

# CTA: A CORRELATION-TOLERANT ANALYSIS OF THE DEADLINE-FAILURE PROBABILITY OF DEPENDENT TASKS



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Filip Marković, Pierre Roux, Sergey Bozhko,  
Alessandro V. Papadopoulos, and Björn B. Brandenburg



MAX-PLANCK-GESELLSCHAFT



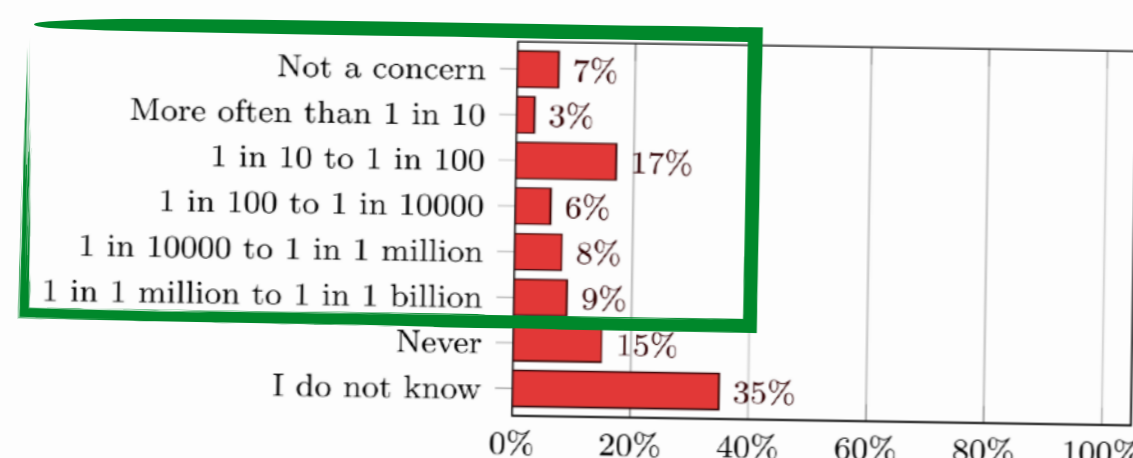
MAX PLANCK INSTITUTE  
FOR SOFTWARE SYSTEMS



# PROBABILISTIC ANALYSIS OF REAL-TIME SYSTEMS

Why is it relevant?

**Question 14** For the most time-critical functions in the system, roughly how frequently can the deadline of a function be missed without causing a system failure. (n = 101)



[1]

Many soft real-time systems **do not benefit** from deterministic analysis as it would **unnecessarily** over-provision system resources

Many **safety standards** are defined in terms of **failure probabilities**

Many systems have **soft** real-time guarantees rather than **hard** ones

Many systems are **not statically** analyzable but rather **statistically**

The total utilization of that system goes above 100%. Using response time analysis in such situation automatically yields unbounded (infinite) worst-case response times.

[2]

SIL	Low demand mode prob. failure on demand	Continuous/High demand mode prob. failure per hour
1	$\geq 10^{-2}$ to $< 10^{-1}$	$\geq 10^{-6}$ to $< 10^{-5}$
2	$\geq 10^{-3}$ to $< 10^{-2}$	$\geq 10^{-7}$ to $< 10^{-6}$
3	$\geq 10^{-4}$ to $< 10^{-3}$	$\geq 10^{-8}$ to $< 10^{-7}$
4	$\geq 10^{-5}$ to $< 10^{-4}$	$\geq 10^{-9}$ to $< 10^{-8}$

Table 1: IEC 61508: Permitted Failure Probabilities

[3]

[1] Akesson et al. *RTSJ* (2022)

[2] Rivas et al. *WATERS* (2016) [3] Agrawal et al. *ICCAD* (2020)

# OPEN PROBLEM: DEPENDENCE

Real-time systems run **intrinsically dependent** tasks, while plenty of analyses assume **independent** task execution.



“Unfortunately, the computation times of individual requests **are not statistically independent**. In the system studied here, the computation times of requests in each task **are correlated** with that of requests in many other tasks...” [1]



“Issues of **dependence** are of **great importance** in probabilistic schedulability analysis.” [2]



We present a **Correlation-Tolerant Analysis**

“... As a consequence, the probability of meeting deadlines thus computed may be **overly optimistic**.” [1]

“... Analyses **are needed** that can address **dependencies**.” [2]

## CTA: A Correlation-Tolerant Analysis of the Deadline-Failure Probability of Dependent Tasks

Filip Marković<sup>1</sup> Pierre Roux<sup>2</sup> Sergey Bozhko<sup>1,3</sup> Alessandro V. Papadopoulos<sup>4</sup> Björn B. Brandenburg<sup>1</sup>  
<sup>1</sup>Max Planck Institute for Software Systems (MPI-SWS), Germany  
<sup>2</sup>ONERA/DTIS, Université de Toulouse, France  
<sup>3</sup>Saarbrücken Graduate School of Computer Science, Saarland University, Germany  
<sup>4</sup>Mälardalen University (MDU), Sweden

**Abstract**—Estimating the worst-case deadline failure probability (WCDFP) of a real-time task is notoriously difficult, primarily because a task’s execution time typically depends on prior activations (i.e., history dependence) and the execution of other tasks (e.g., via shared inputs). Previous analyses have either assumed that execution times are probabilistically independent (which is unrealistic and unsafe), or relied on complex upper-bounding abstractions such as probabilistic worst-case execution time (pWCET), which mask dependencies with pessimism. Exploring an analytically novel direction, this paper proposes the first closed-form upper bound on WCDFP that accounts for dependent execution times. The proposed correlation-tolerant analysis (CTA), based on Cantelli’s inequality, targets fixed-priority scheduling and requires only two basic summary statistics of each task’s ground-truth execution time distribution: upper bounds on the mean and standard deviation (for any possible job-arrival sequence). Notably, CTA does not use pWCET, nor does it require the full execution-time distribution to be known. Core parts of the analysis have been verified with the Coq proof assistant. Empirical comparison with state-of-the-art WCDFP analyses reveals that CTA can yield significantly improved bounds (e.g., a lower WCDFP than any pWCET-based method for ~70% of the workloads tested at 90% pWCET utilization and 60% average utilization). Beyond accuracy gains, the favorable results highlight the potential of the previously unexplored analytical direction underlying CTA.

**1. INTRODUCTION**

Probabilistic analysis of real-time systems holds the promise of addressing the central challenge of modern hardware and software architectures: *unavoidable uncertainty* in the execution behavior of real-time tasks. Such uncertainty, deeply embedded in the fabric of modern computing systems, more often than not precludes meaningful (classical) worst-case analysis, leaving a stochastic perspective as the only viable option.

One of the most pressing open problems in this space is the issue of *dependent* execution times (also referred to as *execution-time correlation*). Specifically, when bounding a task’s worst-case deadline-failure probability (WCDFP), it is crucial to account for possible dependencies on both previous activations (*inter-task dependence*) and other tasks in the system (*intra-task dependence*). If such dependencies are ignored, the WCDFP may be severely under-approximated.

These observations are not new: the lack of independence in practice was recognized as a safety problem already more than 25 years ago by Tia et al. [49] in one of the first works on probabilistic schedulability analysis. Unfortunately, only little progress has been made on this issue since Tia et al.’s

observation, with Davis and Cucu-Grosjean noting in the closing remarks of their recent survey [19]: “*Issues of dependence are of great importance in probabilistic schedulability analysis [...] Analyses are needed that can address dependencies*”.

Prior attempts at tackling dependence in state-of-the-art WCDFP analyses have relied on over-approximation. The common idea in this line of work is to “pad” the ground-truth execution-time distributions with “sufficient pessimism,” to the point that task behavior can be safely assumed to be independent. The primary mechanism for realizing such an analysis in a sound manner is the concept of a *probabilistic worst-case execution time* (pWCET) distribution [5, 8, 14, 17, 18], which can be determined for each task either via static analyses (e.g., 4, 6, 16, 31) or with measurement-based techniques such as *extreme value theory* (EVT) [e.g., 32, 35, 46, 47].

Specifically, the pWCET approach promises that the analysis may model execution times with independent random variables following the pWCET distribution, provided the pWCET distribution is suitably determined [19]. However, a significant limitation of such *independence-assuming analysis* (IAA) lies in its inherent over-approximation of the ground truth, which can lead to considerable pessimism compared to actual behavior.

**This paper.** Exploring a fundamentally different direction, we propose a novel *correlation-tolerant analysis* (CTA) of WCDFP under fixed-priority scheduling. CTA is based on *Cantelli’s inequality* [9] and departs from the state of the art in three major ways: first, CTA does not use pWCET, nor does it otherwise require ground-truth distributions to be pessimistically padded; second, unlike traditional methods, CTA does not require full knowledge of the ground-truth distributions, as it uses only bounds on their means and standard deviations (under any possible job-arrival sequence); and last but not least, CTA is safe in the presence of arbitrarily dependent execution times. Notably, CTA also does not require the degree of inter- or intra-task correlation to be quantified, which is desirable in practice.

In developing CTA, we make the following contributions:

- We convey the core idea with a simple example (Sec. II).
- From Cantelli’s inequality [9] we derive, and verify with Coq [13, 41], an upper bound on the sum of random variables with unknown degrees of correlation (Sec. IV).
- We formally model the execution of a stochastic sporadic real-time workload under preemptive uniprocessor fixed-

[1] Tia et al. *RTAS* (1995)

[2] Davis and Cucu-Grosjean *LITES* (2019)

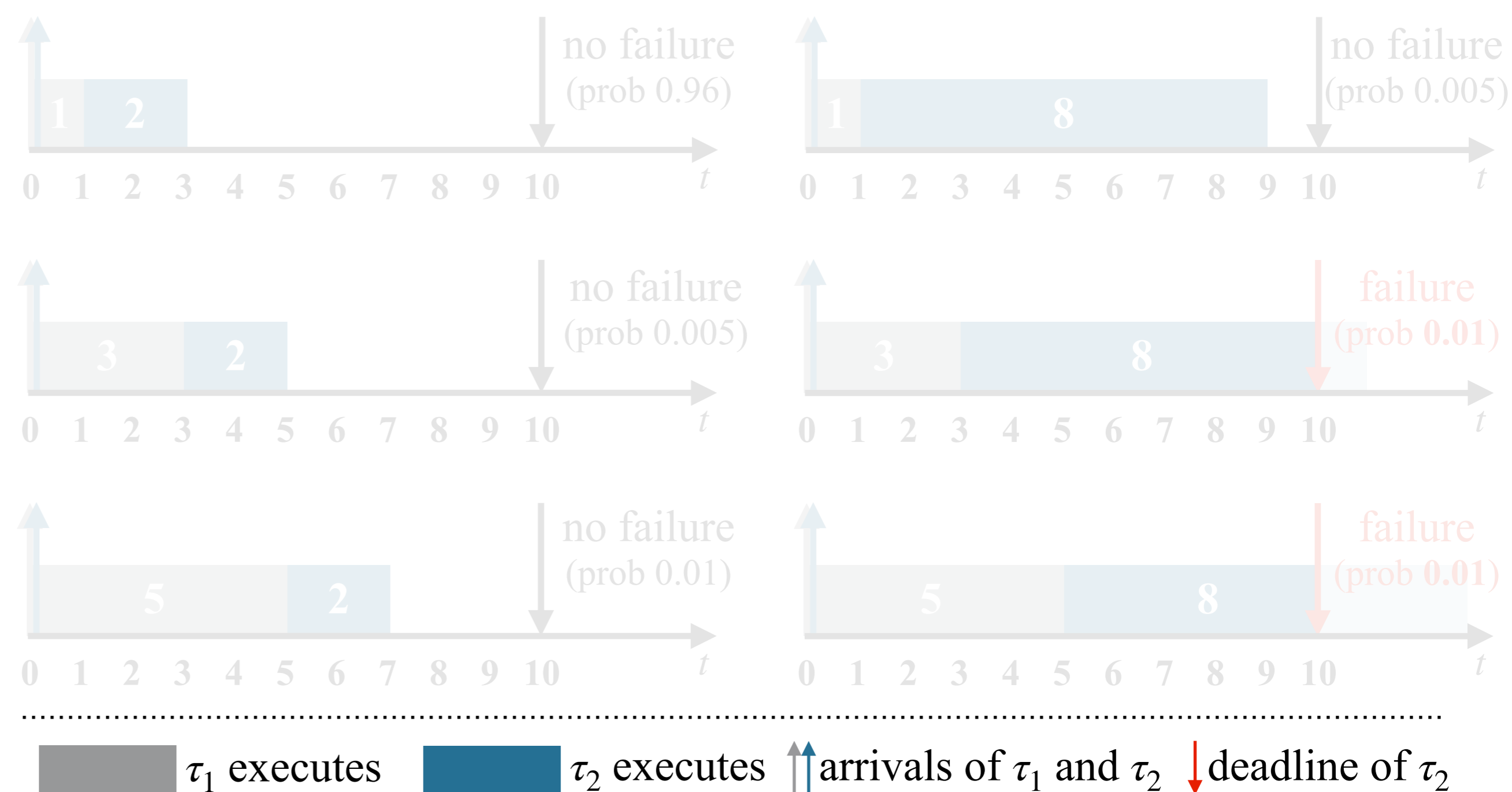
# DEADLINE-FAILURE PROBABILITY (DFP)

*The probability that a job of a task fails to complete before its deadline.*

Consider a simple system comprising two tasks

- grey task (high priority),
- blue task (low priority).

## Ground-truth behavior: all possible evolutions



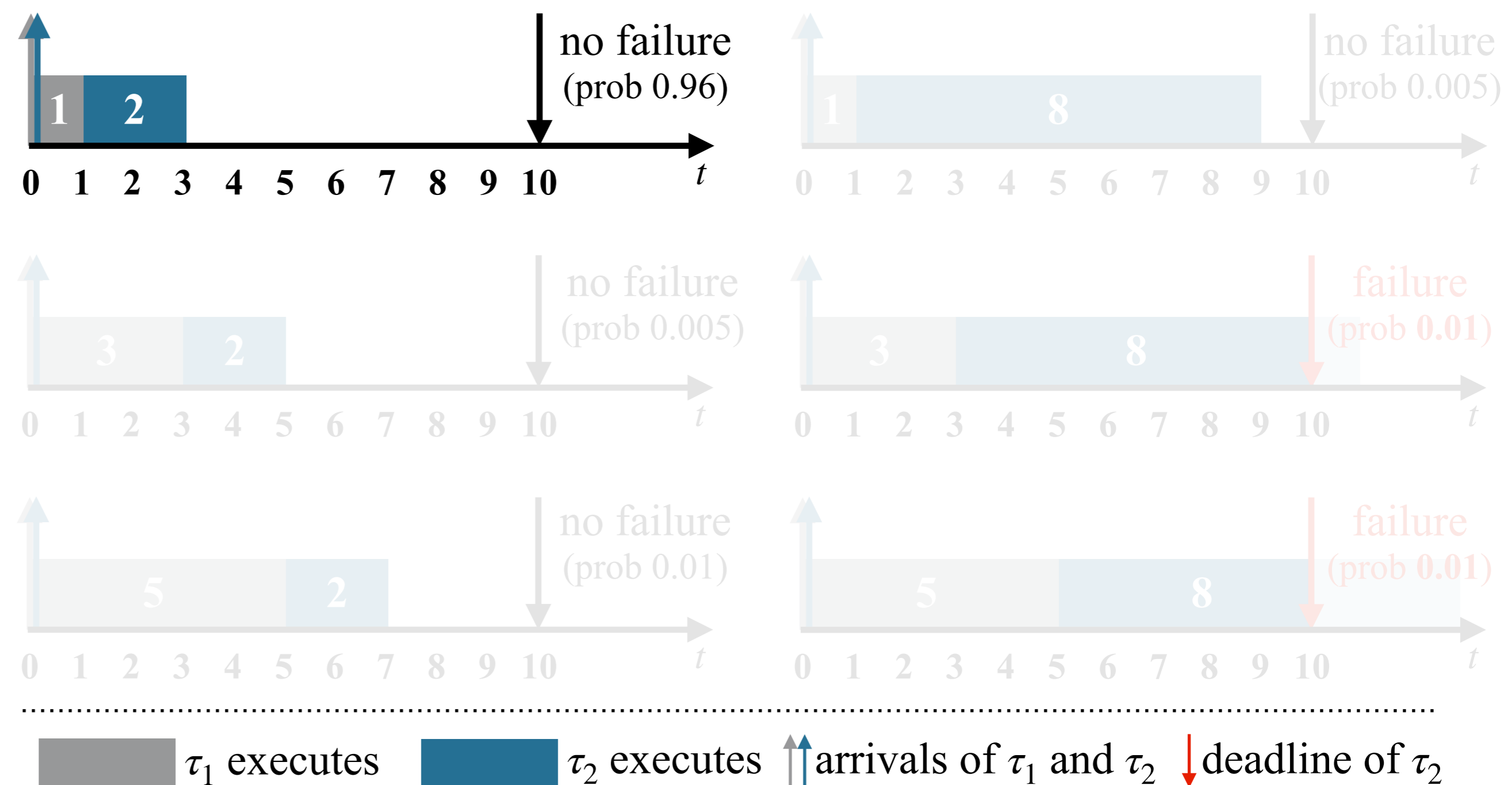
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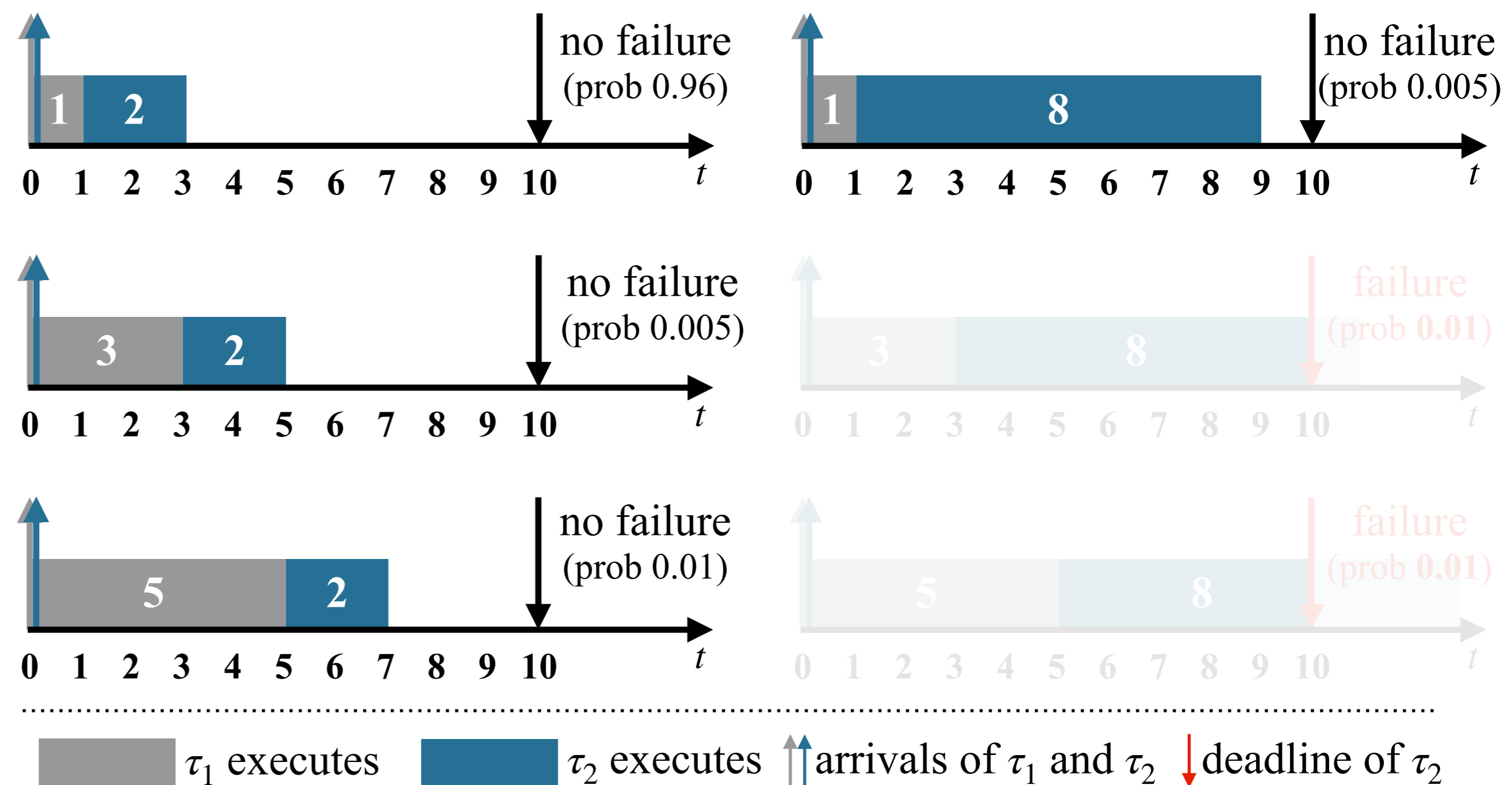
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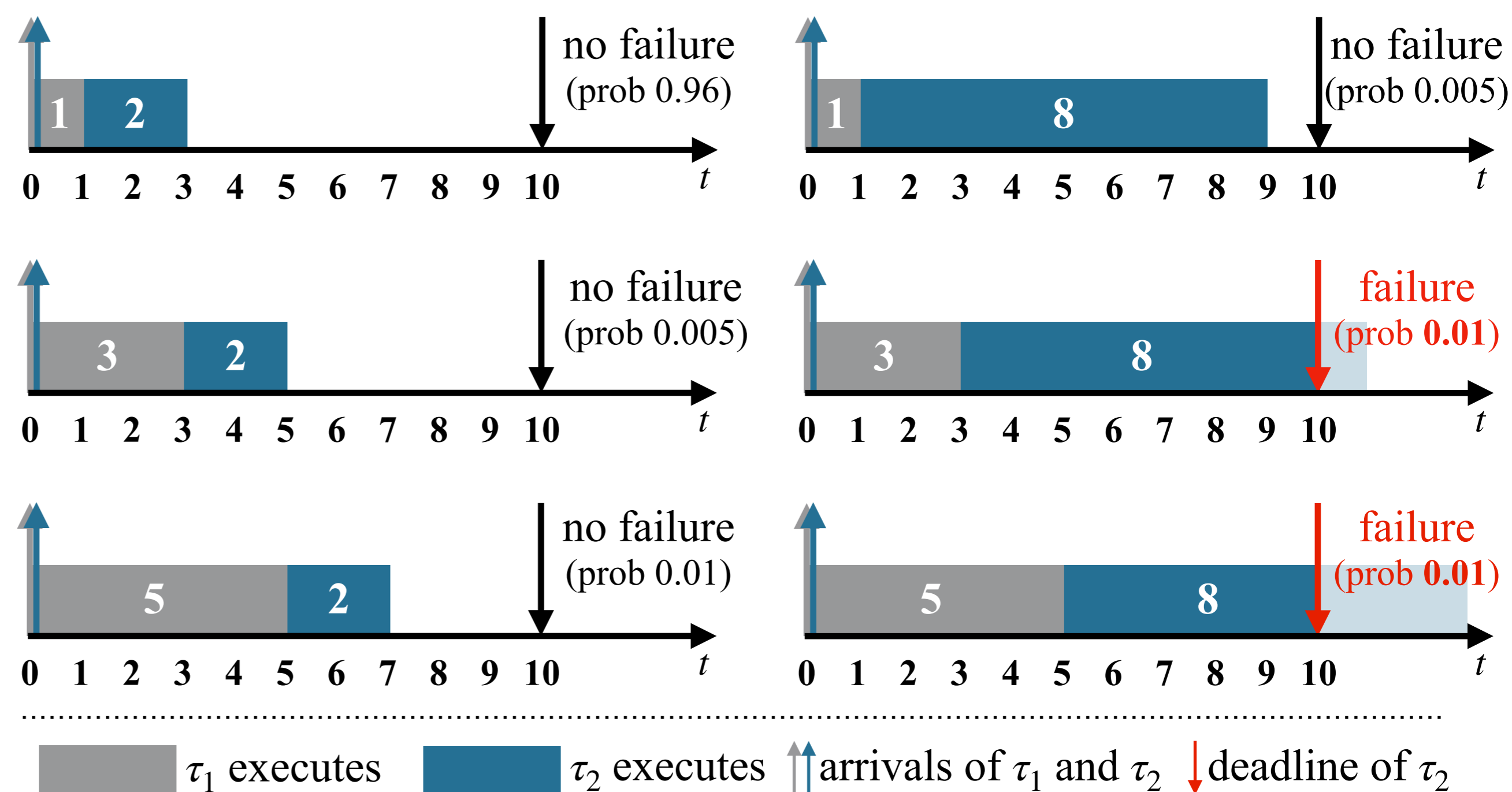
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The probability that a job of a task fails to complete before its deadline.

Consider a simple system comprising two tasks

- grey task (high priority),
- blue task (low priority).

## Ground-truth behavior: all possible evolutions



The **ground-truth DFP** of the **blue task** is **0.02**.

# DFP ANALYSIS

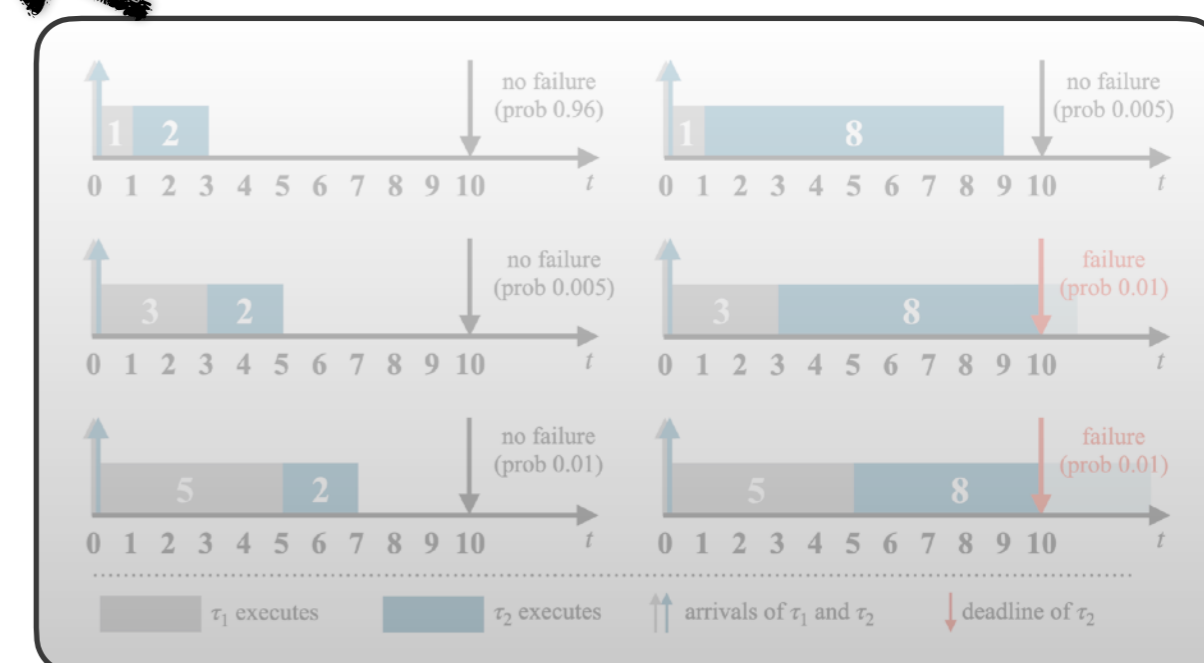
Analysis that derives an upper bound on the DFP of any job of a task.

**Input:** model parameters ← the easier to obtain, the better

**Output:** DFP upper bound

**Goal:** Efficient and accurate DFP

Ground-truth behavior

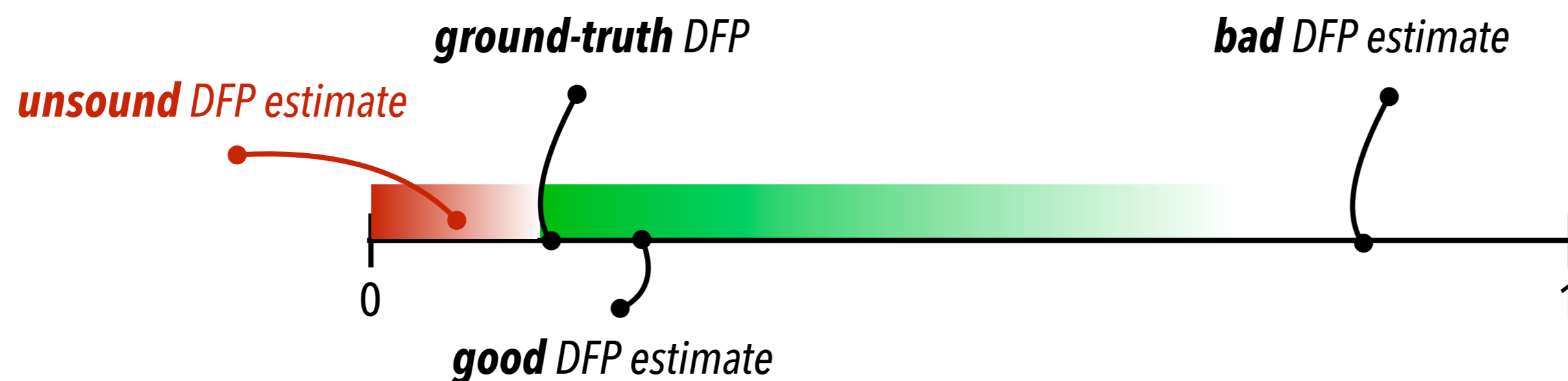


**Efficient:** minimize



space and time complexity

**Accurate:** minimize over-approximation





# **PRIOR WORK ON DFP ANALYSIS**

# DFP ANALYSIS ASSUMING INDEPENDENCE

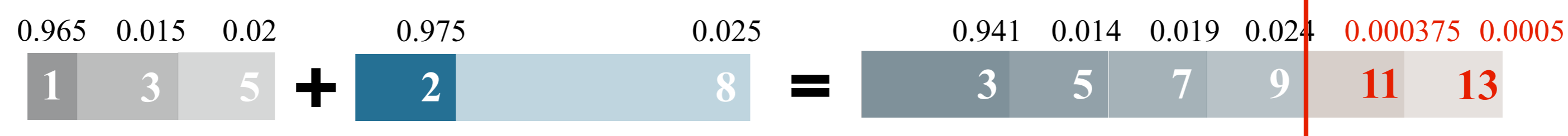
Computation of the DFP (**blue task**) using per-task distributions and assuming independence

**Input:** measured per-task execution-time distributions



*deadline = 10*

**Analysis:** assumes independence



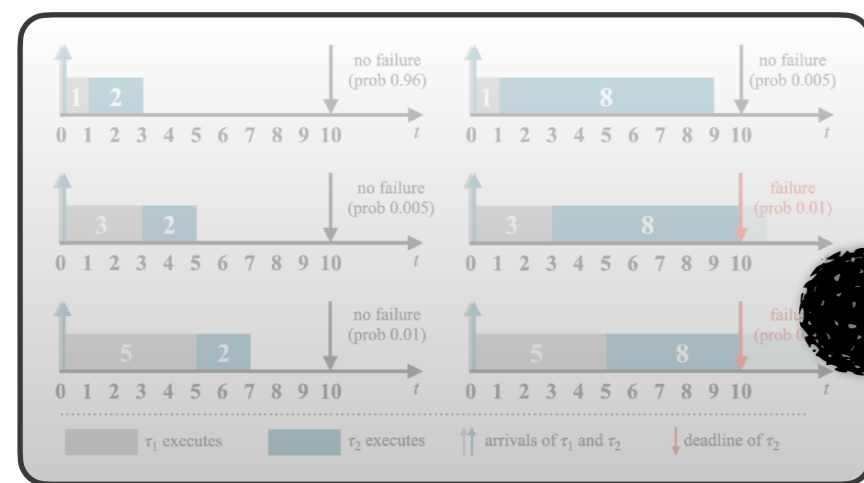
**Output:** **0.000875** < 0.02 (ground-truth DFP)

**Conclusion:** Ignoring task dependence (correlation) risks **unsound** DFP estimation.

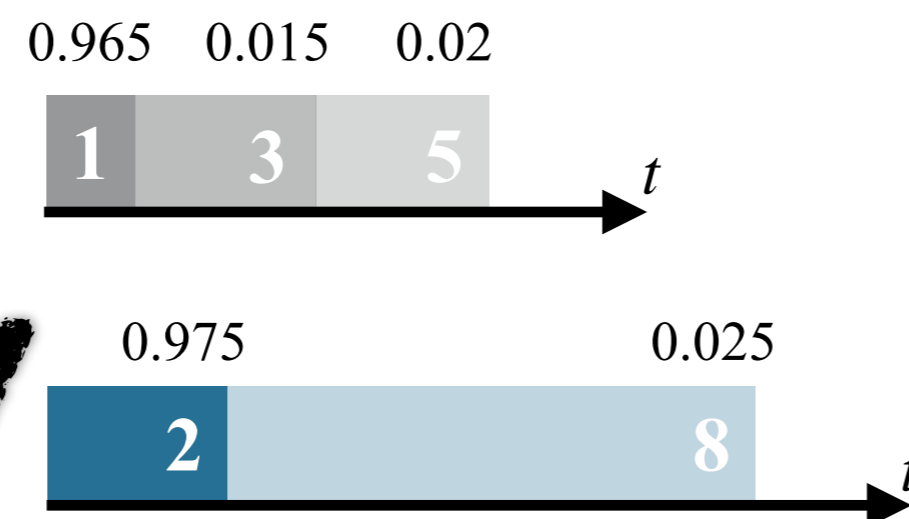
# pWCET: PESSIMISM "BAKED IN"

A distribution designed to "hide" dependence while being analytically convenient.

**Input:** Probabilistic Worst-Case Execution Time (pWCET)

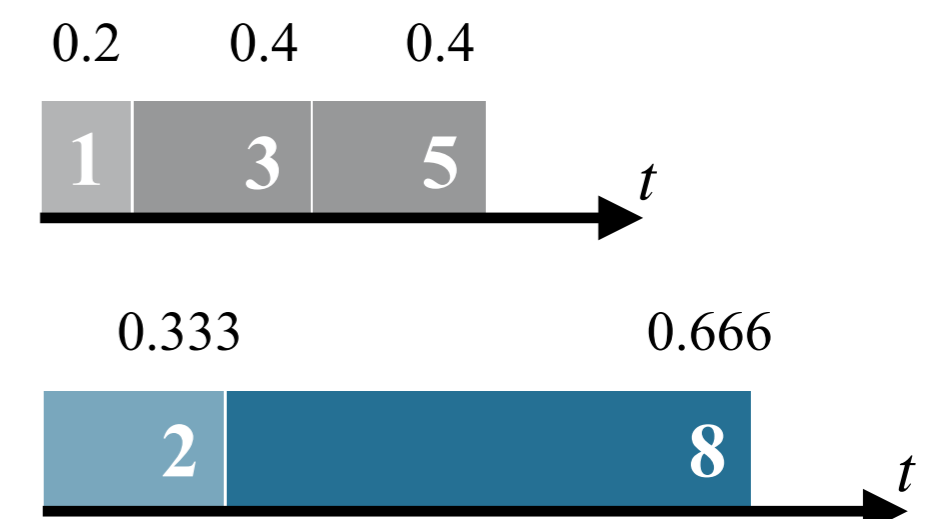


Ground-truth behavior



per-task ground-truth execution-time distributions

distribution padding



pWCET distributions

**Analysis:** assumes independence



**Output:** **0.533333** > 0.02

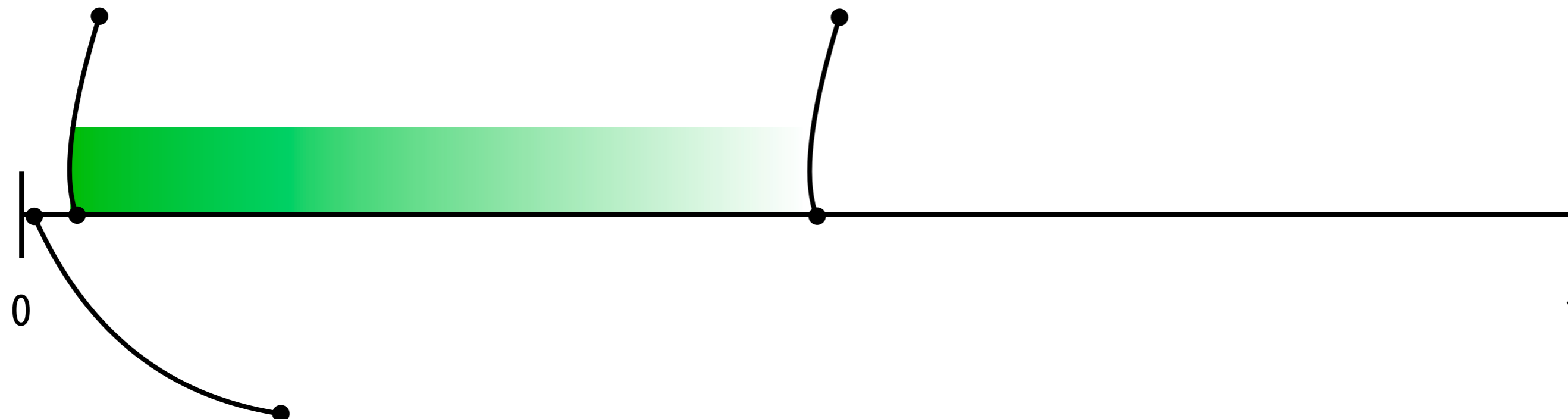
**Conclusion:** pWCET-based analysis can be inherently pessimistic.

# SUMMARY

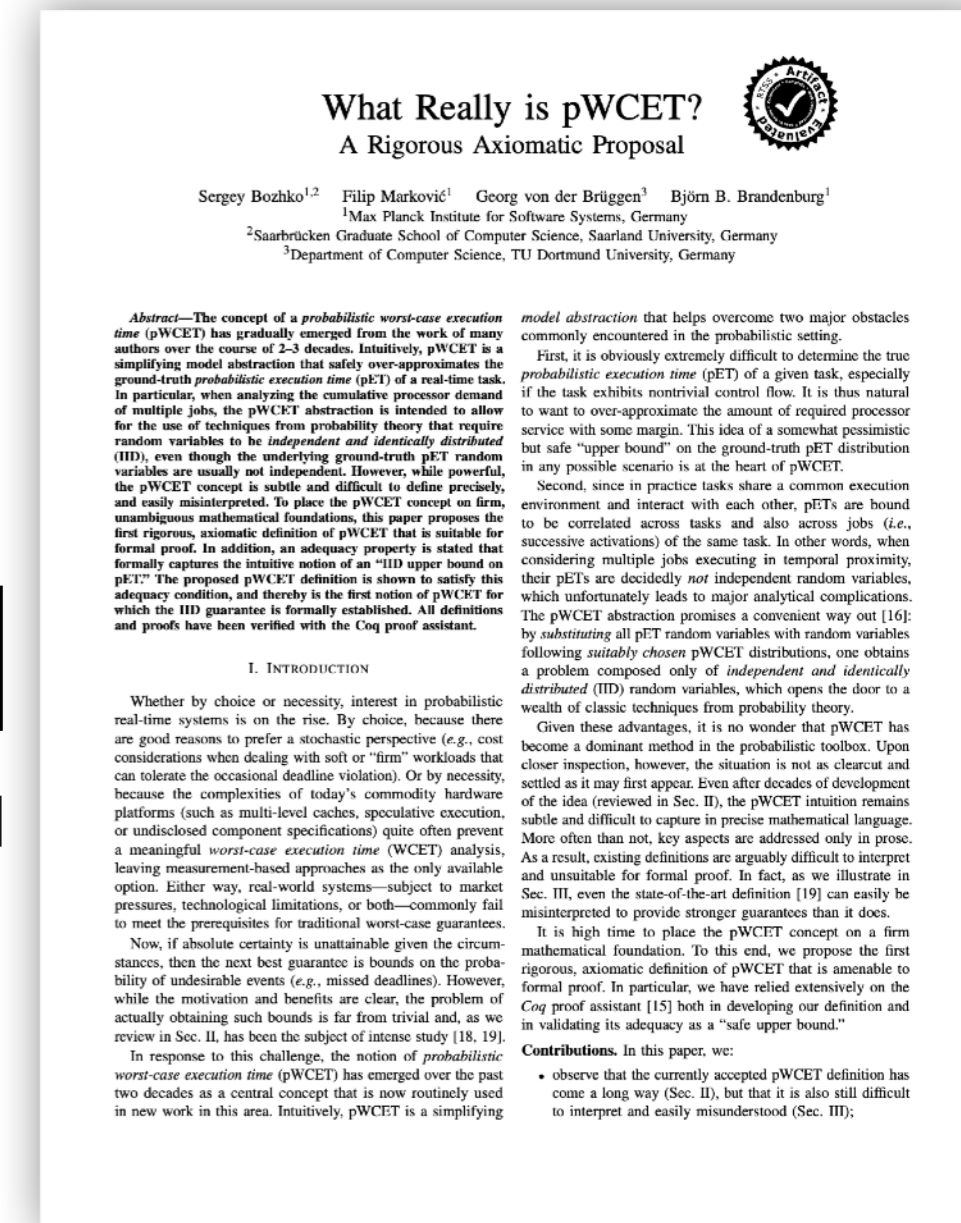
Ignoring correlation can be **unsound**, while pWCET-based approaches can be **overly pessimistic**.

ground-truth DFP: **0.02**

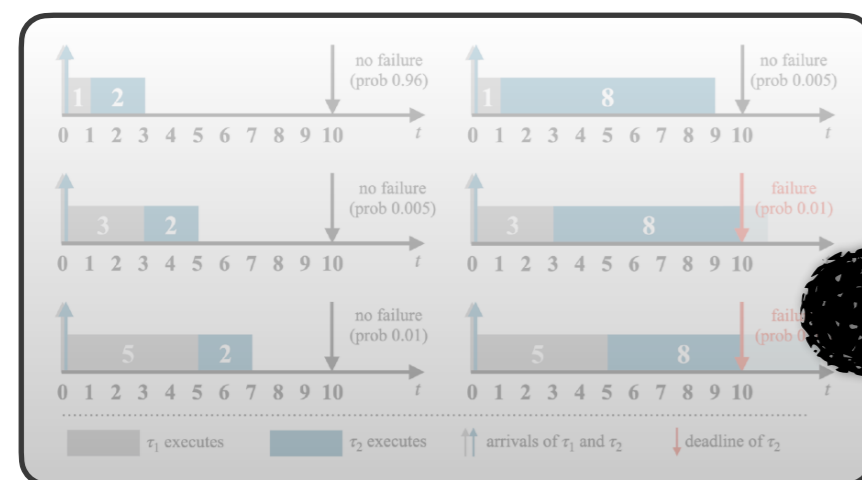
pWCET-based DFP: **0.5333 (overly pessimistic)**



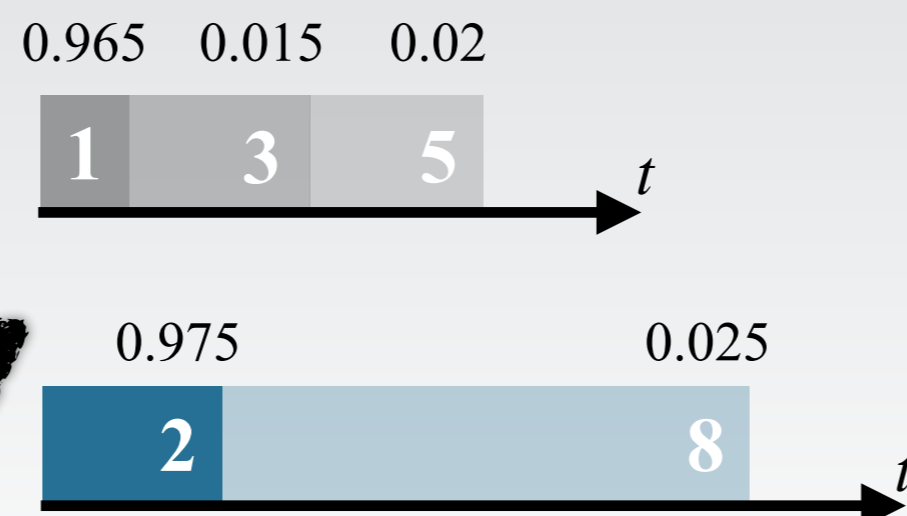
independence-assuming DFP: **0.000875 (unsound estimate)**



## Also, pWCET derivation is not trivial

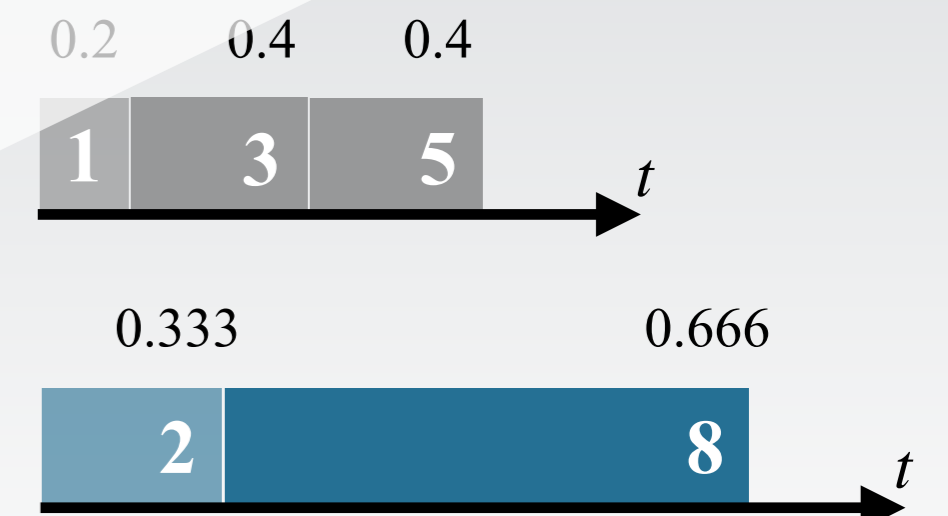


Ground-truth behavior



per-task ground-truth execution distributions

What is enough padding?



pWCET distributions

**THIS PAPER:**  
**A CORRELATION-TOLERANT ANALYSIS**

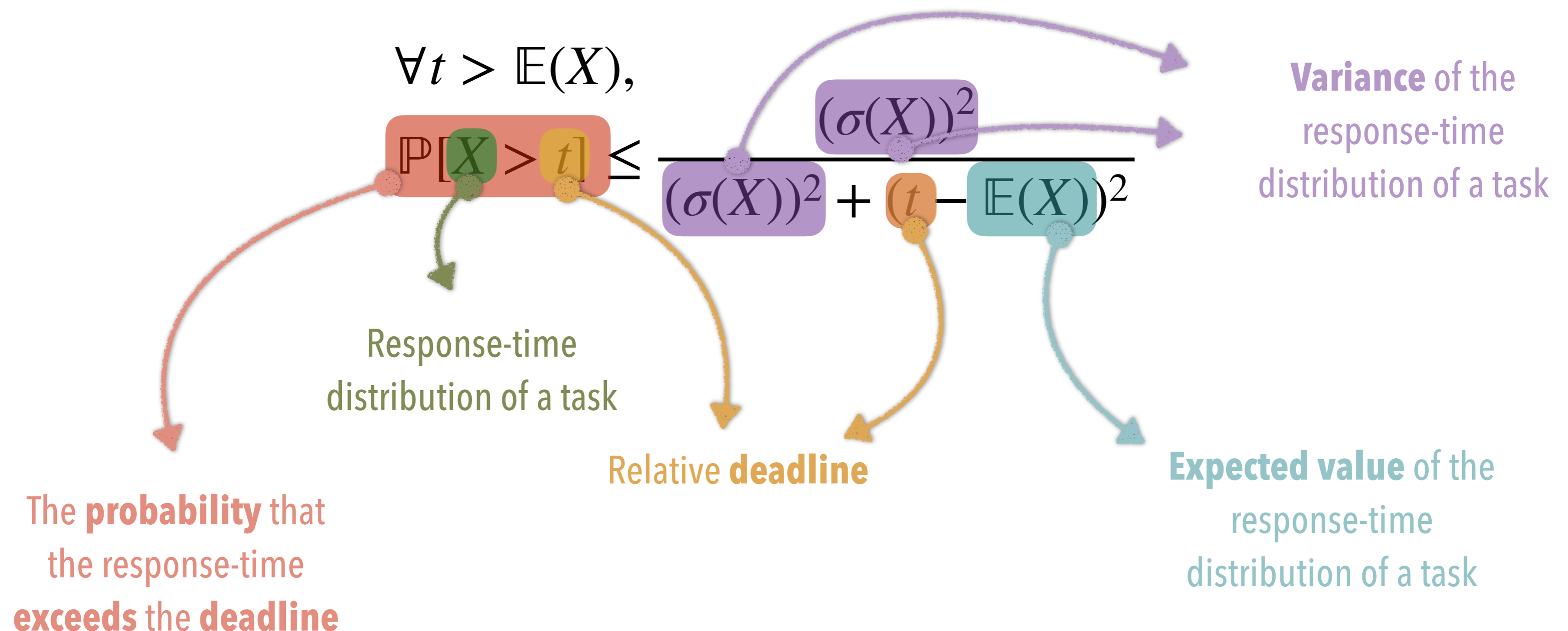
*(Not using  $pWCET!$ )*

# CANTELLI'S INEQUALITY

Given a random variable with a known expected value and standard deviation and some threshold  $t$ , it **bounds the exceedance probability**.

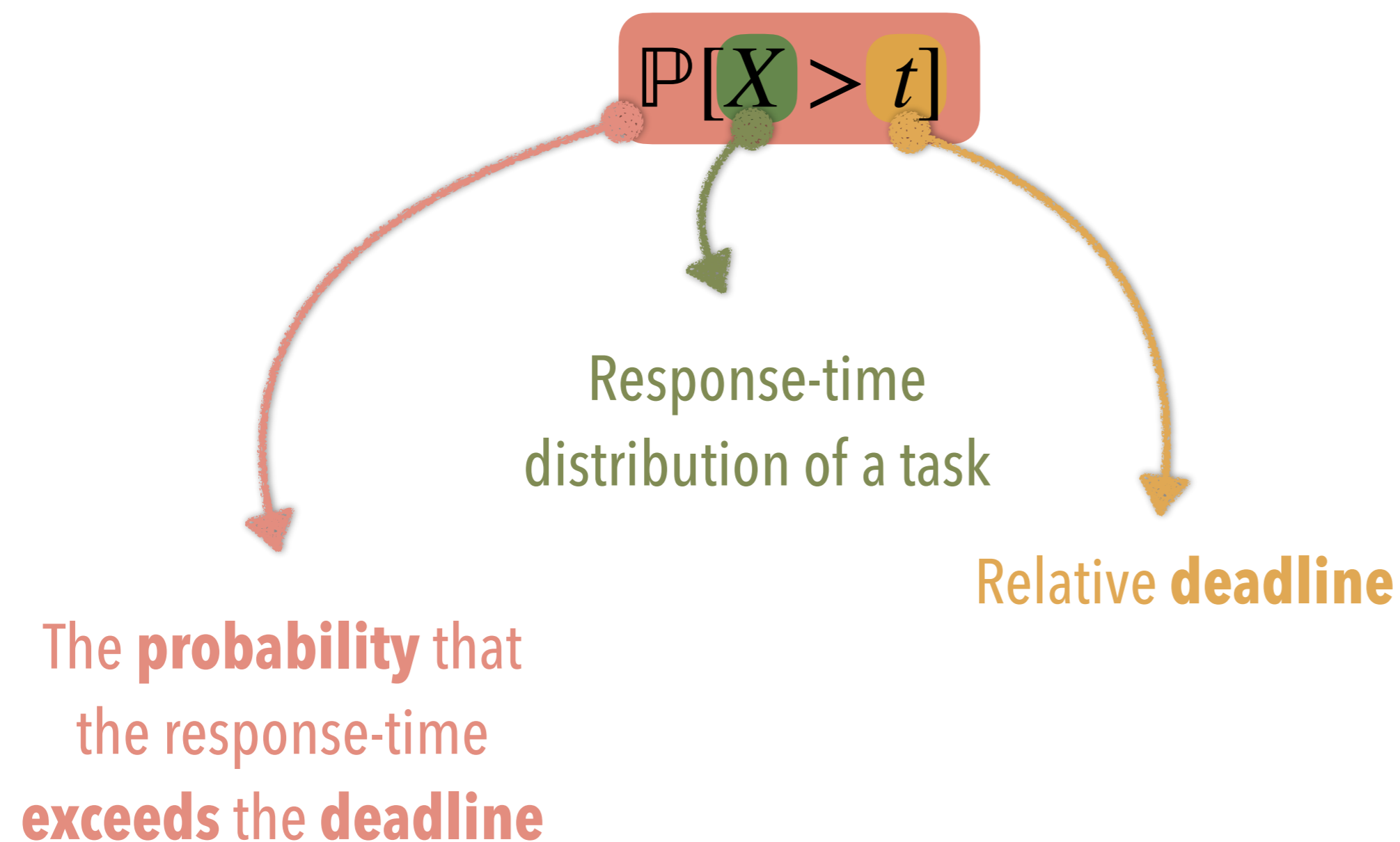
**Input:** expected value and variance of some random variable  $X$ .

But how does this translate to our **RT problem**?



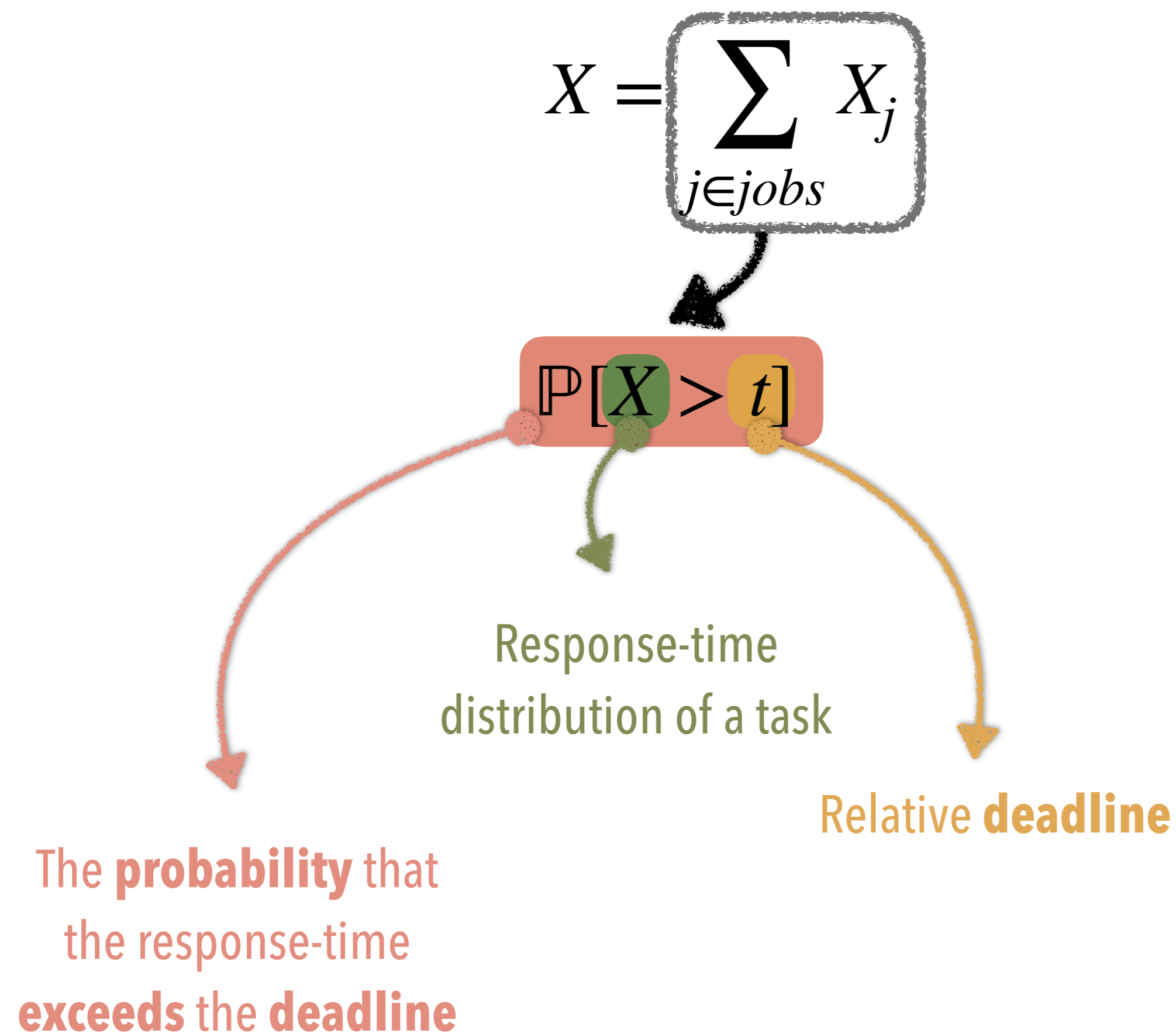
# A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

*We can apply Cantelli's Inequality to a sum of possibly correlated random variables.*



# A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

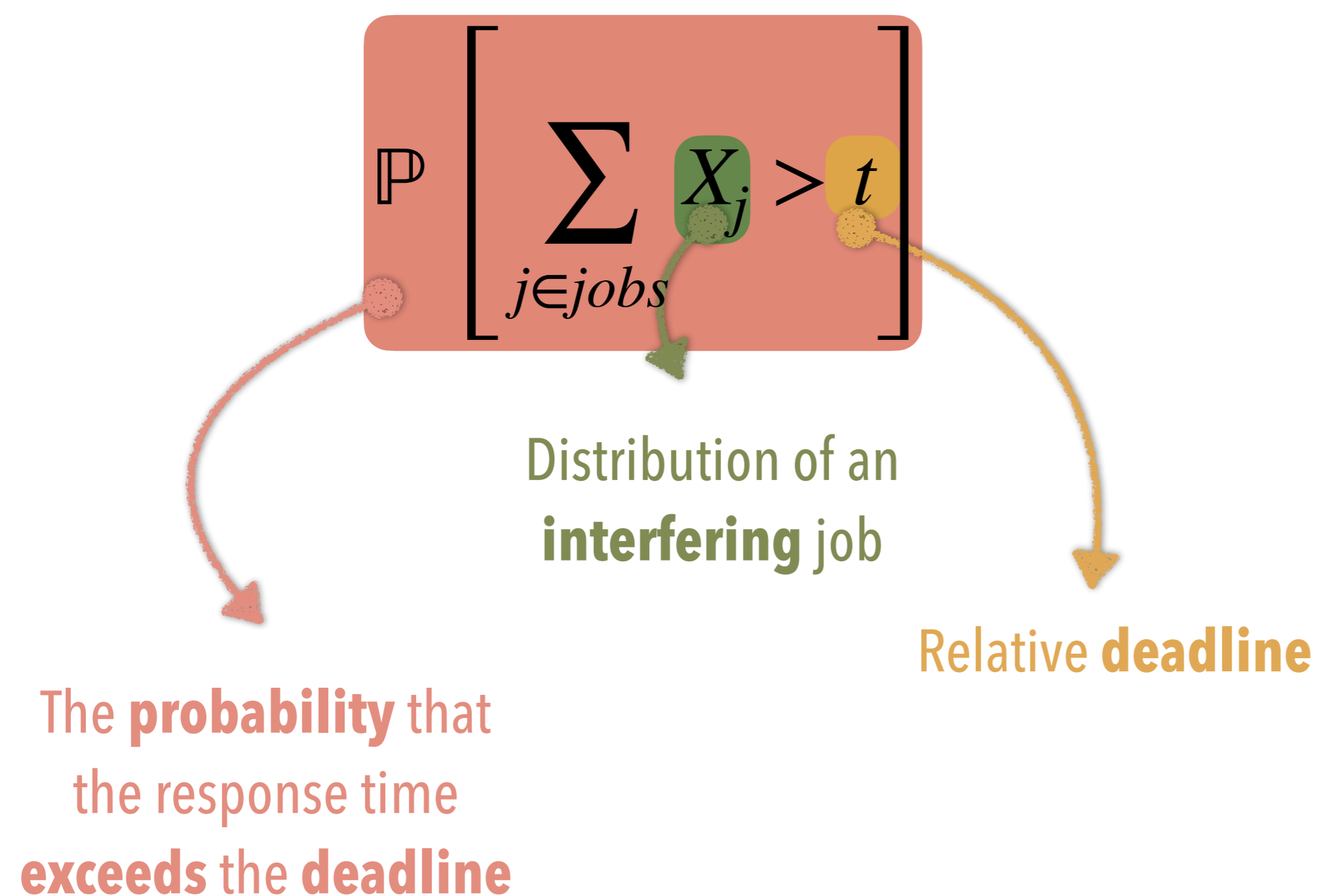
We can apply Cantelli's Inequality to a sum of possibly correlated random variables.





# A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

*We apply the same substitution to the rest of the inequality.*



# A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

We apply the same substitution to the rest of the inequality.

$$X = \sum_{j \in \text{jobs}} X_j$$

$$\forall t > \mathbb{E}(X),$$

$$\mathbb{P} \left[ \sum_{j \in \text{jobs}} X_j > t \right] \leq \frac{(\sigma[X])^2}{(\sigma[X])^2 + (t - \mathbb{E}[X])^2}$$

Distribution of an interfering job

Relative **deadline**

The **probability** that the response time **exceeds the deadline**

# A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

We apply the same substitution to the rest of the inequality. ✓

$$\forall t > \mathbb{E} \left[ \sum_{j \in \text{jobs}} X_j \right],$$

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Distribution of an  
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Relative **deadline**

The **probability** that  
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$$\leq \frac{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2}{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2 + \left( t - \mathbb{E} \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2}$$

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Distribution of an interfering job

Relative **deadline**

The **probability** that the response time **exceeds the deadline**

# EXPECTATION OF THE SUM

What is the expected value of the sum of possibly correlated random variables?

Expected value of the response time distribution

$$\forall t > \mathbb{E} \left[ \sum_{j \in \text{jobs}} X_j \right],$$

$$\mathbb{P} \left[ \sum_{j \in \text{jobs}} X_j > t \right]$$

Distribution of an interfering job

$$\leq \frac{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2}{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2 + \left( t - \mathbb{E} \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2}$$

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Distribution of an interfering job

Relative deadline

$$\leq \frac{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2}{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2 + \left( t - \mathbb{E} \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2}$$

$$= \sum_{j \in \text{jobs}} \mathbb{E} \left[ X_j \right]$$

Expected value of an interfering job

The **probability** that the response time exceeds the **deadline**

# EXPECTATION OF THE SUM

What is the expected value of the sum of possibly correlated random variables?

**Equal to** the sum of per-RV expected values.

Expected value of the response time distribution

$$\forall t > \mathbb{E} \left[ \sum_{j \in \text{jobs}} X_j \right],$$

$$\mathbb{P} \left[ \sum_{j \in \text{jobs}} X_j > t \right]$$

Distribution of an interfering job

Relative deadline

$$\leq \frac{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2}{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2 + \left( t - \mathbb{E} \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2}$$

= linearity of expectation

$$\sum_{j \in \text{jobs}} \mathbb{E} \left[ X_j \right]$$

Expected value of an interfering job

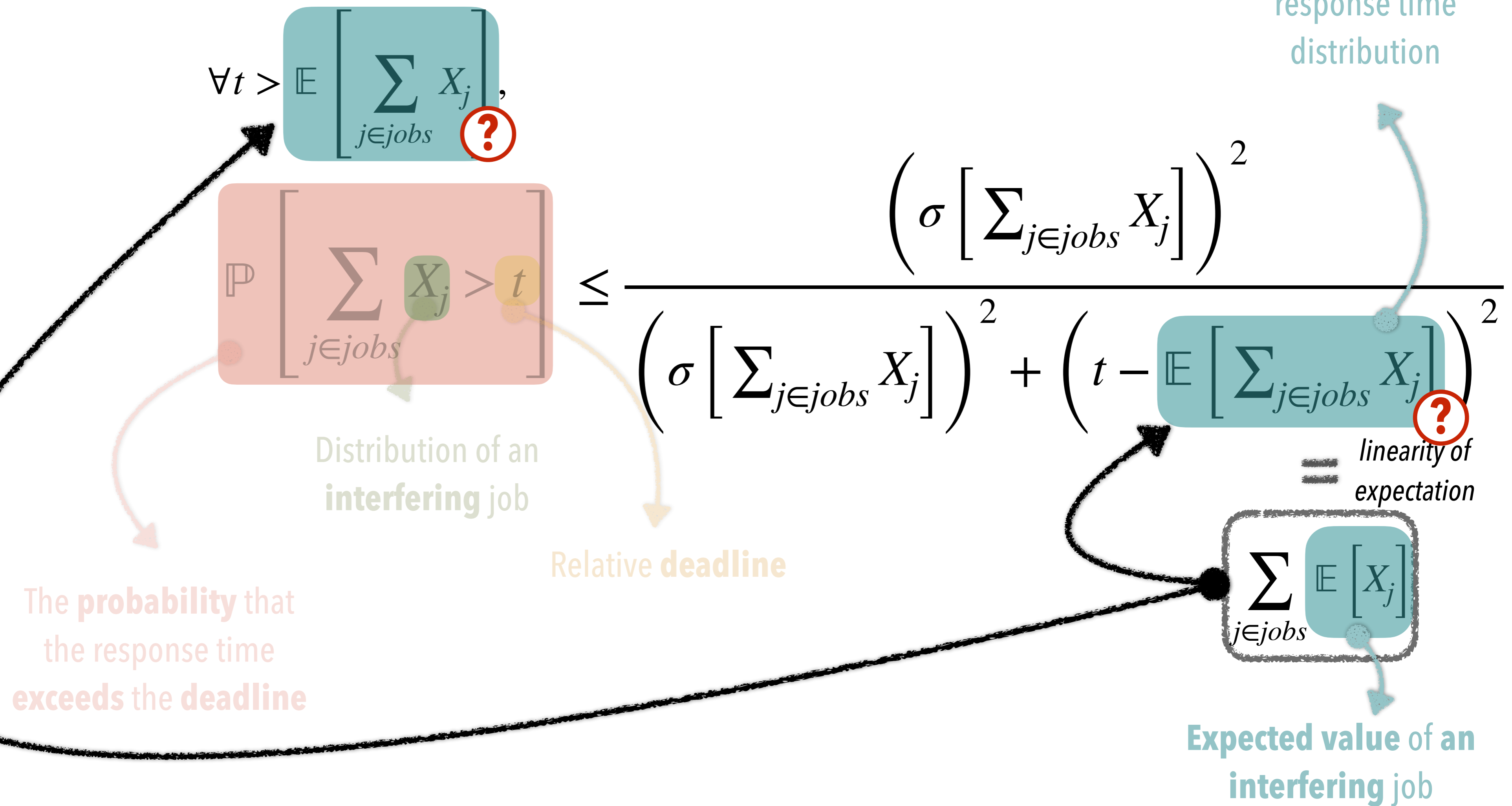
The **probability** that the response time exceeds the **deadline**

# EXPECTATION OF THE SUM

What is the expected value of the sum of possibly correlated random variables?

**Equal to** the sum of per-RV expected values.

Expected value of the response time distribution





# EXPECTATION OF THE SUM

What is the expected value of the sum of possibly correlated random variables?

**Equal to** the sum of per-RV expected values. ✓

$$\forall t > \sum_{j \in \text{jobs}} \mathbb{E}[X_j],$$

$$\mathbb{P} \left[ \sum_{j \in \text{jobs}} X_j > t \right] \leq \frac{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2}{\left( \sigma \left[ \sum_{j \in \text{jobs}} X_j \right] \right)^2 + \left( t - \sum_{j \in \text{jobs}} \mathbb{E}[X_j] \right)^2}$$

Distribution of an interfering job

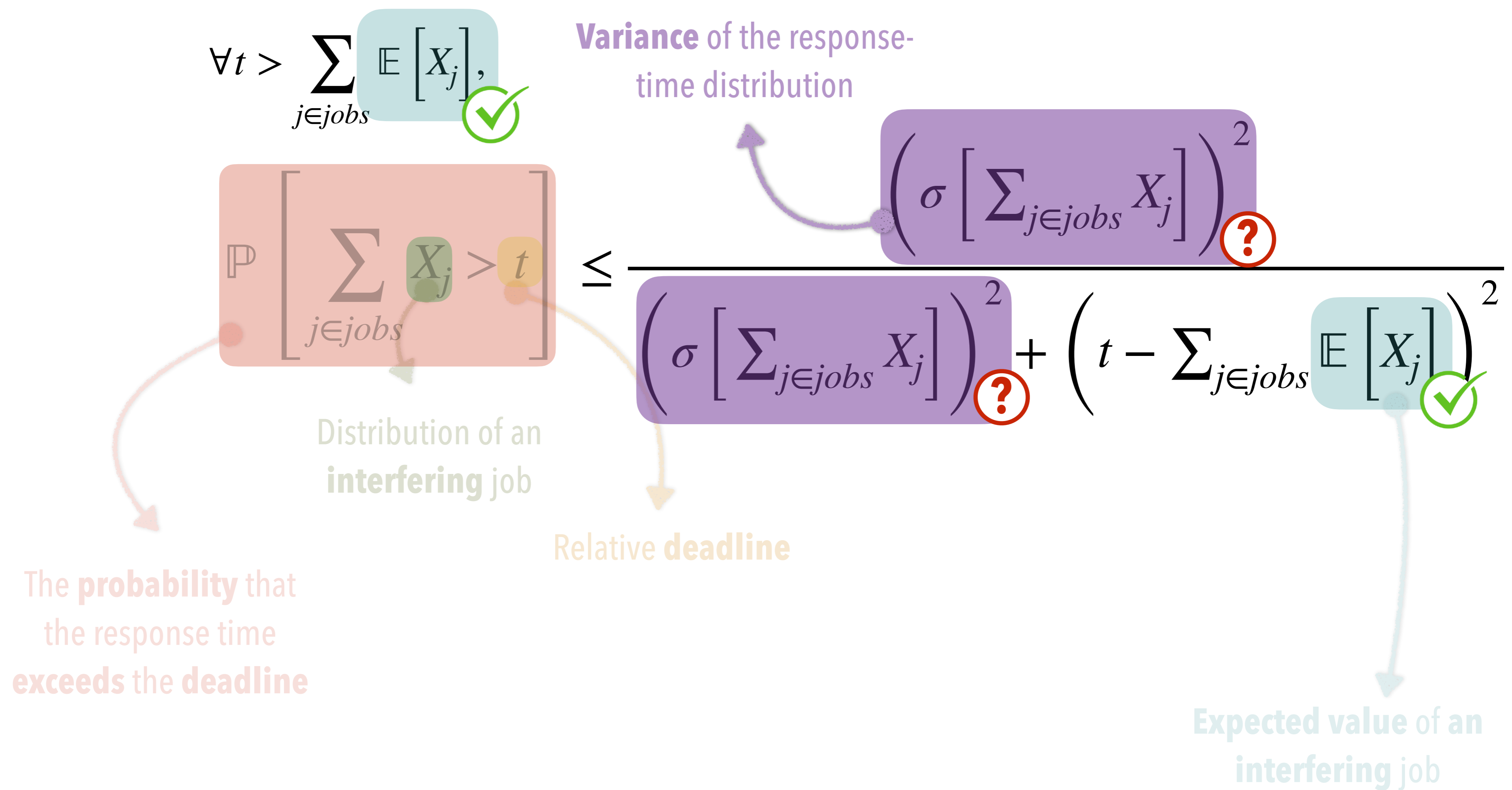
Relative **deadline**

The **probability** that the response time **exceeds the deadline**

Expected value of an interfering job

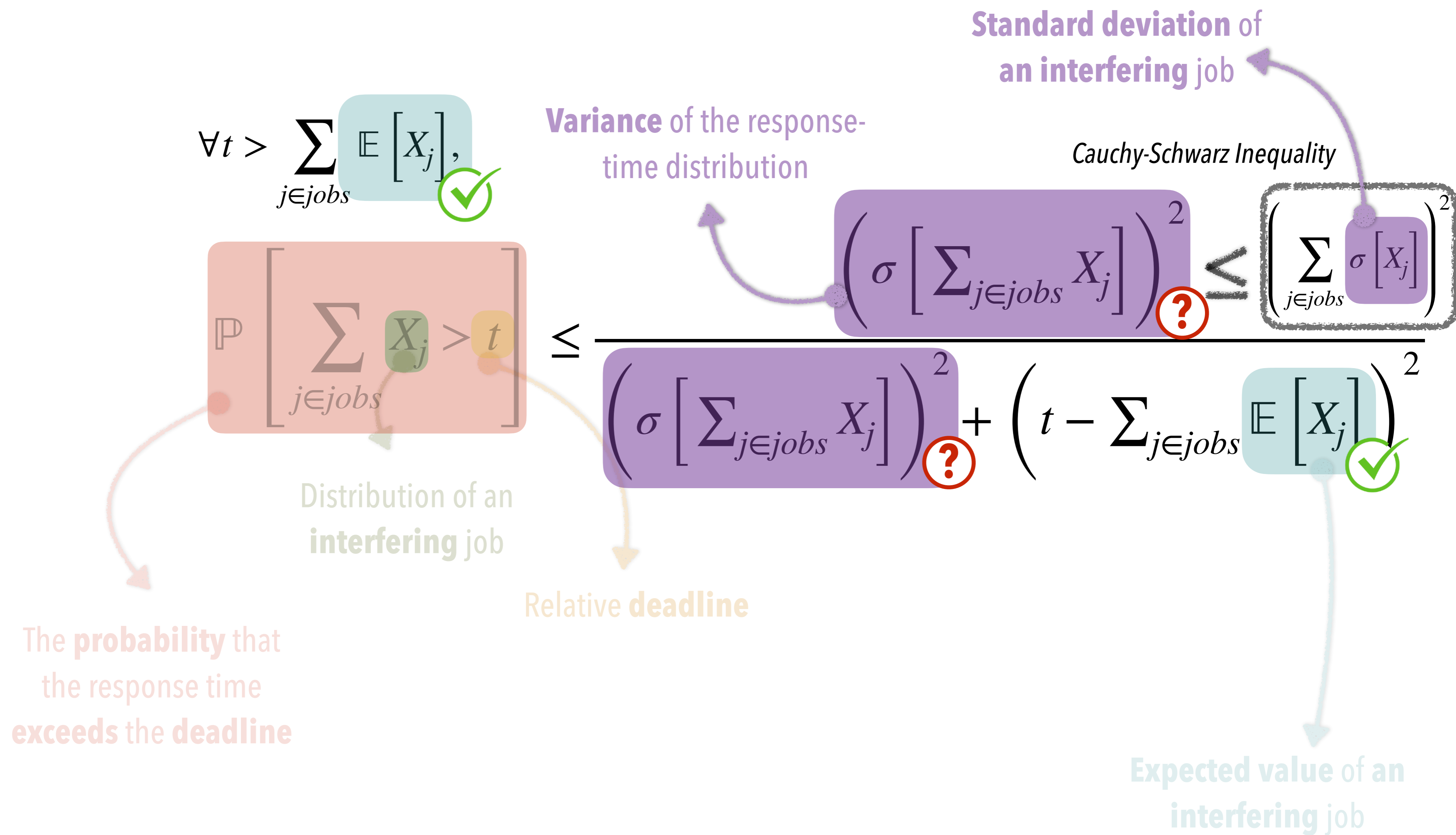
# VARIANCE OF THE SUM

What is the value of the variance of the sum of possibly correlated random variables?



# VARIANCE OF THE SUM

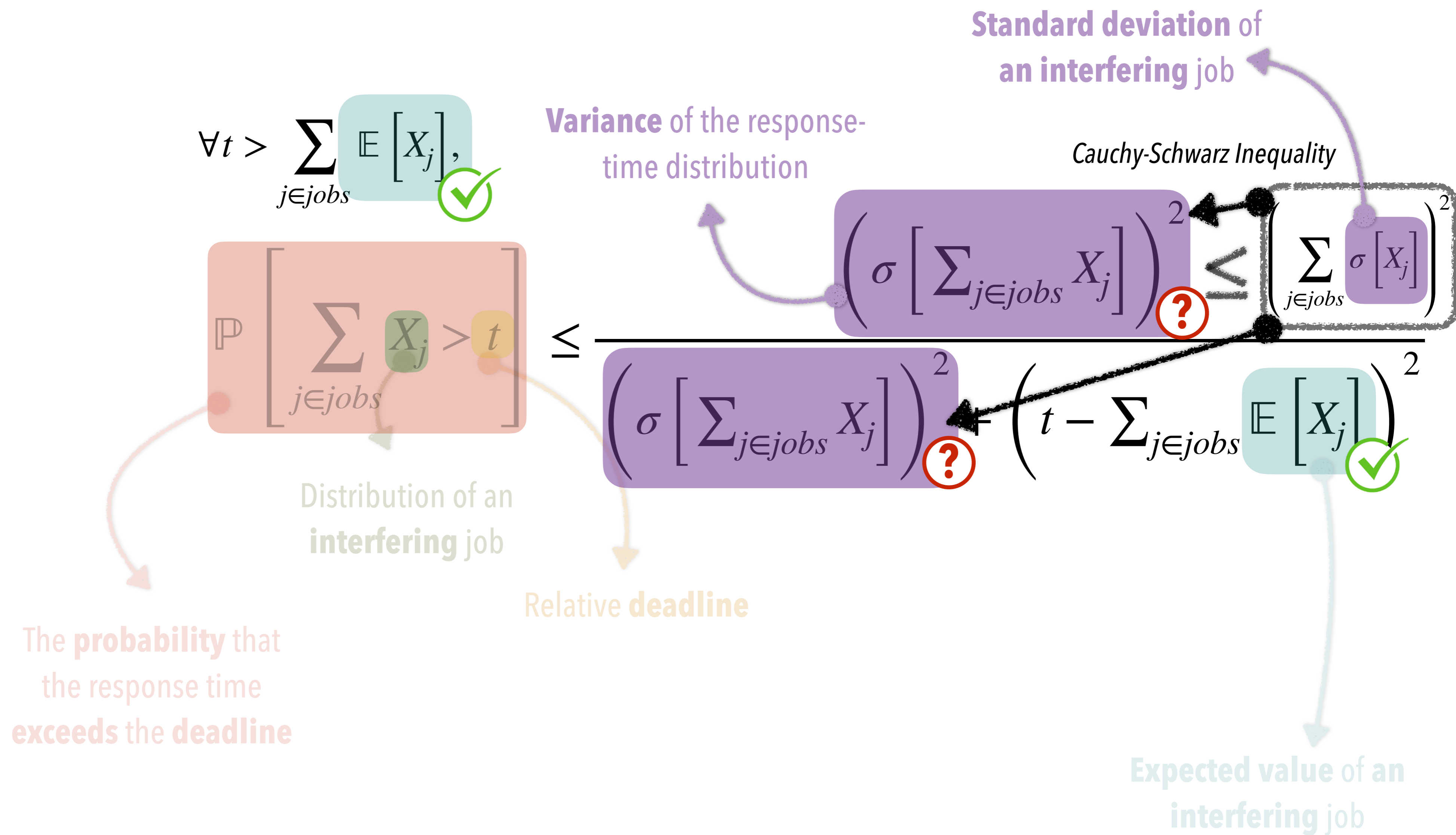
What is the value of the variance of the sum of possibly correlated random variables?



# VARIANCE OF THE SUM

What is the value of the variance of the sum of possibly correlated random variables?

**Less than or equal to the sum of per-RV standard deviations.**



# VARIANCE OF THE SUM

What is the value of the variance of the sum of possibly correlated random variables?

**Less** than the sum of per-RV standard deviations. ✓

✓  
Standard deviation of  
an interfering job

$$\forall t > \sum_{j \in \text{jobs}} \mathbb{E}[X_j],$$

$$\mathbb{P} \left[ \sum_{j \in \text{jobs}} X_j > t \right] \leq \frac{\left( \sum_{j \in \text{jobs}} \sigma[X_j] \right)^2}{\left( \sum_{j \in \text{jobs}} \sigma[X_j] \right)^2 + \left( t - \sum_{j \in \text{jobs}} \mathbb{E}[X_j] \right)^2}$$

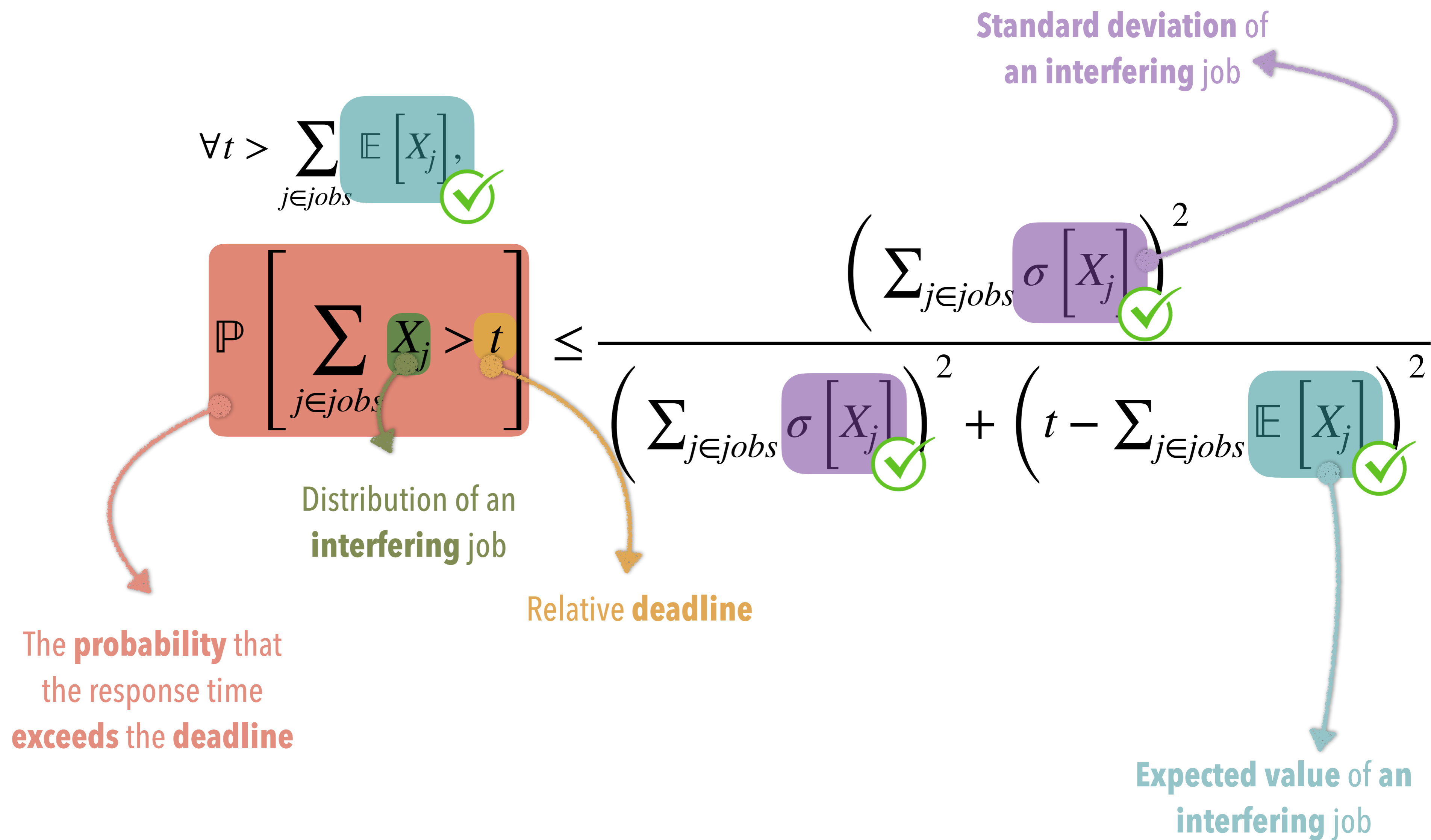
Distribution of an interfering job  
 Relative deadline  
 The **probability** that the response time exceeds the **deadline**  
 Expected value of an interfering job

# CORRELATION-TOLERANT INEQUALITY

Now, we can use the expected values and standard deviations of the **individual, possibly correlated** random variables.

$$\forall t > \sum_{j \in \text{jobs}} \mathbb{E}[X_j],$$

$$\mathbb{P} \left[ \sum_{j \in \text{jobs}} X_j > t \right] \leq \frac{\left( \sum_{j \in \text{jobs}} \sigma[X_j] \right)^2}{\left( \sum_{j \in \text{jobs}} \sigma[X_j] \right)^2 + \left( t - \sum_{j \in \text{jobs}} \mathbb{E}[X_j] \right)^2}$$



The **probability** that the response time exceeds the **deadline**  
 Distribution of an **interfering job**  
 Standard deviation of an interfering job  
 Expected value of an interfering job  
 Relative **deadline**

# THERE IS MUCH MORE IN THE PAPER

## Concentration Inequality

- based on Cantelli's Ineq,
- Verified in Coq.

## Ground-Truth System Model

- jobs aborted upon deadline miss
- fully-preemptive scheduling
- fixed priority, uniprocessor

## Correlation-Tolerant Analysis (CTA)

- connects the other two pillars
- soundly-upper bounds DFP
- closed-form solution

$$\forall t > E(X), \\ P[X > t] \leq \frac{(\sigma(X))^2}{(\sigma(X))^2 + (t - E(X))^2}$$

 The Coq Proof Assistant

[coq.inria.fr](http://coq.inria.fr)

# LET US APPLY CTA TO THE EXAMPLE

**Input:** expected value and standard deviation upper bounds on task execution time distributions.

$$\mathbb{E}(\overset{0.965}{\boxed{1}} \overset{0.015}{\boxed{3}} \overset{0.02}{\boxed{5}}) = 1.11 \leq 1.12$$

$$\sigma(\overset{0.965}{\boxed{1}} \overset{0.015}{\boxed{3}} \overset{0.02}{\boxed{5}}) \approx 0.606 \leq 0.61$$

$$\mathbb{E}(\overset{0.975}{\boxed{2}} \overset{0.025}{\boxed{8}}) = 2.15 \leq 2.16$$

$$\sigma(\overset{0.975}{\boxed{2}} \overset{0.025}{\boxed{8}}) \approx 0.937 \leq 0.94$$

*deadline = 10*



# LET US APPLY CTA TO THE EXAMPLE

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$$\mathbb{E}(\overset{0.975}{\mathbf{2}} \overset{0.025}{\mathbf{8}}) = 2.15 \leq 2.16$$

$$\sigma(\overset{0.975}{\mathbf{2}} \overset{0.025}{\mathbf{8}}) \approx 0.937 \leq 0.94$$

*deadline = 10*

**Analysis:**

$$\mathbb{P}[X > t] \leq \frac{(\sigma(X))^2}{(\sigma(X))^2 + (t - \mathbb{E}(X))^2}$$

# LET US APPLY CTA TO THE EXAMPLE

**Input:** expected value and standard deviation upper bounds on task execution time distributions.

$$\mathbb{E}(\overset{0.965}{\text{1}} \overset{0.015}{\text{3}} \overset{0.02}{\text{5}}) = 1.11 \leq 1.12$$

$$\sigma(\overset{0.965}{\text{1}} \overset{0.015}{\text{3}} \overset{0.02}{\text{5}}) \approx 0.606 \leq 0.61$$

$$\mathbb{E}(\overset{0.975}{\text{2}} \overset{0.025}{\text{8}}) = 2.15 \leq 2.16$$

$$\sigma(\overset{0.975}{\text{2}} \overset{0.025}{\text{8}}) \approx 0.937 \leq 0.94$$

*deadline = 10*

**CTA:**

$$\mathbb{P}[\overset{0.965}{\text{1}} \overset{0.015}{\text{3}} \overset{0.02}{\text{5}} + \overset{0.975}{\text{2}} \overset{0.025}{\text{8}} > 10] \leq \frac{(0.61 + 0.94)^2}{(0.61 + 0.94)^2 + (10 - (1.12 + 2.16))^2} \approx 0.05$$

# LET US APPLY CTA TO THE EXAMPLE

The CTA-derived DFP over-approximates the ground-truth DFP, being more accurate than pWCET-DFP.

**Input:** expected value and standard deviation upper bounds on task execution time distributions.

$$\mathbb{E}(\overset{0.965}{1} \overset{0.015}{3} \overset{0.02}{5}) = 1.11 \leq 1.12$$

$$\sigma(\overset{0.965}{1} \overset{0.015}{3} \overset{0.02}{5}) \approx 0.606 \leq 0.61$$

$$\mathbb{E}(\overset{0.975}{2} \overset{0.025}{8}) = 2.15 \leq 2.16$$

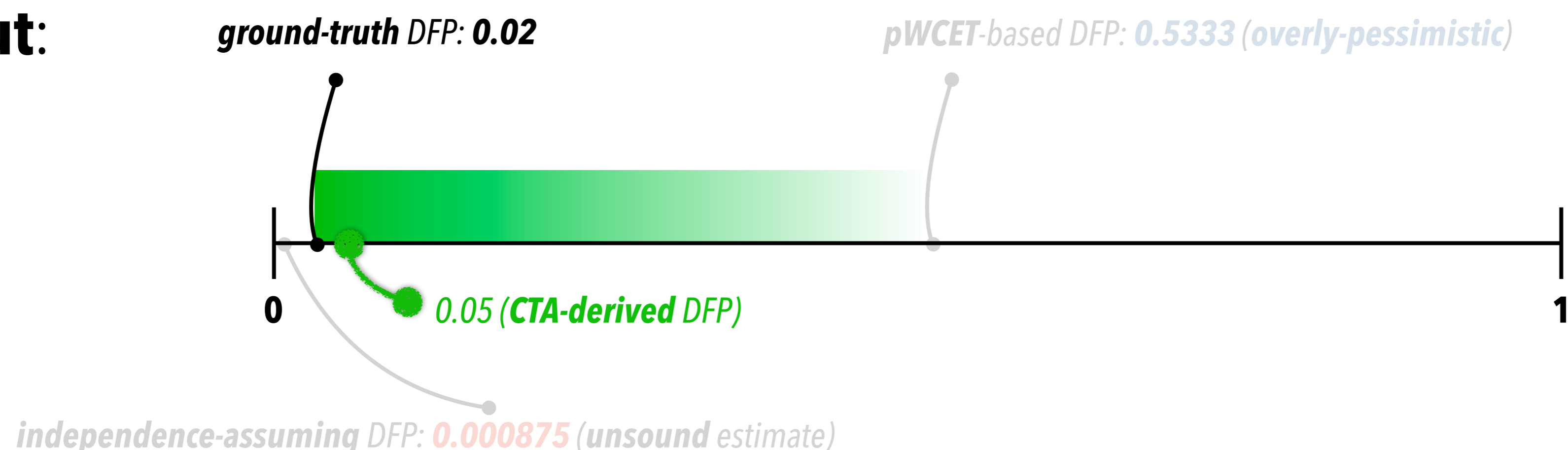
$$\sigma(\overset{0.975}{2} \overset{0.025}{8}) \approx 0.937 \leq 0.94$$

deadline = 10

**CTA:**

$$\mathbb{P}[\overset{0.965}{1} \overset{0.015}{3} \overset{0.02}{5} + \overset{0.975}{2} \overset{0.025}{8} > 10] \leq \frac{(0.61 + 0.94)^2}{(0.61 + 0.94)^2 + (10 - (1.12 + 2.16))^2} \approx 0.05$$

**Output:**



# **EVALUATION**

# EVALUATION

How does CTA compare to the  $p$ WCET-based analyses in general?

We compared **CTA** to the following baselines:

- **Berry-Essen**: DFP lower bound computed with the Berry-Esseen theorem.
- **Chernoff**: DFP upper bound computed with Chernoff bound



# EVALUATION SETUP

*Synthetic task sets were randomly generated to highlight differences between pWCET and CTA analysis.*

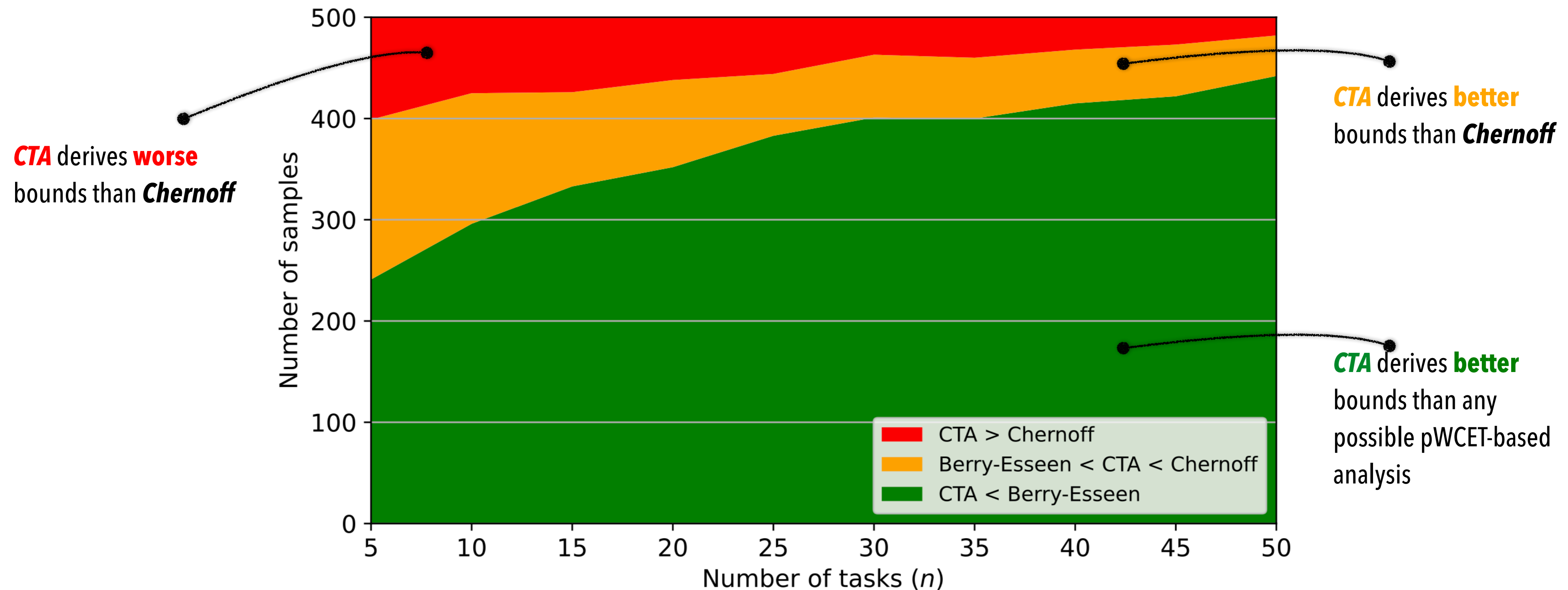
**Four experiments** were conducted to investigate:

1. Influence of the **task set size** on DFP,
2. The influence of the **expected utilization** according to pWCET distributions,
3. The influence of the **expected utilization** according to CTA inputs,
4. The influence of the **maximum standard deviation** on CTA.

*In this talk, we focus on (1)*

# EVALUATION, EXPERIMENT 1

Investigating the influence of the task-set size



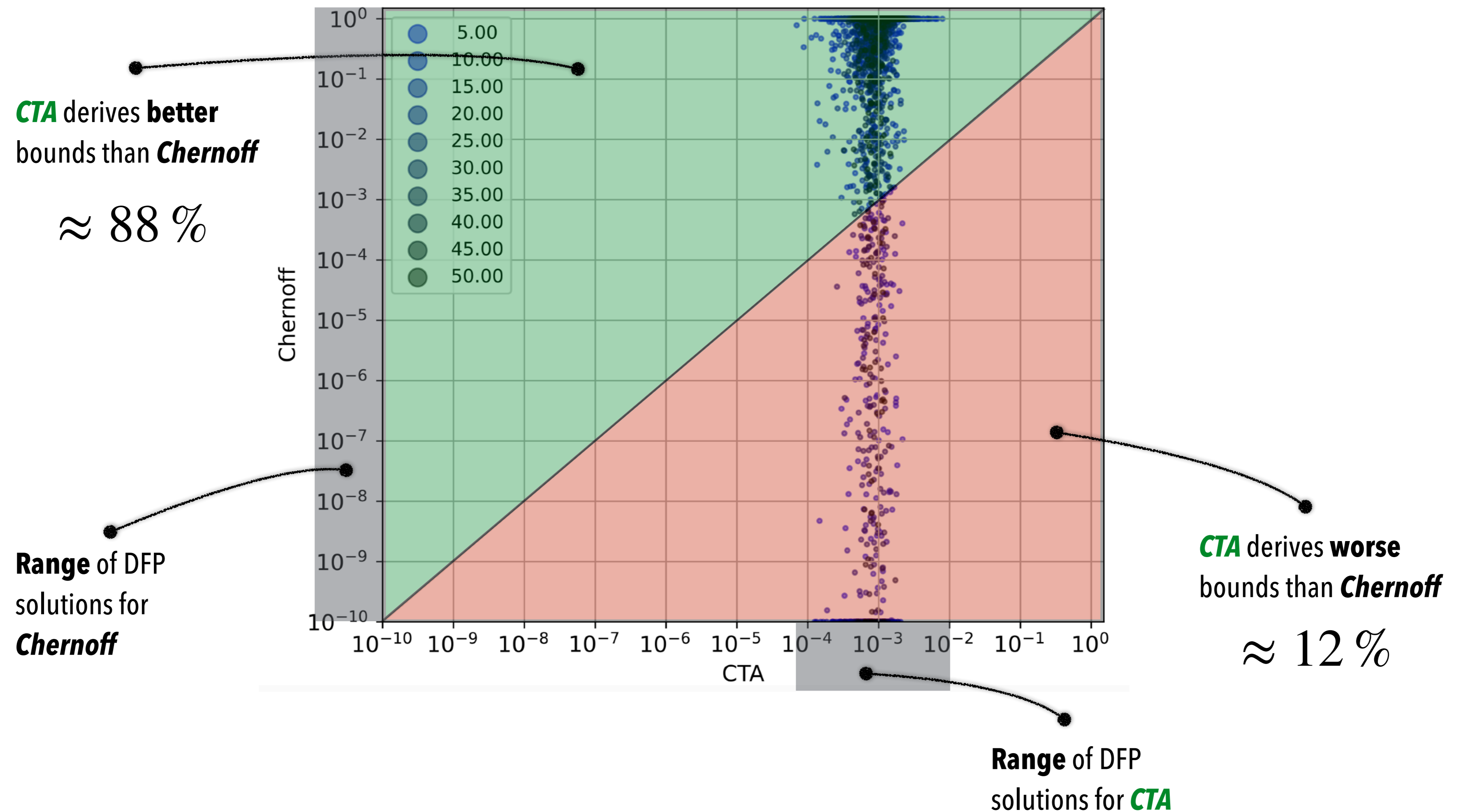
As the number of tasks in a set **increases**, the CTA method's **advantage** over pWCET-based baselines **grows**.

This is because pWCET can be overly pessimistic in the presence of correlations.

The level of pessimism increases at a faster rate than the expectation used by CTA with new interfering tasks.

# EVALUATION, EXPERIMENT 1

Investigating the influence of the task-set size



CTA typically offers **lower bounds**,  
 but its reliance on simple summary statistics **can limit the range** of obtainable DFP bounds.



# **SUMMARY**

# SUMMARY

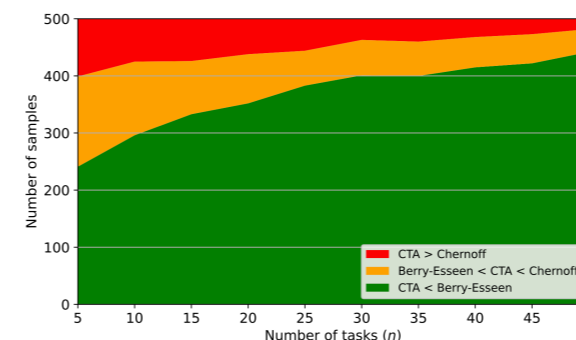
**Efficient:** minimize



**space** and **time** complexity

- CTA relies on a **closed-form** expression; its run-time and space complexity are **negligible**
- CTA **tolerates dependence** by construction
- CTA **does not** require pWCET nor any similar independence-implying construct

**Accurate:** minimize over-approximation



- The results are promising, but pWCET can still be useful under certain conditions.

# A novel analysis with a lot of potential.