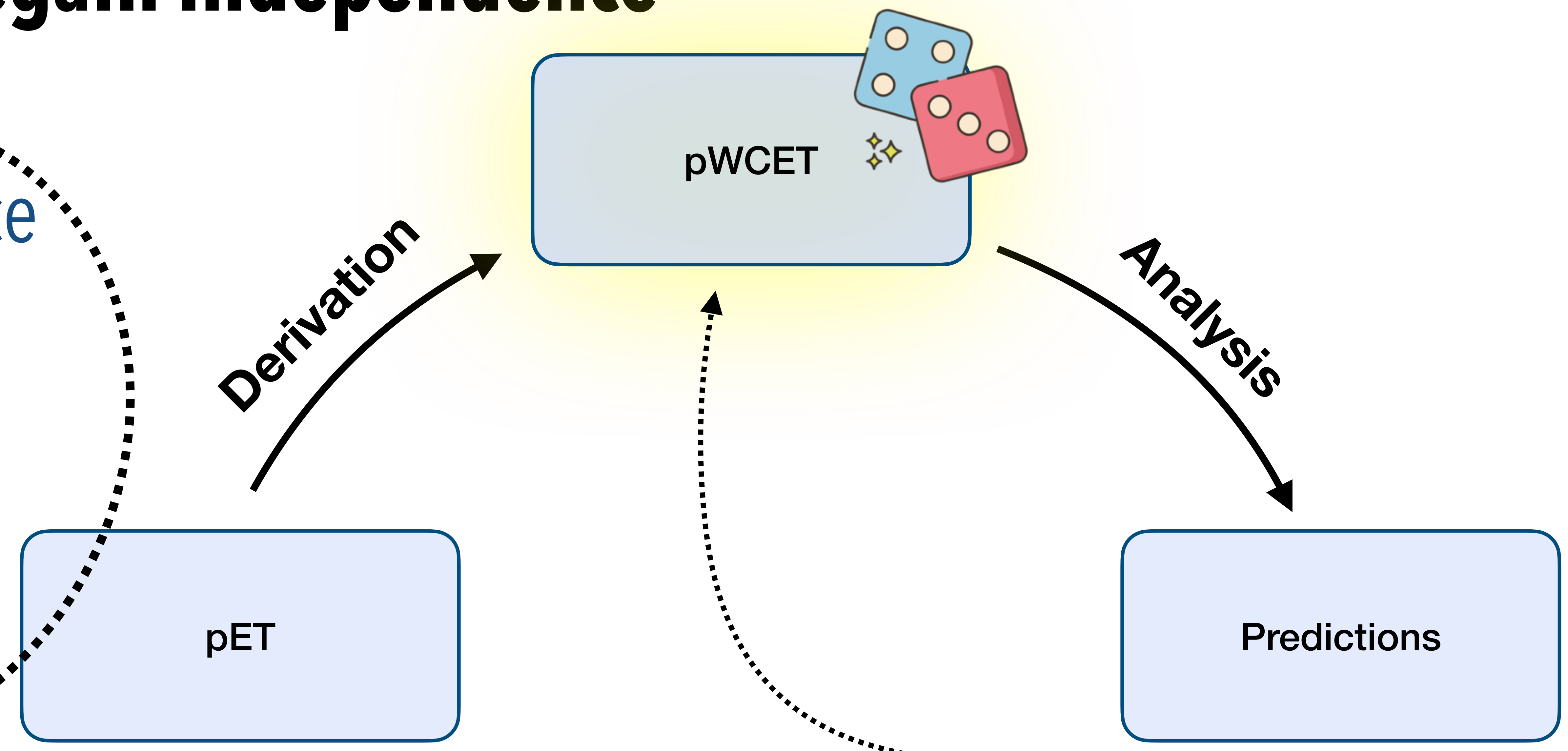
- ➔ .... but when **exactly** is a pWCET distribution "appropriate"?



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▶ **Definition 2.** The *probabilistic Worst-Case Execution Time (pWCET)* distribution for a task is the least upper bound, in the sense of the greater than or equal to operator $\succeq$ (defined below), on the execution time distribution of the jobs of the task for every valid scenario of operation, where a *scenario* of operation is defined as an infinitely repeating sequence of input states (including both input values and software state variables) and initial hardware states that characterise a feasible way in which recurrent execution of the task may occur. [1]

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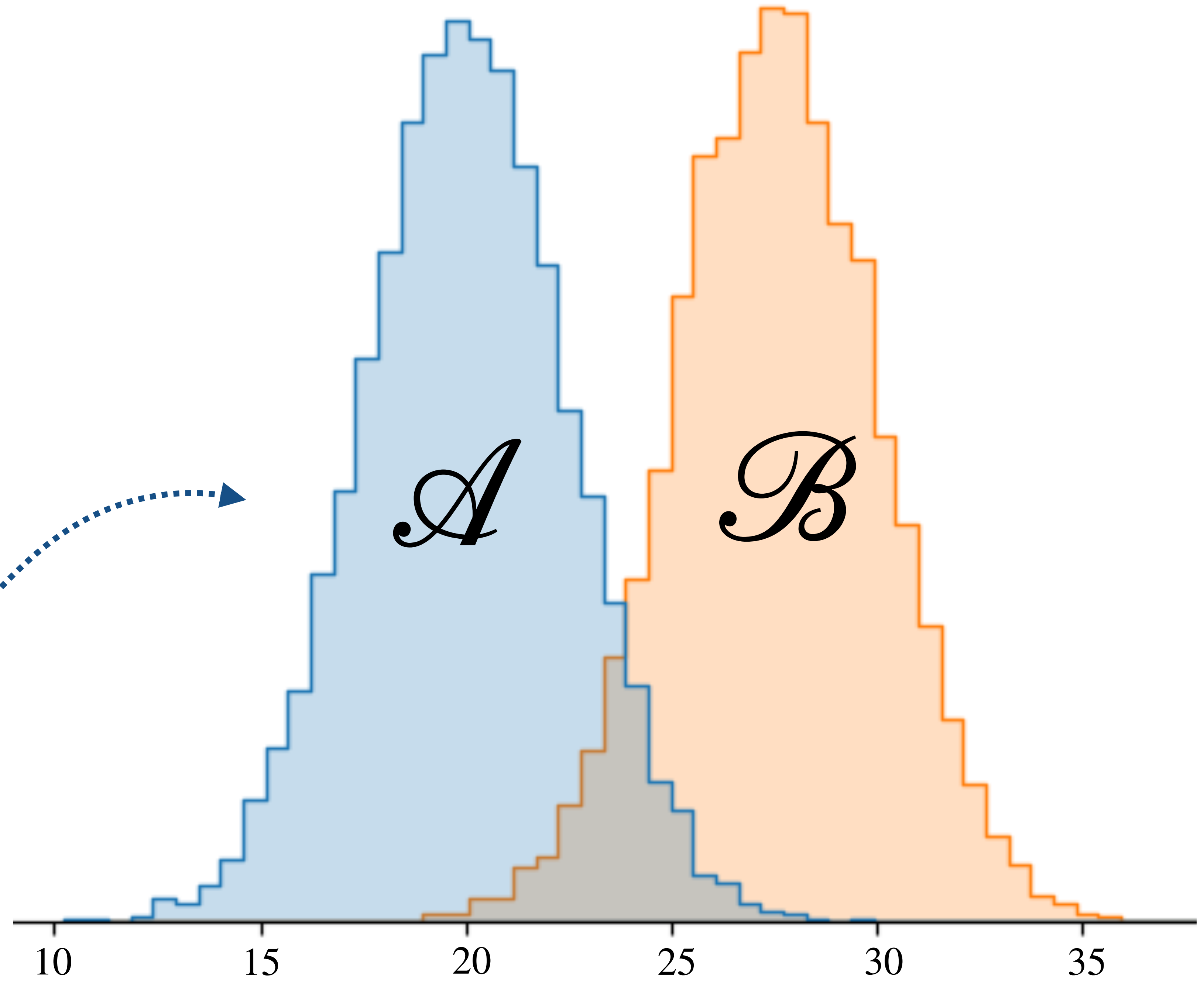
Side note: dominance relation $\preceq$

- ➔ Proposed by Diaz *et al.* in 2004 [2]
- ➔ Partial order on random variables
  - ➔ Similar to stochastic dominance
- ➔ $\mathscr{A} \preceq \mathscr{B} := \forall x, \mathbb{P}[\mathscr{A} \leq x] \geq \mathbb{P}[\mathscr{B} \leq x]$



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- ➔ **Does not** necessarily enable IID-based analyses

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SOTA pWCET *DOES NOT* ENABLE IID ANALYSIS

Already noted in [1]

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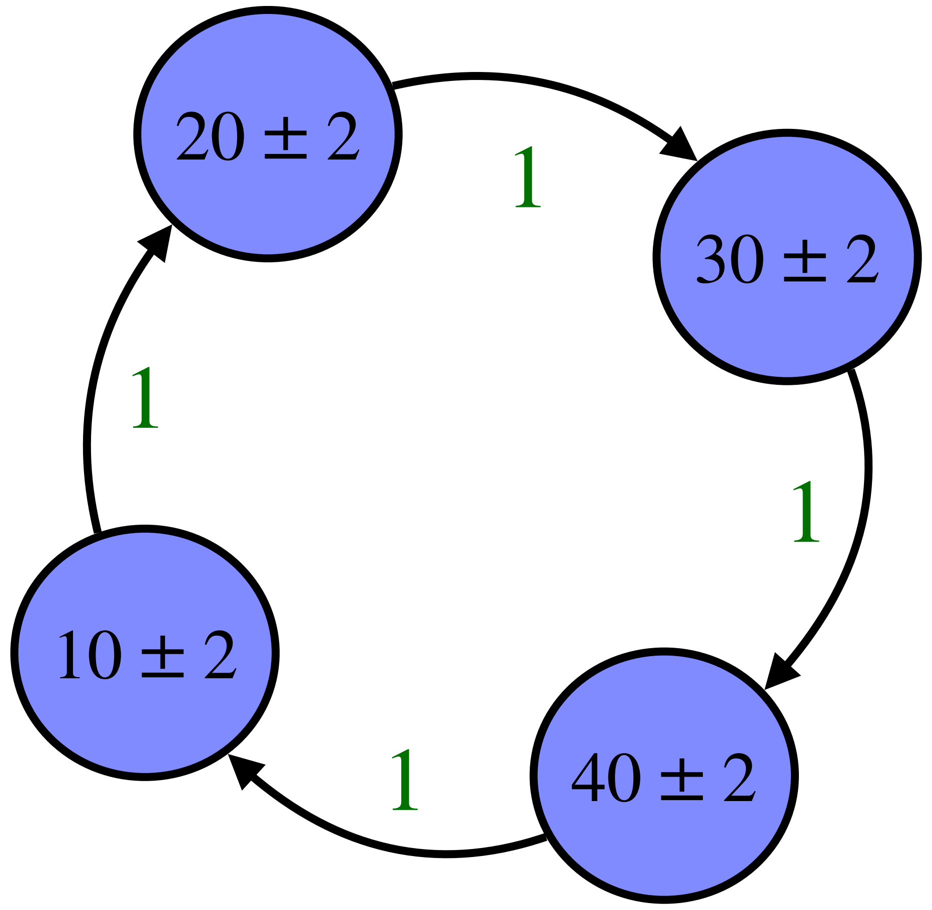
A toy system: [1]

- ➔ Time-predictable hardware
- ➔ System has **four** states
- ➔ State **cycling through** its four possible values
- ➔ Small variability in each of the states
- ➔ Starts with random state



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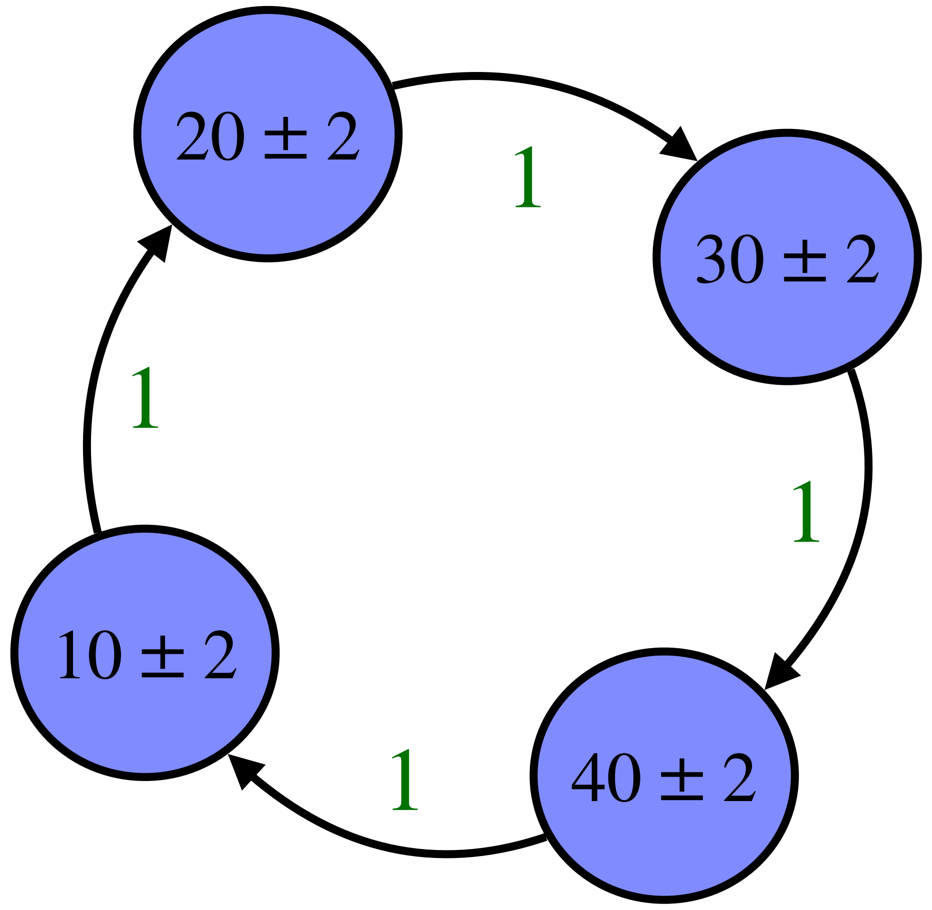

- Resulting pET distribution: [1]

$$\begin{pmatrix} 10 \pm 2 & 20 \pm 2 & 30 \pm 2 & 40 \pm 2 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}$$

- Valid pWCET distribution: [1]

$$\begin{pmatrix} 12 & 22 & 32 & 42 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}$$

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- Small variability in each of the states
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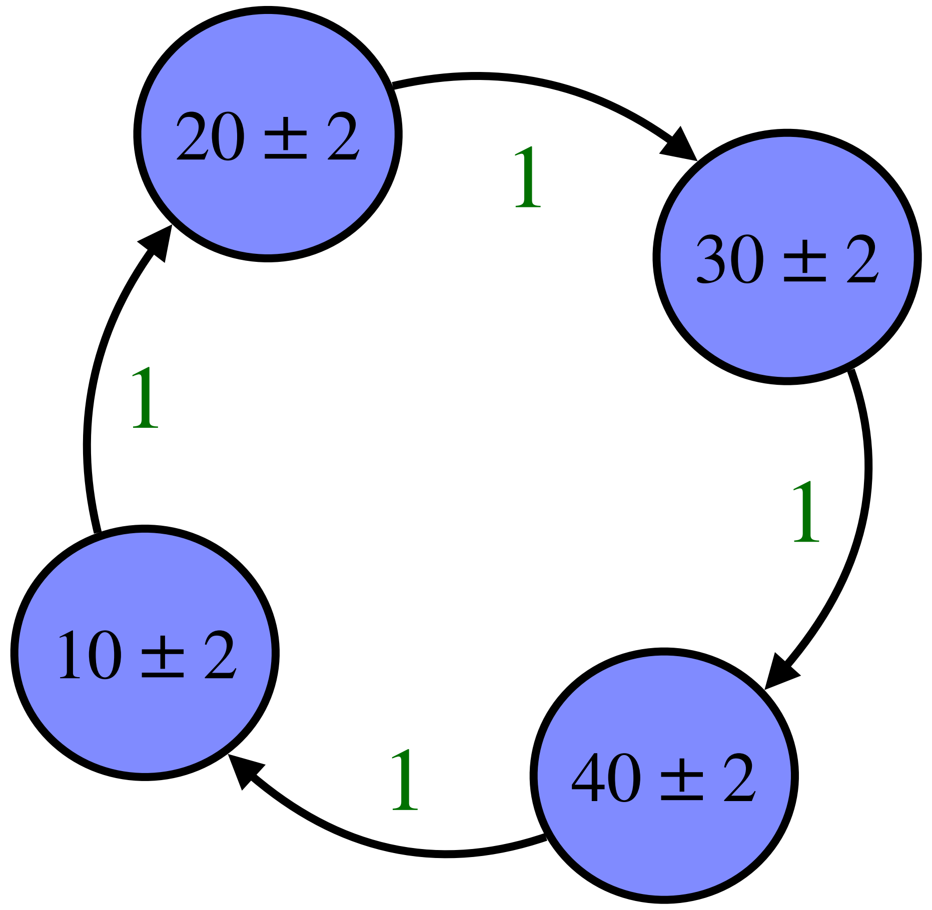

- Resulting pET distribution: [1]

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$$\begin{pmatrix} 12 & 22 & 32 & 42 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}$$

Except that ....

- Smallest workload of four consecutive jobs:
  - $(10 - 2) + (20 - 2) + (30 - 2) + (40 - 2) = 92$

$$\mathbb{P}\left[\sum_4 \text{pET} \geq 92\right] = 1$$

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SOTA pWCET *DOES NOT* ENABLE IID ANALYSIS

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A toy system: [1]

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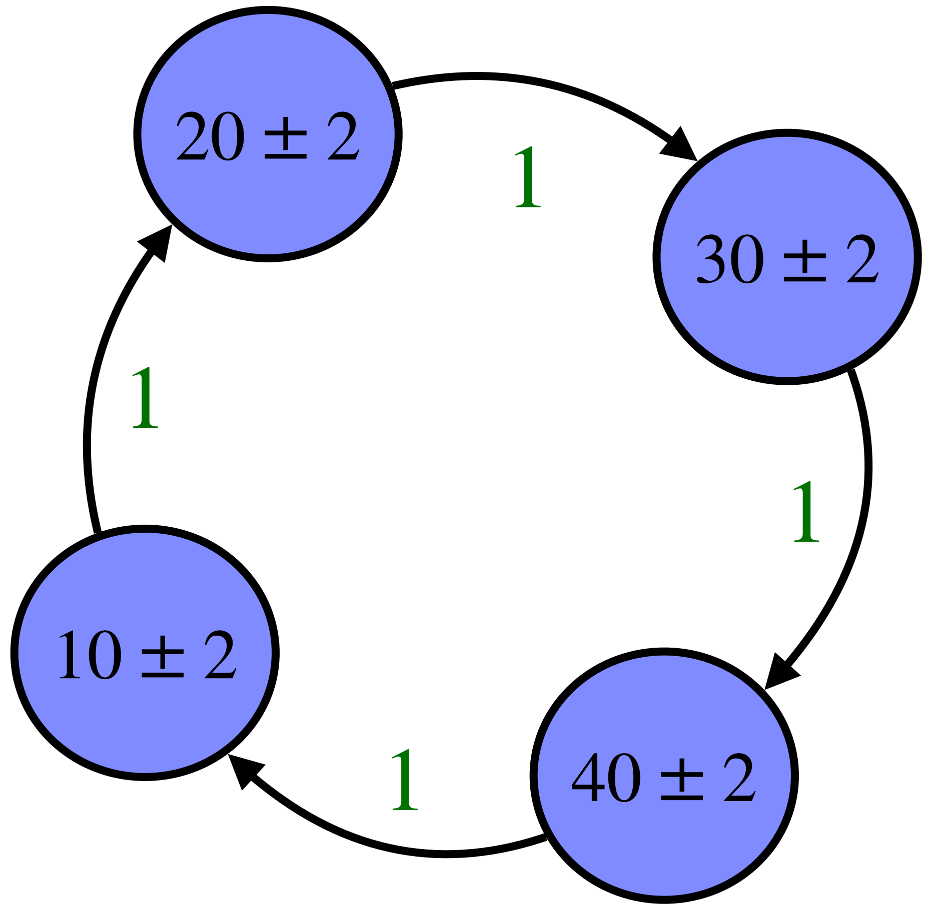

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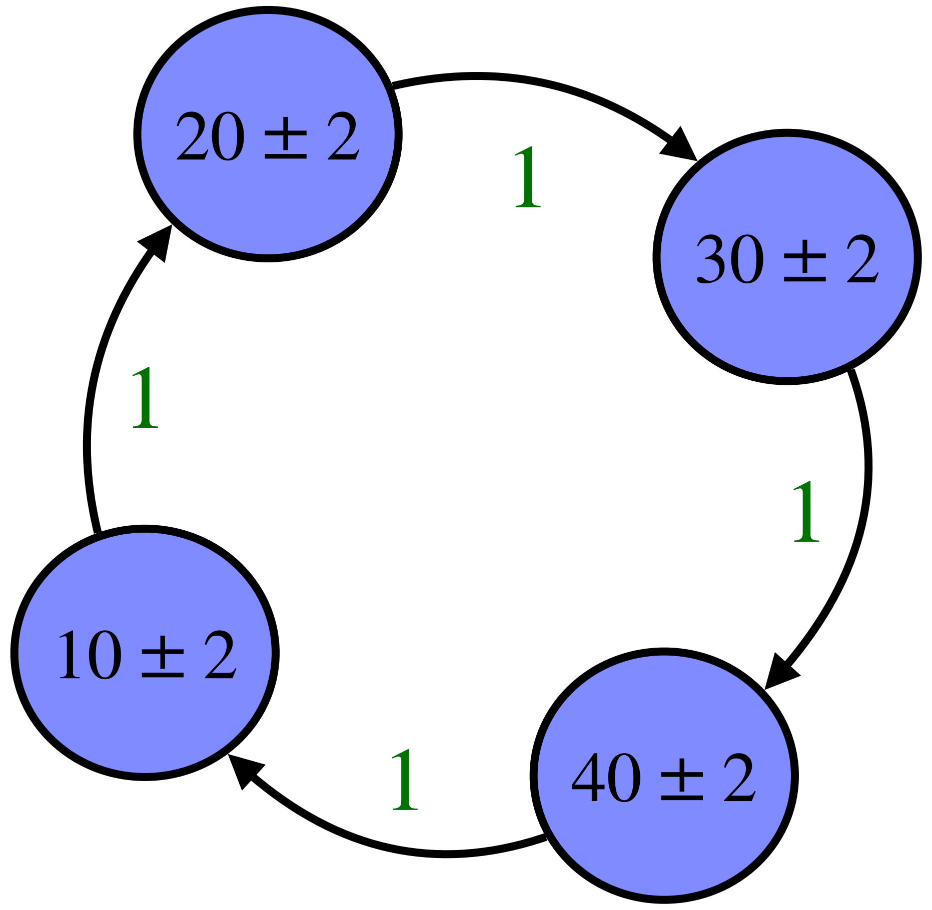

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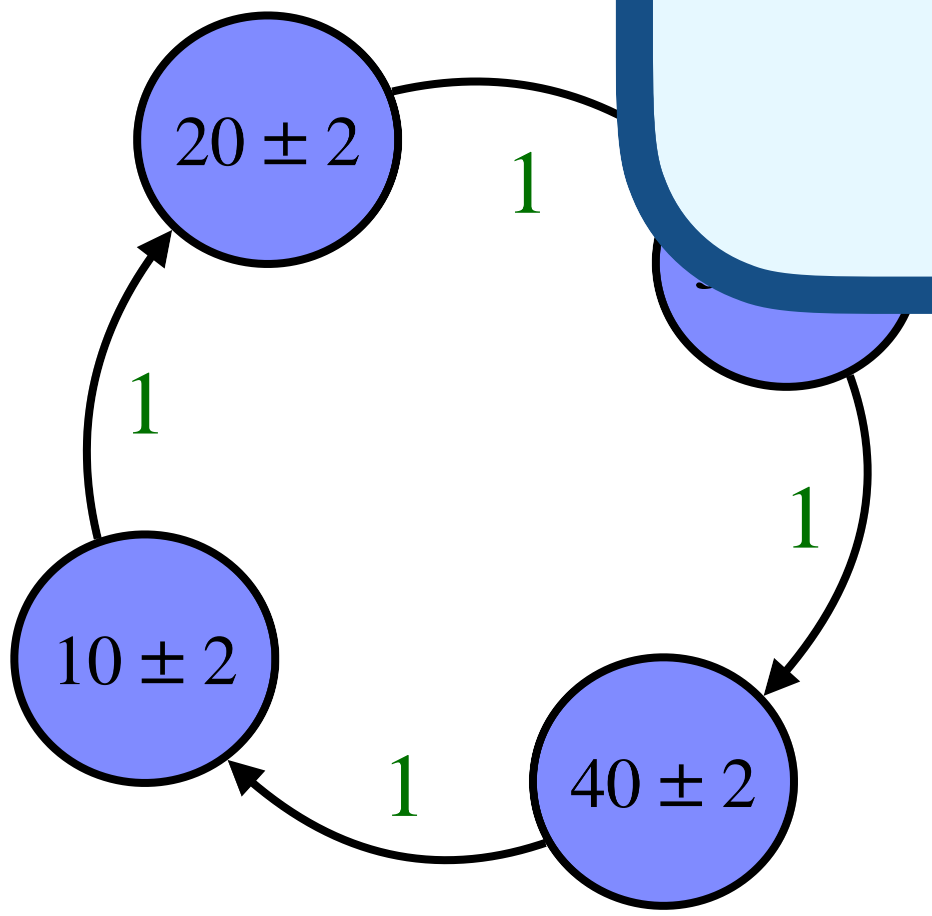

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So, what is "appropriate" pWCET?

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OUR PROPOSAL:
AXIOMATIC pWCET

DESIGN GOALS

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- *Definitions that are mathematically formal and unambiguous*

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Precise enough to be **mechanisable** in Coq

- *pET, pWCET and adequacy property with its proof must be formalisable in Coq proof assistant*

AXIOMATIC pWCET

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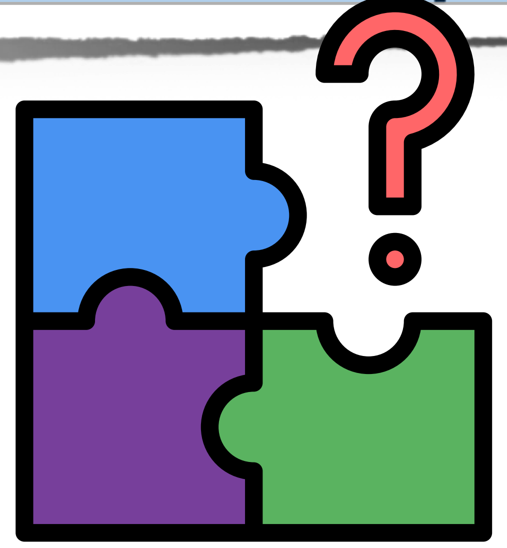

"Appropriate" pWCET part?

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Axiomatic pWCET: *scenario of operation* must ensure that jobs' pETs become independent

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ADEQUACY

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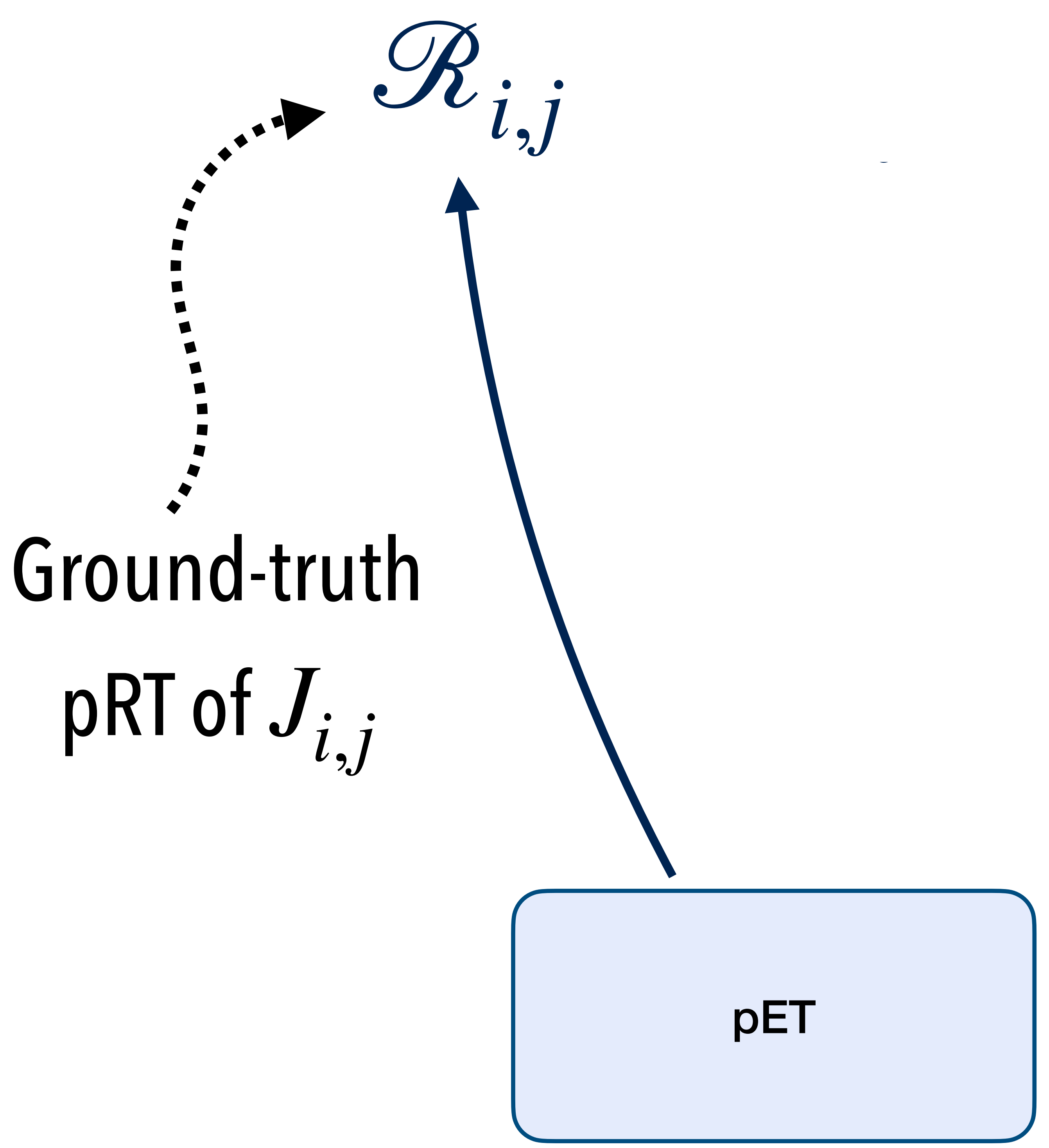

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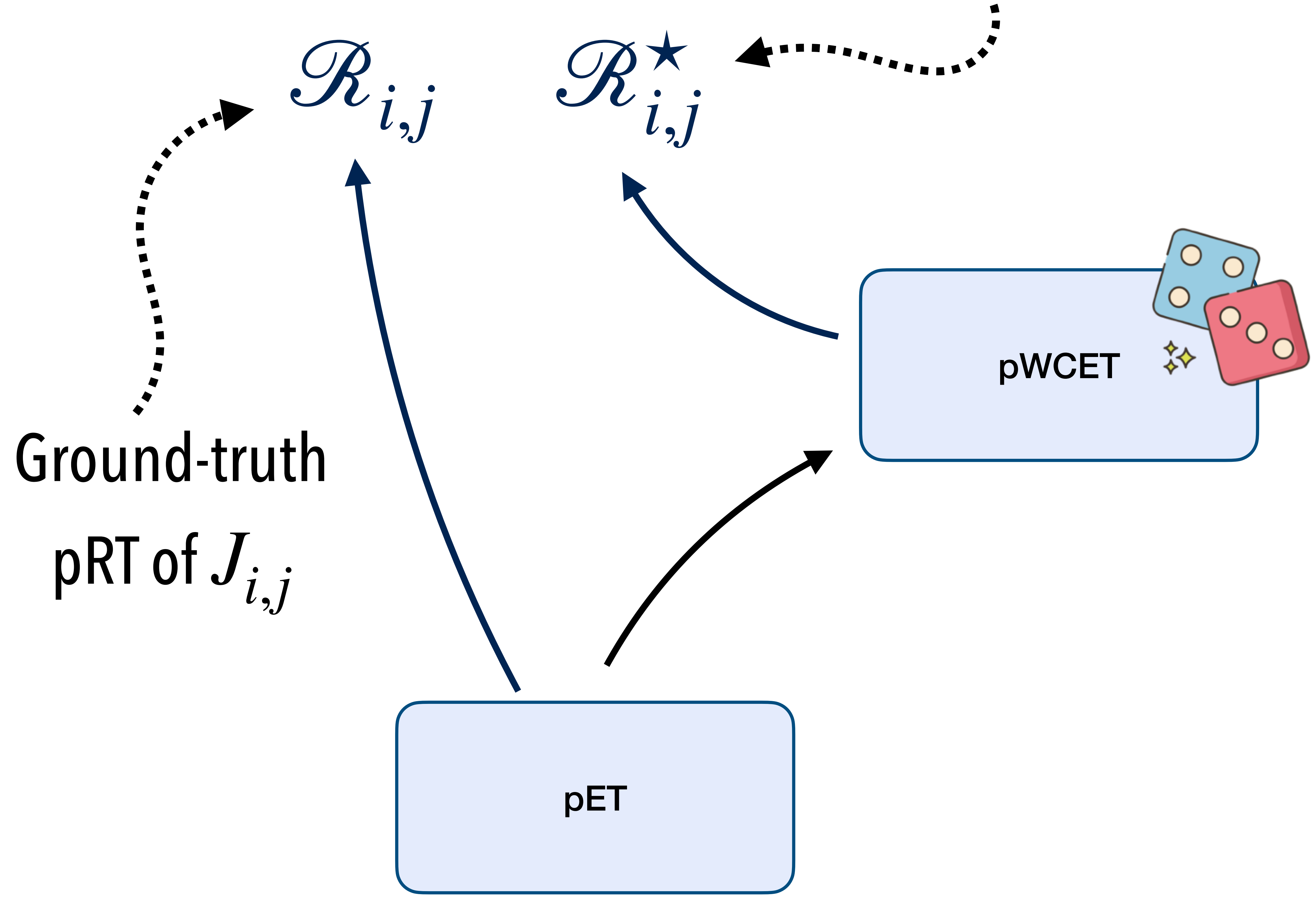

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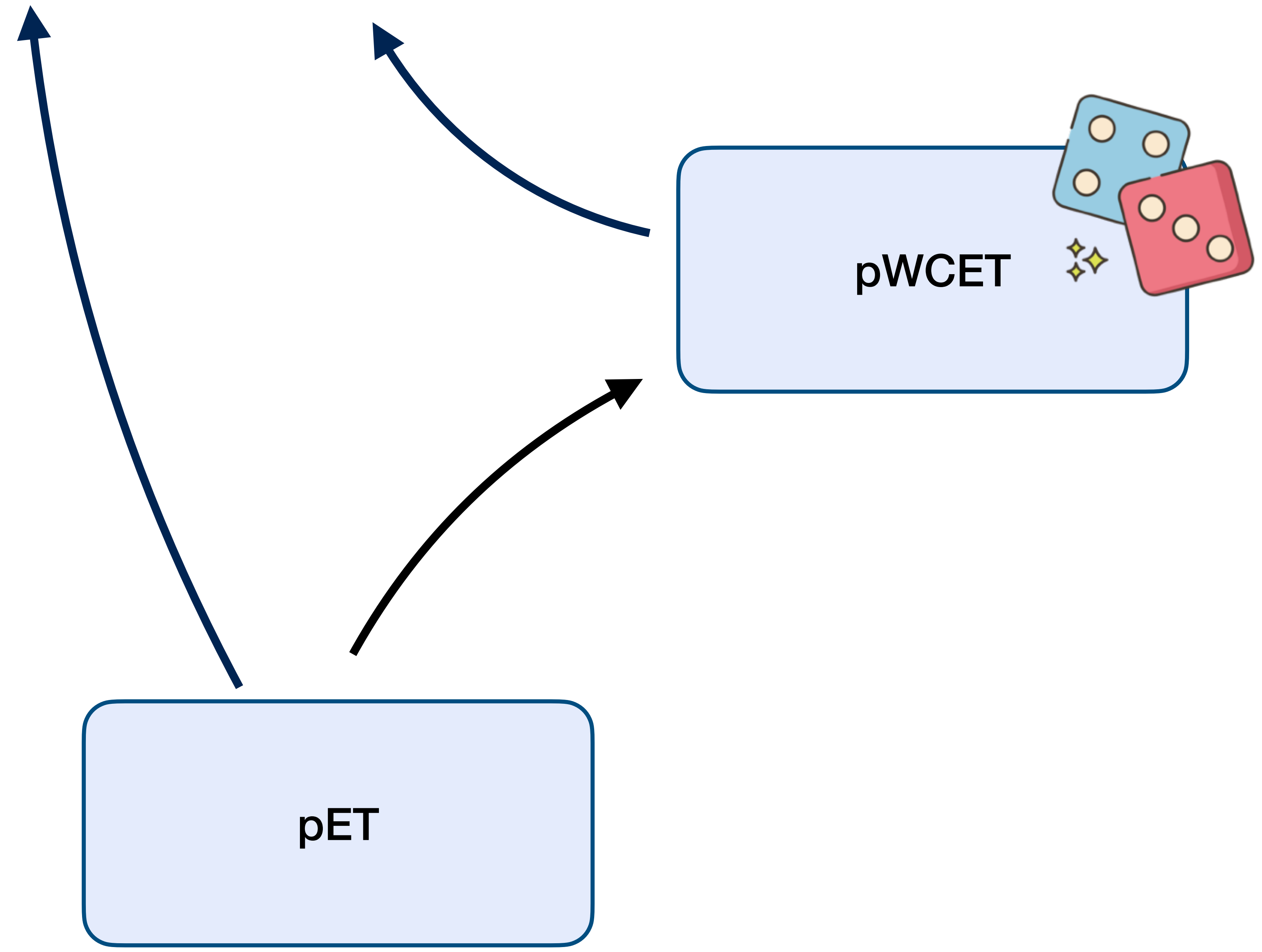

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Formal statement is **surprisingly** tricky and involves the notion of "replacement" of pETs with pWCETs

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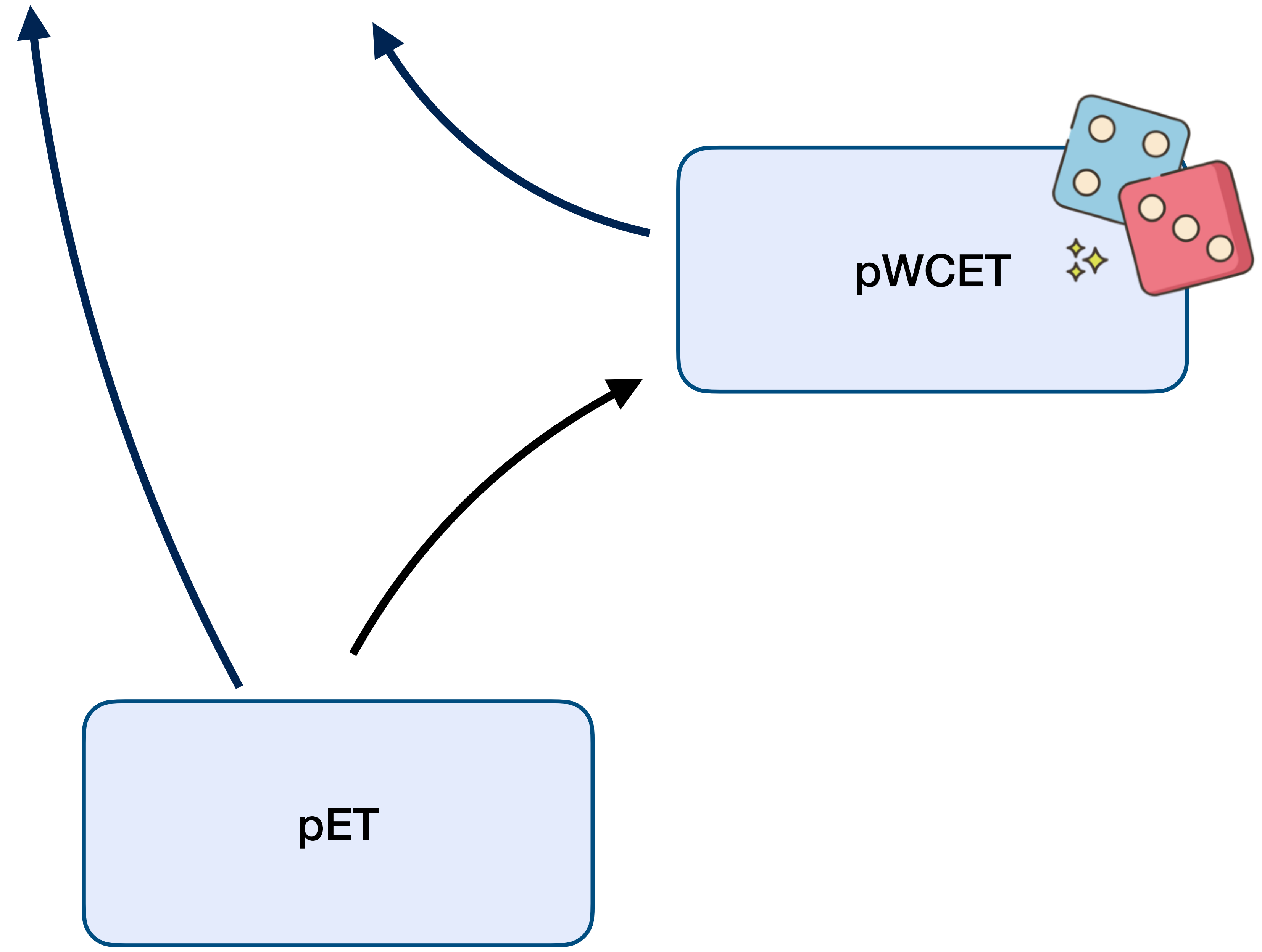

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Theorem *(paraphrased)*. Consider a job $J_{i,j}$. Let $\mathscr{R}_{i,j}$ be the pRT of $J_{i,j}$ in the initial system and $\mathscr{R}^{\star}_{i,j}$ be the pRT of $J_{i,j}$ in a simplified system obtained via pWCET F_i. Then $\mathscr{R}_{i,j} \preceq \mathscr{R}^{\star}_{i,j}$.

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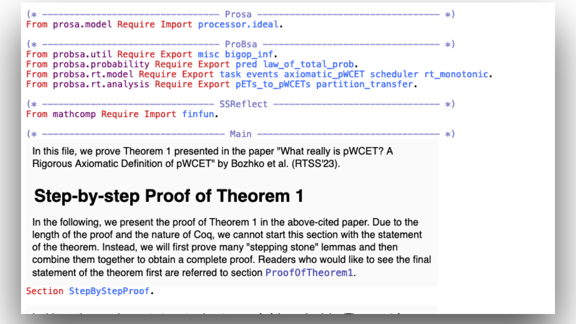

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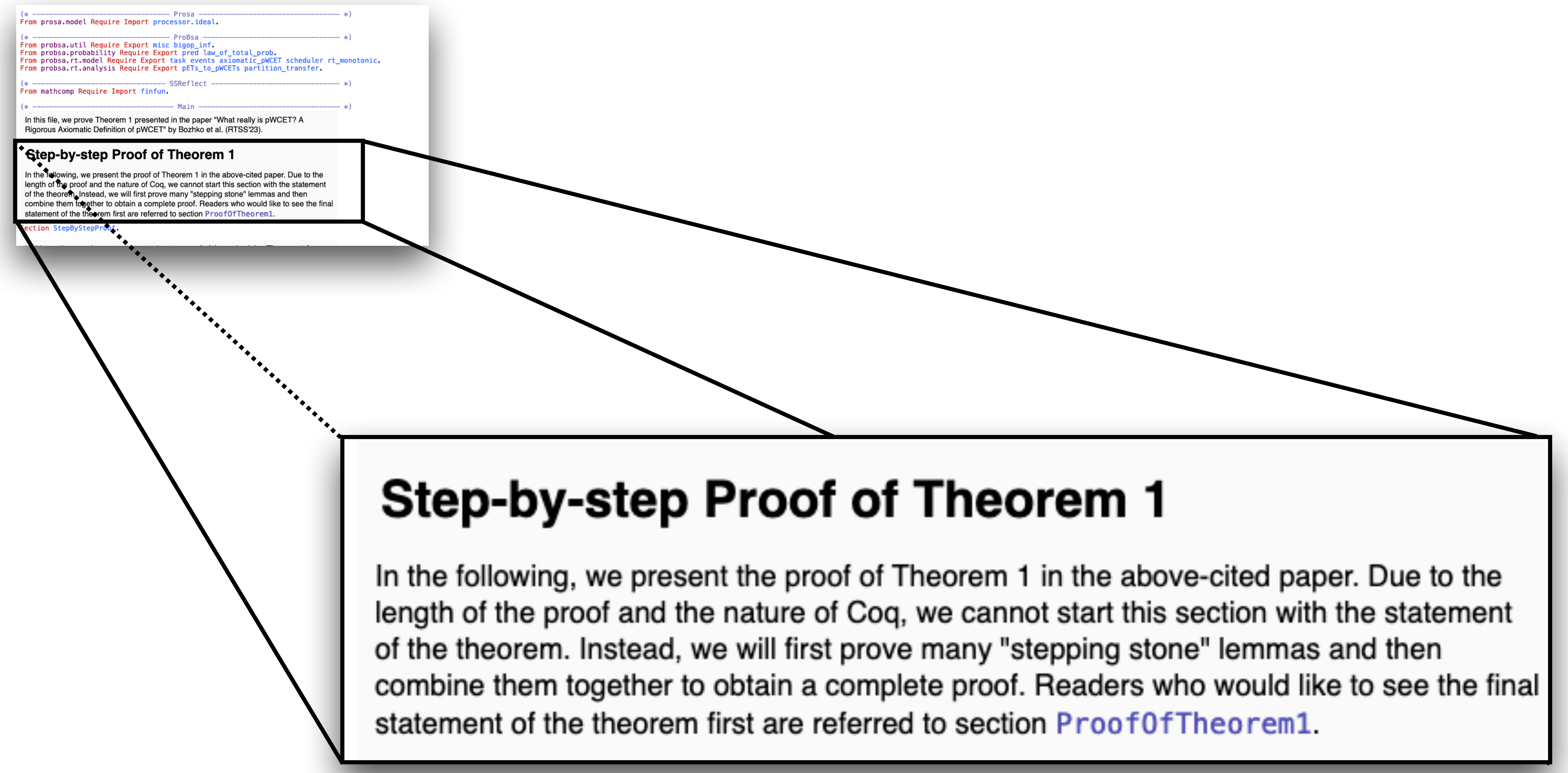

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Theorem *(paraphrased)*. Consider a job $J_{i,j}$. Let $\mathscr{R}_{i,j}$ be the pRT of $J_{i,j}$ in the initial system and $\mathscr{R}^{\star}_{i,j}$ be the pRT of $J_{i,j}$ in a simplified system obtained via pWCET F_i. Then $\mathscr{R}_{i,j} \preceq \mathscr{R}^{\star}_{i,j}$.

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1. Use axiomatic pWCET to construct a "copy" of the initial system, where pETs are replaced with job costs that are, **by construction**, IID and have distribution F_i
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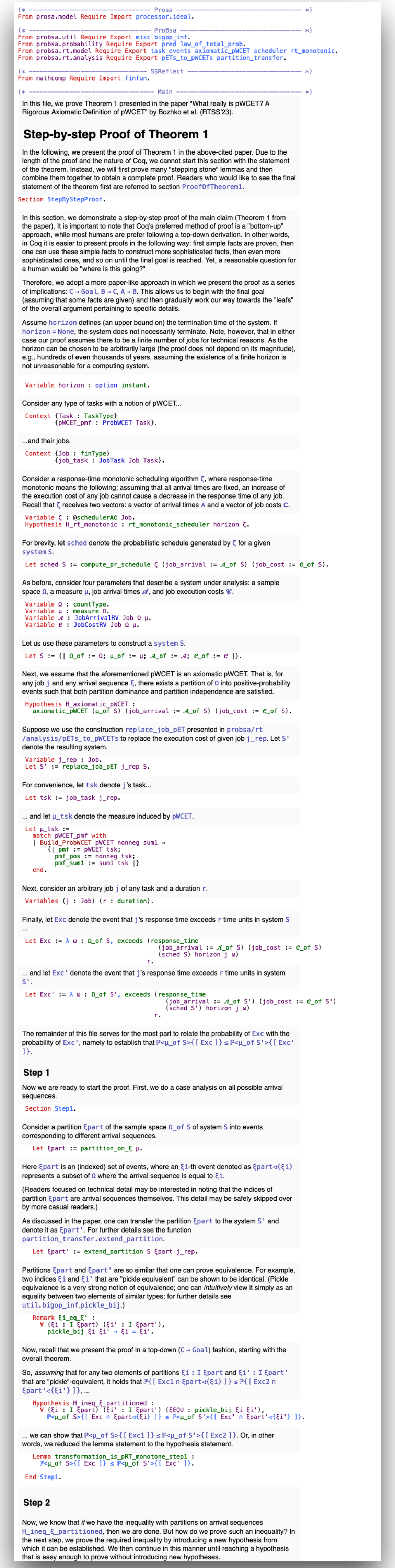

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Step-by-step Proof of Theorem 1

Step 1

Step 2

Step 3

Step 4

Step 5

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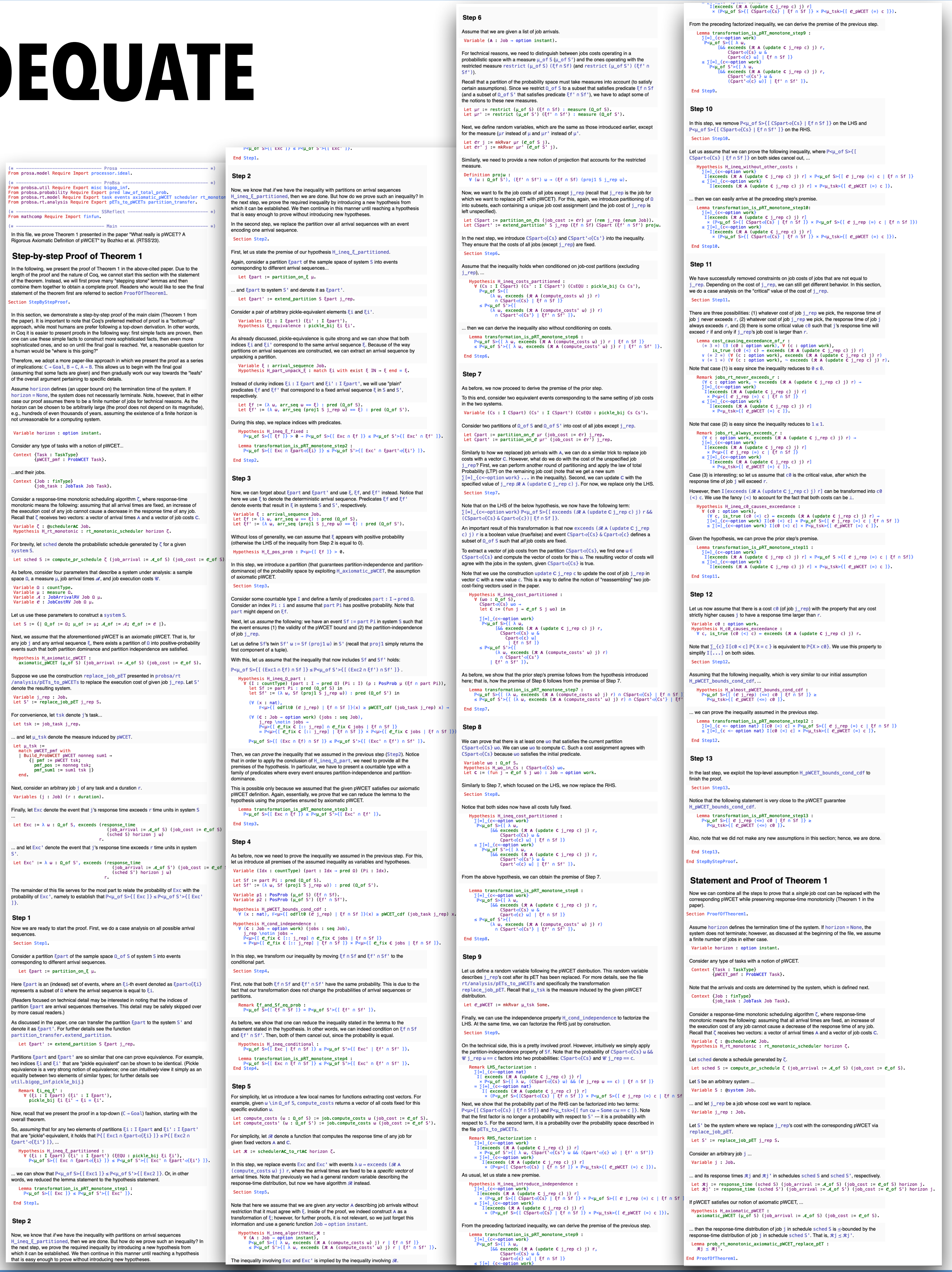

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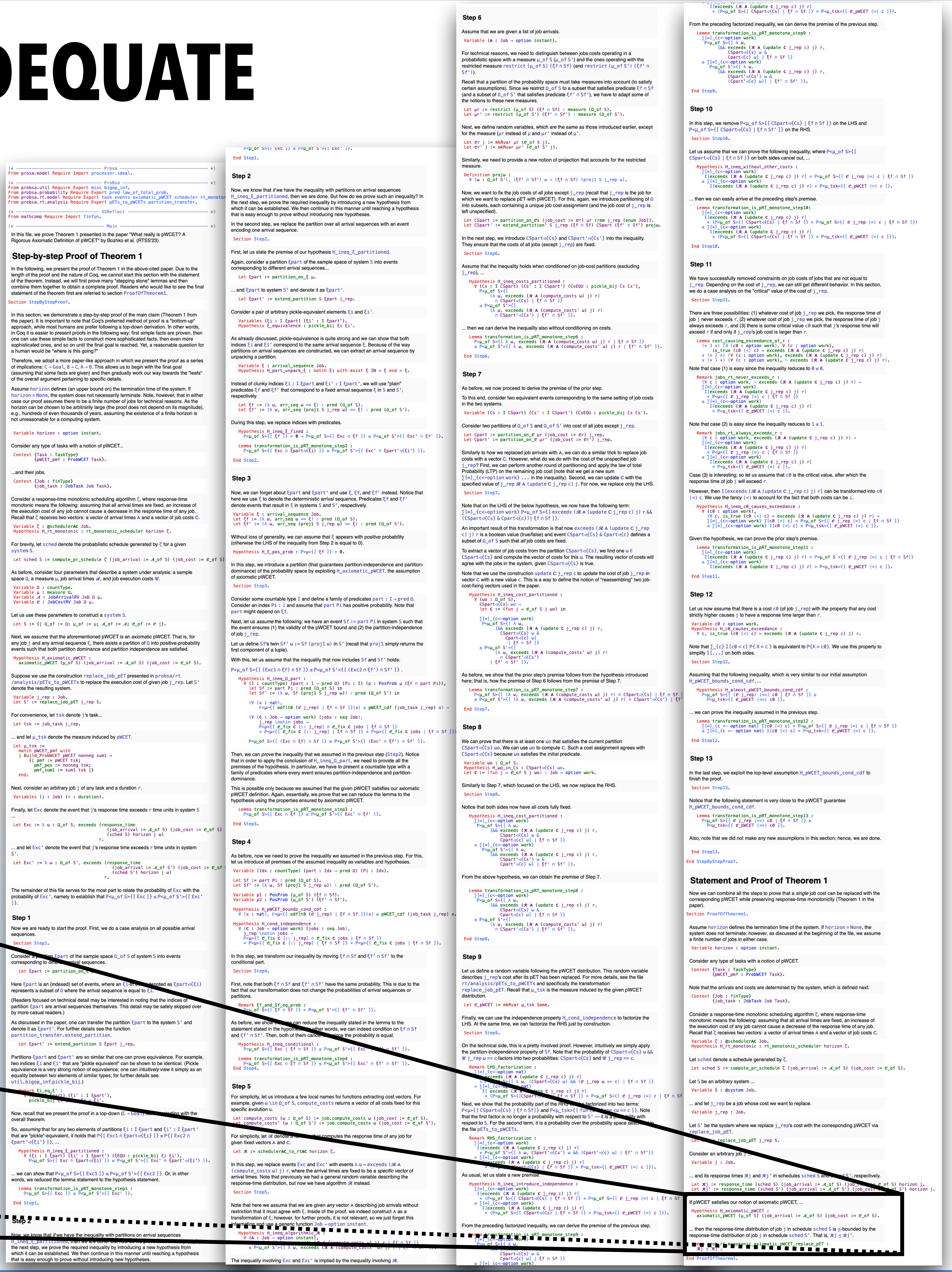
13 steps later ...

If pWCET satisfies our notion of axiomatic pWCET, ...

```
Hypothesis H_axiomatic_pWCET :
  axiomatic_pWCET (μ_of S) (job_arrival := 𝒜_of S) (job_cost := 𝒞_of S).
```

... then the response-time distribution of job j in schedule sched S is ≼-bounded by the response-time distribution of job j in schedule sched S'. That is, ℛj ≼ ℛj'.

```
Lemma prob_rt_monotonic_axiomatic_pWCET_replace_pET :
  ℛj ≼ ℛj'.
```



FORMAL SPECIFICATION AND PROOFS

Clickable links to Coq specification

➔ Each definition, lemma, and proof step is accompanied by a link to the corresponding Coq specification

The LHS and RHS of the inequality can be simplified to $\mathbb{P}[\mathcal{C}_{J_o} > c_0 | \xi \wedge S_l]$ and $\mathbb{P}_f[\widehat{\mathcal{C}}_{J_o} > c_0]$, respectively. Using the fact that $\mathbb{P}[a > b] \leq \mathbb{P}[c > d] \iff \mathbb{P}[a \leq b] \geq \mathbb{P}[c \leq d]$, we transform the inequality to obtain ( ):

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Finally, by construction (Def. 10), $\mathbb{P}_f[\widehat{\mathcal{C}}_{J_o} \leq c_0] = F_i(c_0)$. Hence, we end up with $\mathbb{P}[\mathcal{C}_{J_o} \leq c_0 | \xi \wedge S_l] \geq F_i(c_0)$, which follows ( ) from partition-dominance (Def. 6). □

Step 13

In the last step, we exploit the top-level assumption `H_pWCET_bounds_cond_cdf` to finish the proof.

```
Section Step13.
```

Notice that the following statement is very close to the pWCET guarantee `H_pWCET_bounds_cond_cdf`.

```
  Lemma transformation_is_pRT_monotone_step13 :
    ℙ<μ_of S>{[ 𝓬 j_rep ⟨<=⟩ c0 | ξf ∩ Sf ]} ≥
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```

Also, note that we did not make any new assumptions in this section; hence, we are done.

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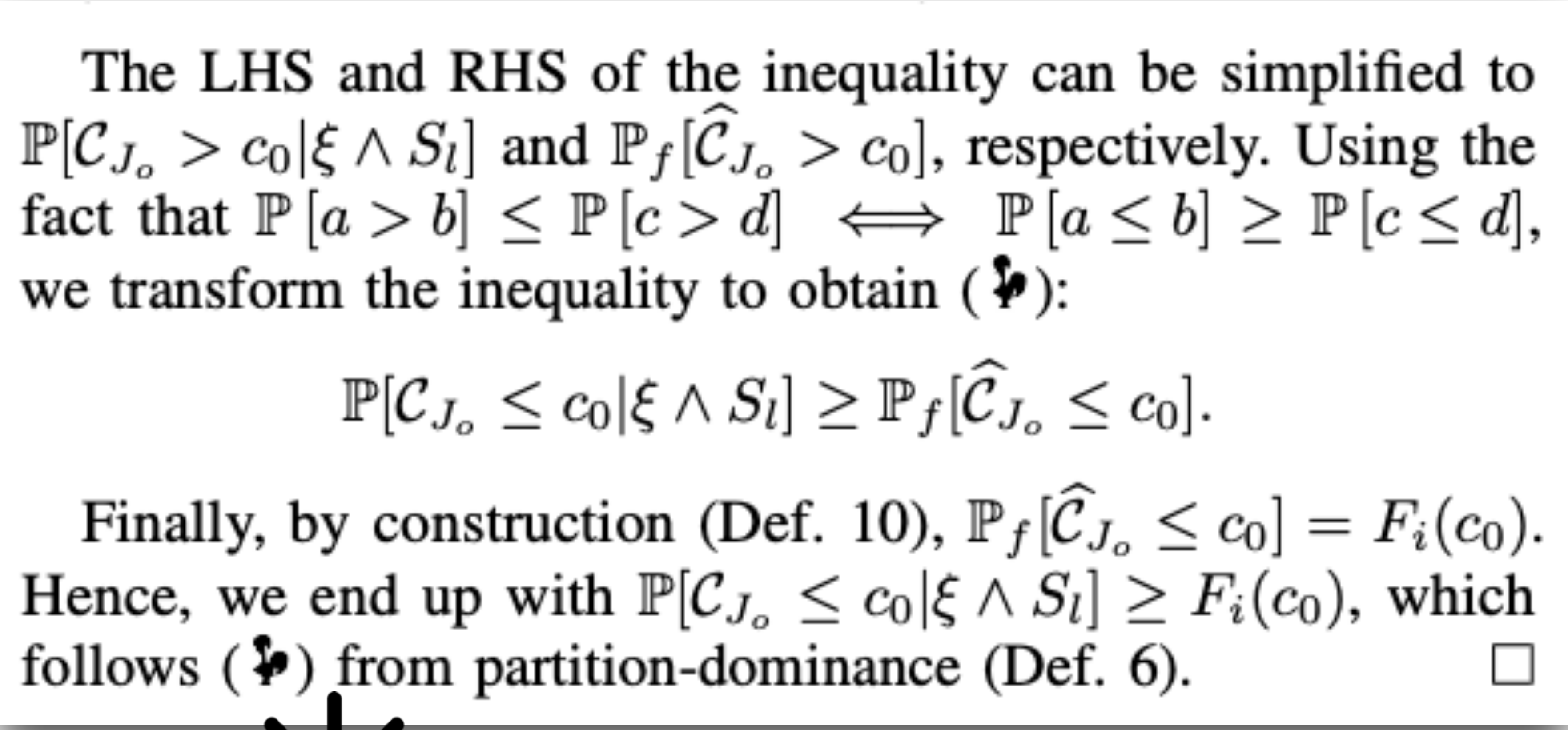
- ➔ Links are directly clickable in the version provided on the author websites



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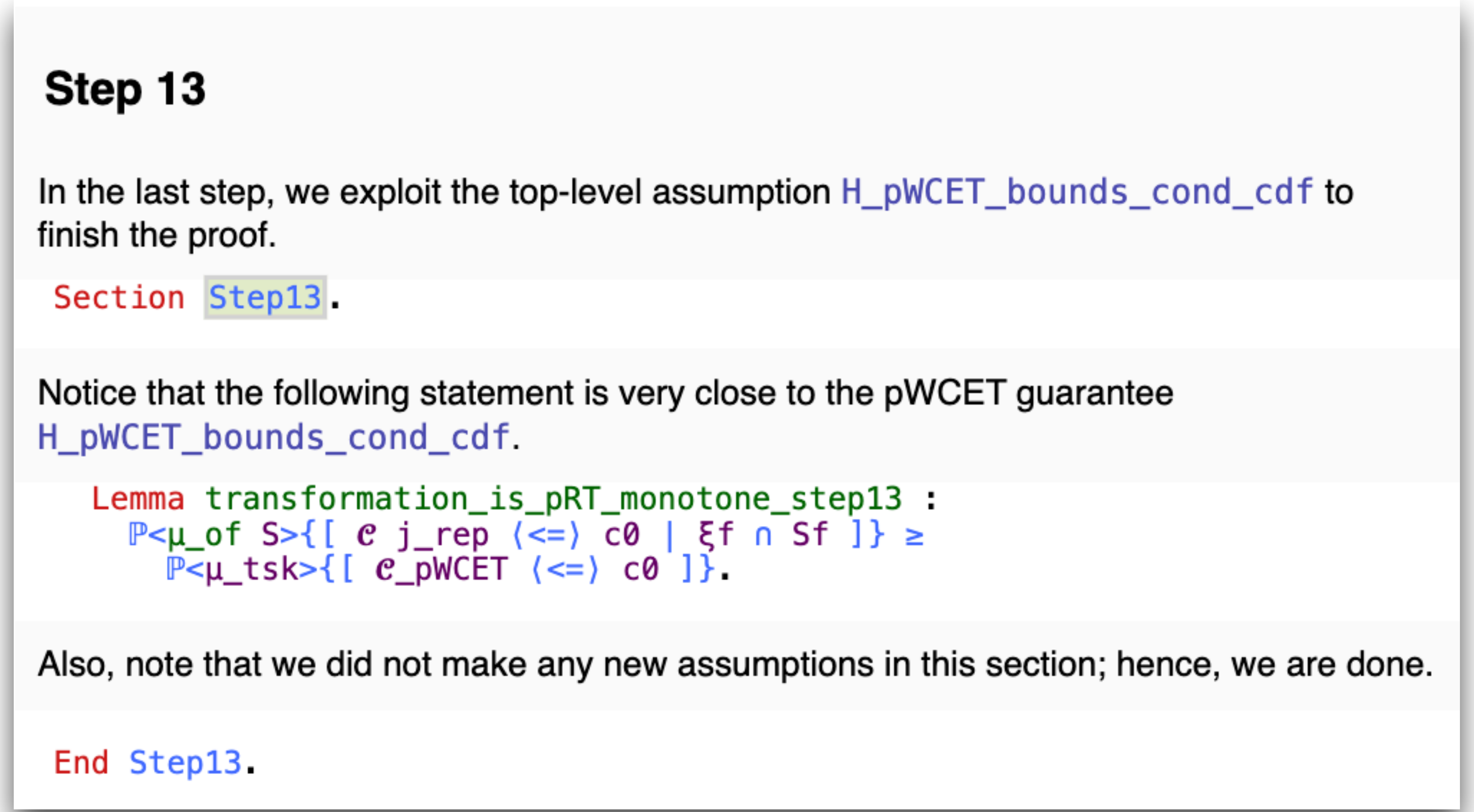

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CONCLUSION AND FUTURE WORK

What we did:

- ➔ First **fully formal** definitions of pET and pWCET
- ➔ **Adequacy property**: formalization of "safe IID upper bound on pET"
- ➔ **Prove** that our pWCET proposal is adequate
- ➔ All **mechanized** with Coq

The Coq Proof Assistant

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RTSS * Artifact * Consistent * Complete * Well Documented * Easy to Reuse * Evaluated

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Still do not like pWCET?

- *Come watch Filip's talk about **pWCET-less Correlation-Tolerant Analysis** on December 8th (Session 11 @ 12:35pm)*

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- *Please propose your preferred definition … and present an adequacy proof*

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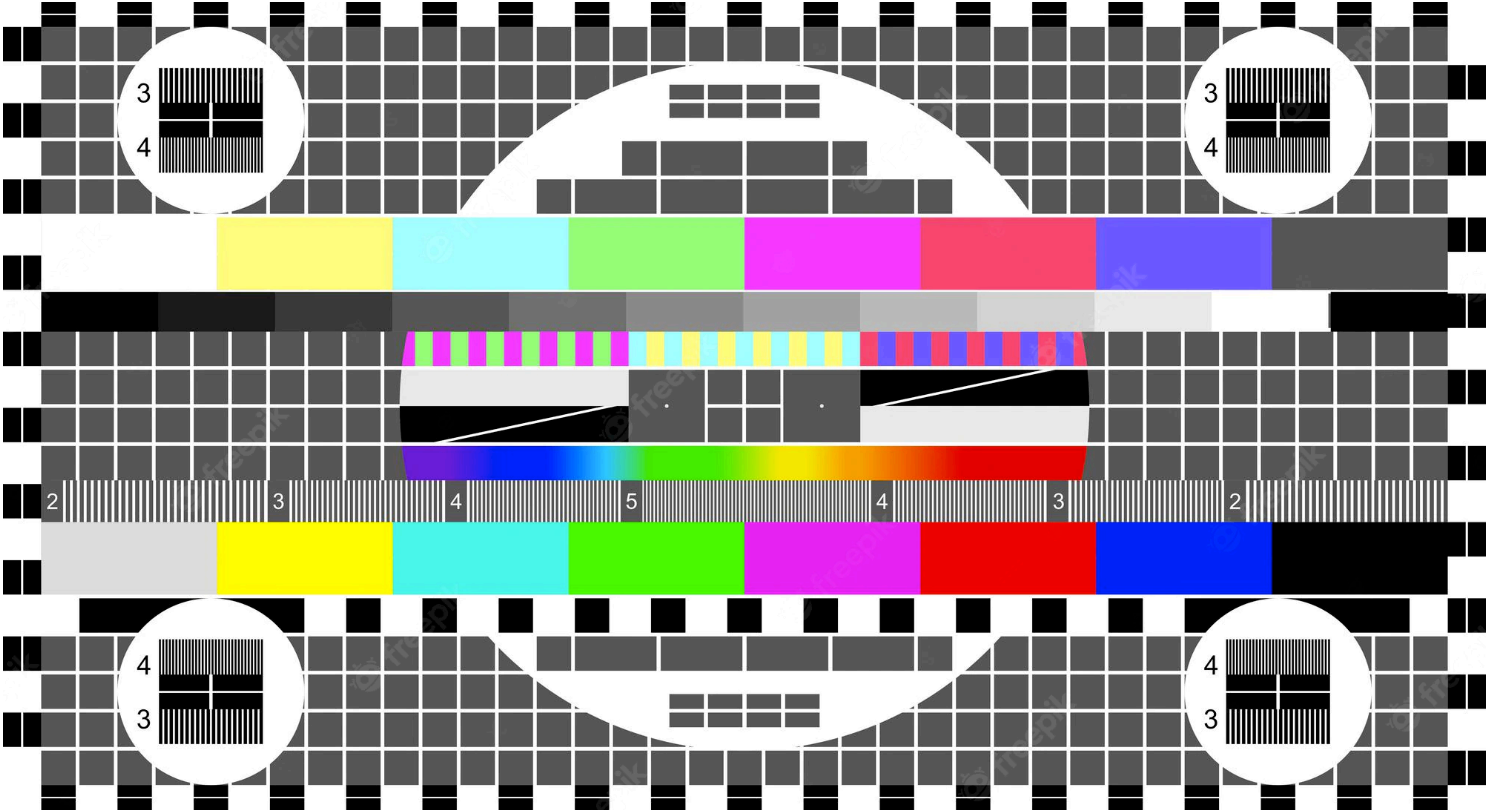
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BACKUP SLIDES

WHY *AXIOMATIC* pWCET?

Theorem *(paraphrased)*. Consider a job $J_{i,j}$. Let $\mathscr{R}_{i,j}$ be the pRT of $J_{i,j}$ in the initial system and $\mathscr{R}^{\star}_{i,j}$ be the pRT of $J_{i,j}$ in a simplified system obtained via pWCET F_i. Then $\mathscr{R}_{i,j} \preceq \mathscr{R}^{\star}_{i,j}$.

Def. 7 (✂). *A monotonically increasing function $F_i : \mathbb{W} \to [0,1]$ with $F_i(0) = 0$ and $\lim_{t\to\infty} F_i(t) = 1$ is an axiomatic pWCET for a task τ_i if, for every $J \in \tau_i$ and every fixed arrival sequence $\xi \in \Xi$, there exists a partition $\mathfrak{S}$ (Def. 4) such that*

1) *$\mathcal{C}_J$ is partition-independent w.r.t. ξ and $\mathfrak{S}$ (Def. 5), and*
2) *F_i $\mathfrak{S}$-dominates $\mathcal{C}_J$ w.r.t. ξ (Def. 6).*

Hint:

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2. Prove that pRT $\mathscr{R}^{\star}_{i,j}$ in the simplified system stochastically dominates the original pRT $\mathscr{R}_{i,j}$

Weakest precondition for which we could find a proof of the adequacy property

TWO TYPES OF pWCET

Dominance pWCET [1]

- $F_i : \mathbb{W} \to [0,1]$
- Given c, $F_i(c)$ defines a bound on probability of a job of task τ_i to have cost exceeding c

If $F_i(50) = 0.999$, then out of $100{,}000$ jobs, at most 100 jobs are expected to have cost greater than 50

Confidence pWCET [2]

- $F_i : \mathbb{W} \to [0,1]$
- Given c, $F_i(c)$ defines a bound on probability that WCET of task τ_i does not exceed c

If $F_i(50) = 0.999$, no job is expected to have cost greater than 50 and we are 99.9% confident about it

[1] Davis, Robert I., et al. "Analysis of probabilistic cache related pre-emption delays."
[2] Edgar, Stewart, and Alan Burns. "Statistical analysis of WCET for scheduling."