CTA: A CORRELATION-TOLERANT ANALYSIS
OF THE
DEADLINE-FAILURE PROBABILITY OF DEPENDENT TASKS

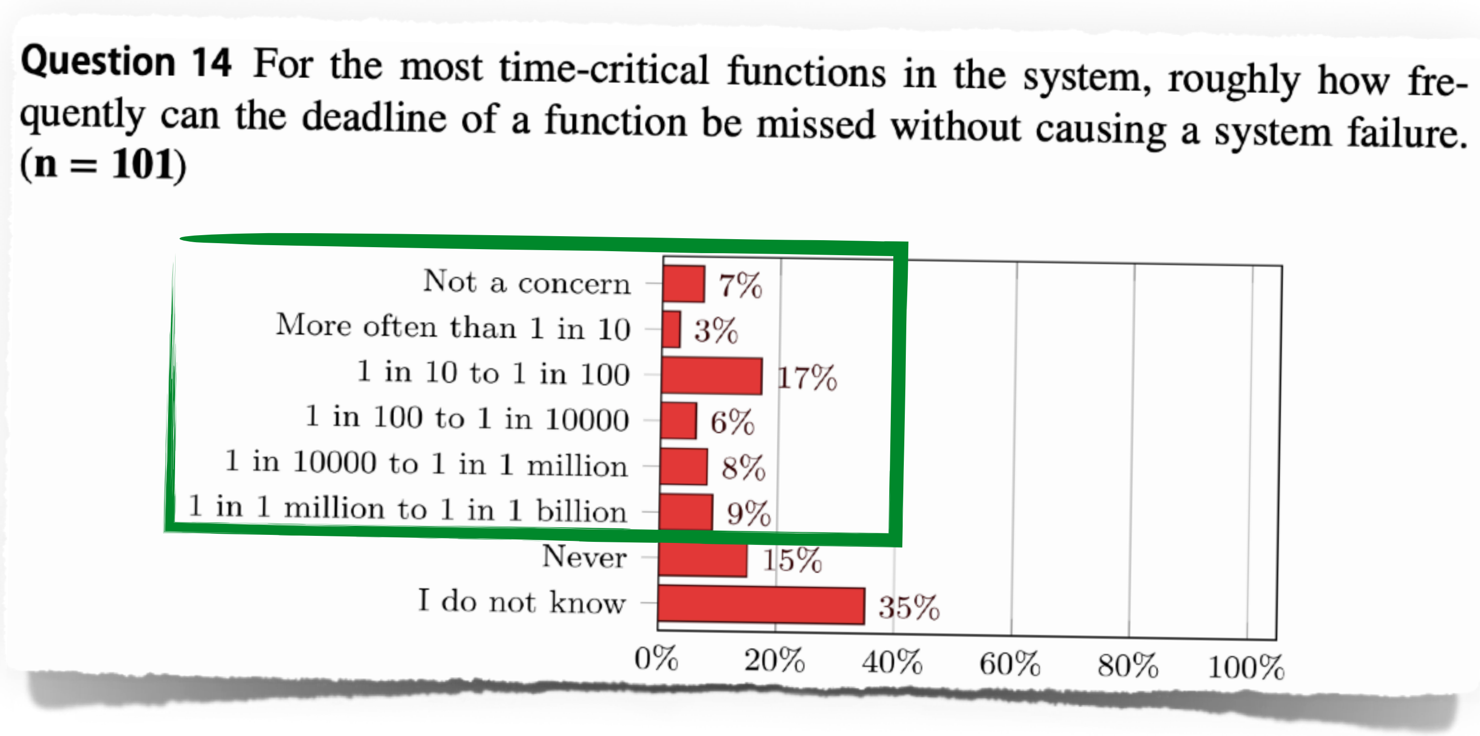
RTSS 2023
December 8, 2022

Filip Marković, Pierre Roux, Sergey Bozhko,
Alessandro V. Papadopoulos, and Björn B. Brandenburg

European Research Council
Established by the European Commission
Vetenskapsrådet
MAX-PLANCK-GESELLSCHAFT
MAX PLANCK INSTITUTE FOR SOFTWARE SYSTEMS
ONERA THE FRENCH AEROSPACE LAB
Mälardalen University

PROBABILISTIC ANALYSIS OF REAL-TIME SYSTEMS

Why is it relevant?



[1]

Many systems have **soft** real-time guarantees rather than **hard** ones

Many systems are **not statically** analyzable but rather **statistically**

Many soft real-time systems **do not benefit** from deterministic analysis as it would **unnecessarily** over-provision system resources

Many **safety standards** are defined in terms of **failure probabilities**

The total utilization of that system goes above 100%. Using response time analysis in such situation automatically yields unbounded (infinite) worst-case response times.

[2]

SIL	Low demand mode prob. failure on demand	Continuous/High demand mode prob. failure per hour
1	$\geq 10^{-2}$ to $< 10^{-1}$	$\geq 10^{-6}$ to $< 10^{-5}$
2	$\geq 10^{-3}$ to $< 10^{-2}$	$\geq 10^{-7}$ to $< 10^{-6}$
3	$\geq 10^{-4}$ to $< 10^{-3}$	$\geq 10^{-8}$ to $< 10^{-7}$
4	$\geq 10^{-5}$ to $< 10^{-4}$	$\geq 10^{-9}$ to $< 10^{-8}$

Table 1: IEC 61508: Permitted Failure Probabilities

[3]

[1] Akesson et al. *RTSJ* (2022)

[2] Rivas et al. *WATERS* (2016)

[3] Agrawal et al. *ICCAD* (2020)

OPEN PROBLEM: DEPENDENCE

Real-time systems run ***intrinsically dependent*** *tasks, while plenty of analyses assume* ***independent*** *task execution.*

1995
Tia et al.
RTAS

"Unfortunately, the computation times of individual requests ***are not statistically independent****. In the system studied here, the computation times of requests in each task* ***are correlated*** *with that of requests in many other tasks…"* [1]

"… As a consequence, the probability of meeting deadlines thus computed may be ***overly optimistic****."* [1]

2019
Davis and Cucu-Grosjean
LITES

"Issues of ***dependence*** *are of* ***great importance*** *in probabilistic schedulability analysis."* [2]

"… Analyses ***are needed*** *that can address* ***dependencies****."* [2]

Today
RTSS

We present a **Correlation-Tolerant Analysis**


CTA: A Correlation-Tolerant Analysis of the Deadline-Failure Probability of Dependent Tasks

[1] Tia et al. *RTAS* (1995)
[2] Davis and Cucu-Grosjean *LITES* (2019)

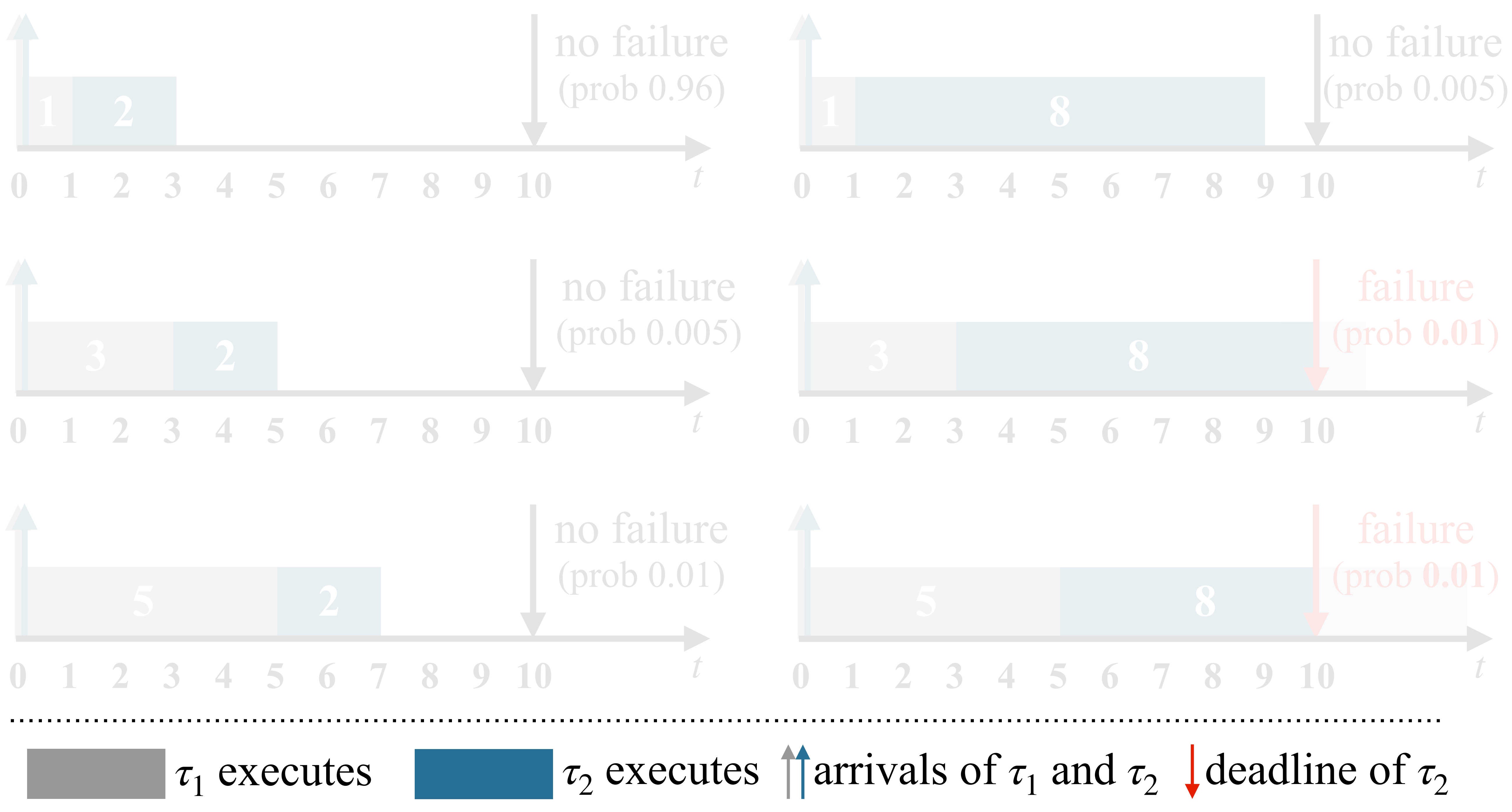
DEADLINE-FAILURE PROBABILITY (DFP)

The probability that a job of a task fails to complete before its deadline.

Consider a simple system comprising two tasks

- grey task (high priority),
- blue task (low priority).

Ground-truth behavior: all possible evolutions



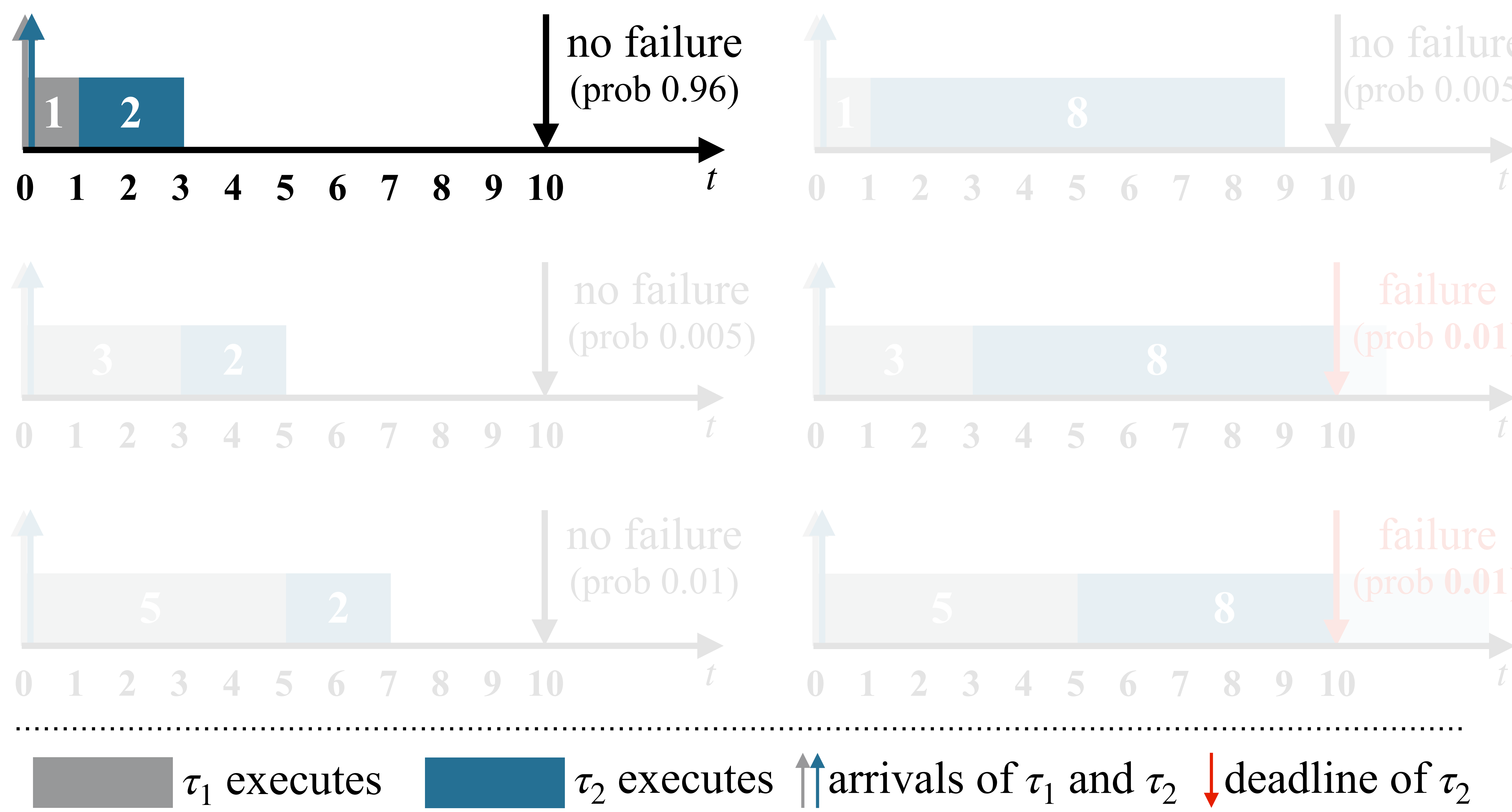
DEADLINE-FAILURE PROBABILITY (DFP)

The probability that a job of a task fails to complete before its deadline.

Consider a simple system comprising two tasks

- grey task (high priority),
- blue task (low priority).

Ground-truth behavior: all possible evolutions



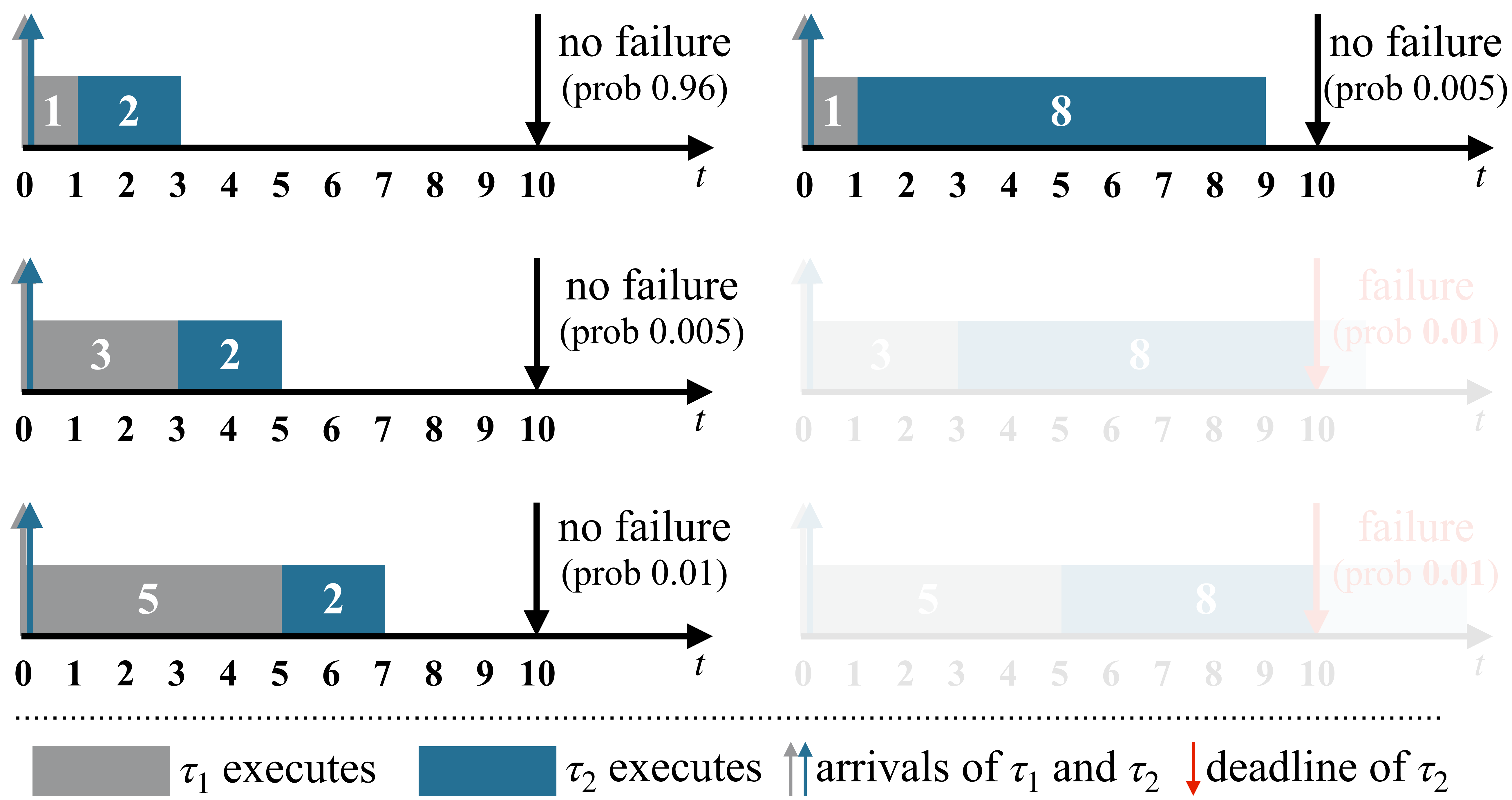
DEADLINE-FAILURE PROBABILITY (DFP)

The probability that a job of a task fails to complete before its deadline.

Consider a simple system comprising two tasks

- grey task (high priority),
- blue task (low priority).

Ground-truth behavior: all possible evolutions



DEADLINE-FAILURE PROBABILITY (DFP)

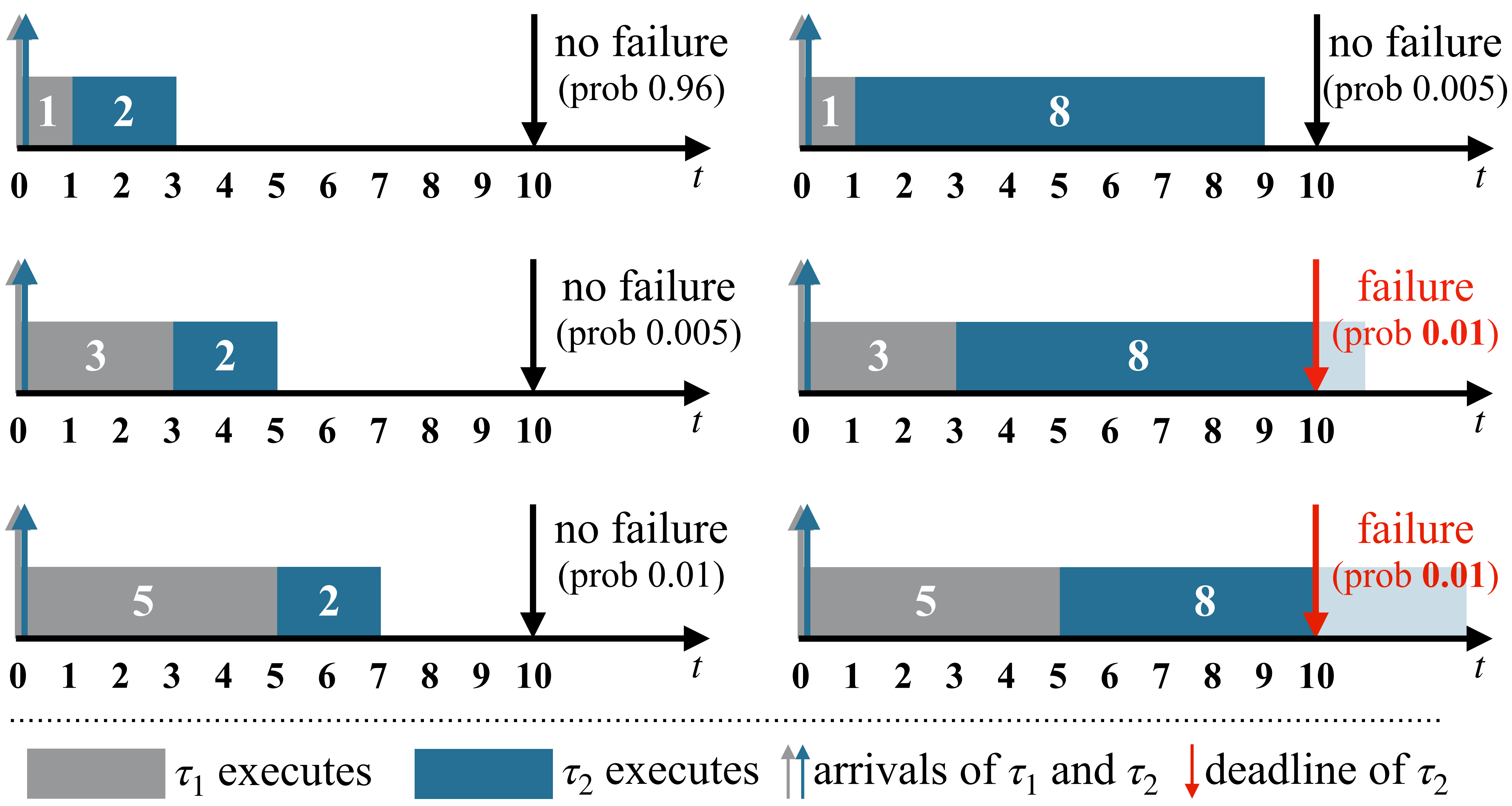
The probability that a job of a task fails to complete before its deadline.

Consider a simple system comprising two tasks

- grey task (high priority),
- blue task (low priority).

The **ground-truth DFP** of the **blue task** is **0.02**.

Ground-truth behavior: all possible evolutions



DFP ANALYSIS

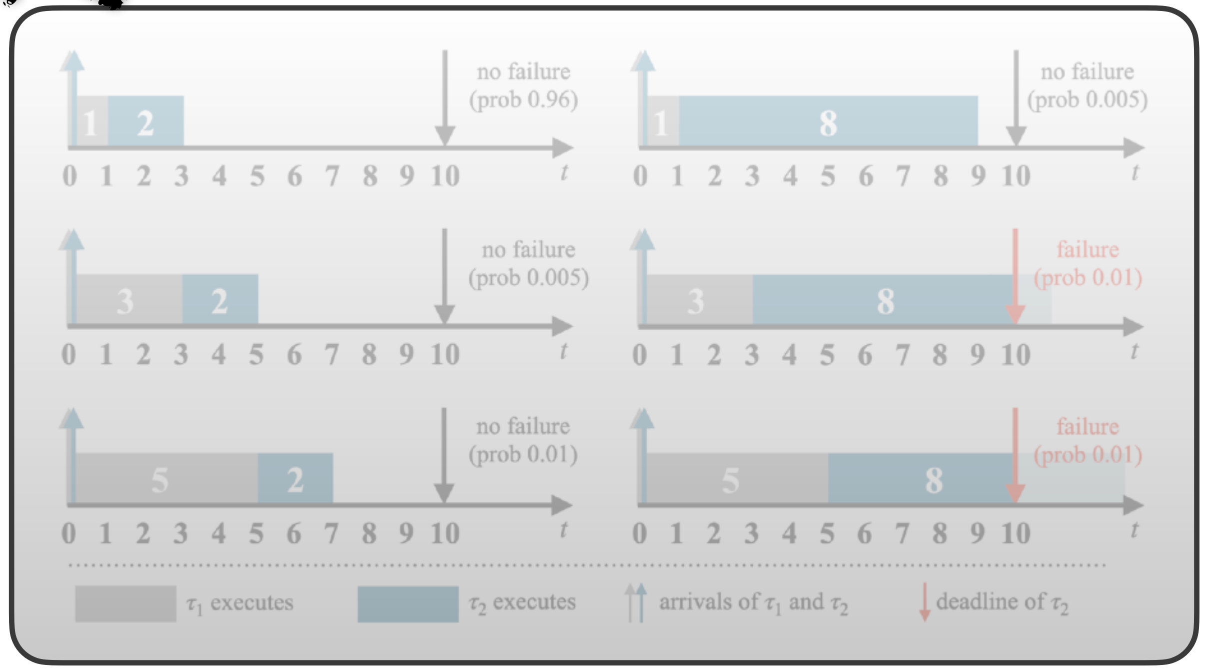
Analysis that derives an upper bound on the DFP of any job of a task.

Input: model parameters ← the easier to obtain, the better

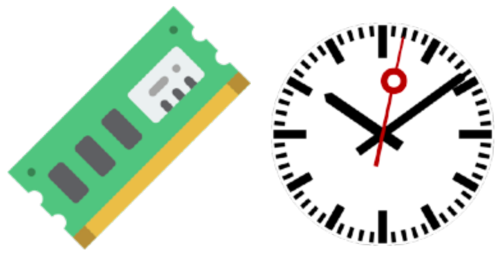
Output: DFP upper bound

Goal: Efficient and accurate DFP

Ground-truth behavior

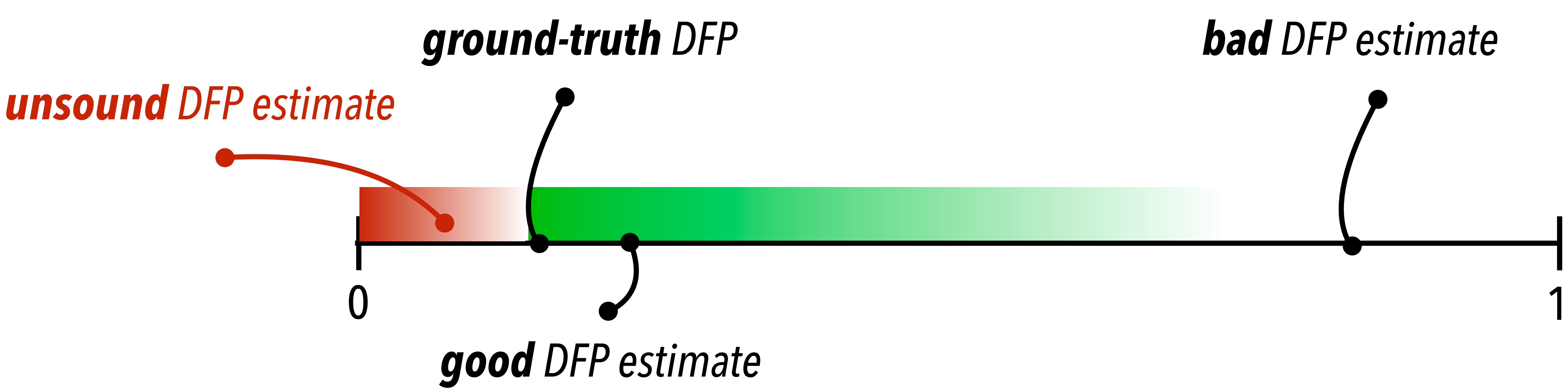


Efficient: minimize



space and **time** complexity

Accurate: minimize over-approximation



PRIOR WORK ON DFP ANALYSIS

DFP ANALYSIS ASSUMING INDEPENDENCE

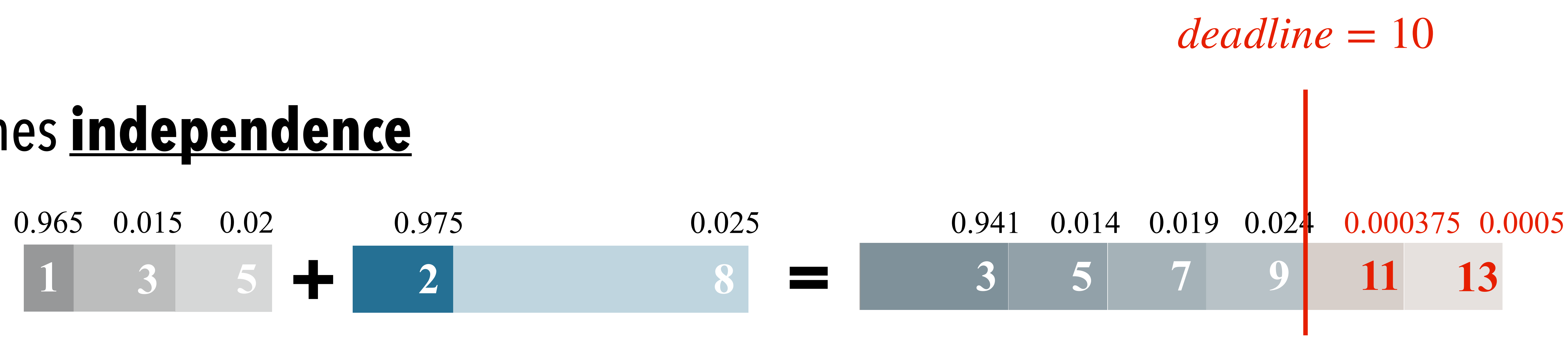
*Computation of the DFP (**blue task**) using per-task distributions and assuming independence*

Input: measured per-task execution-time distributions

Ground-truth behavior

per-task ground-truth execution-time distributions

Analysis: assumes **independence**



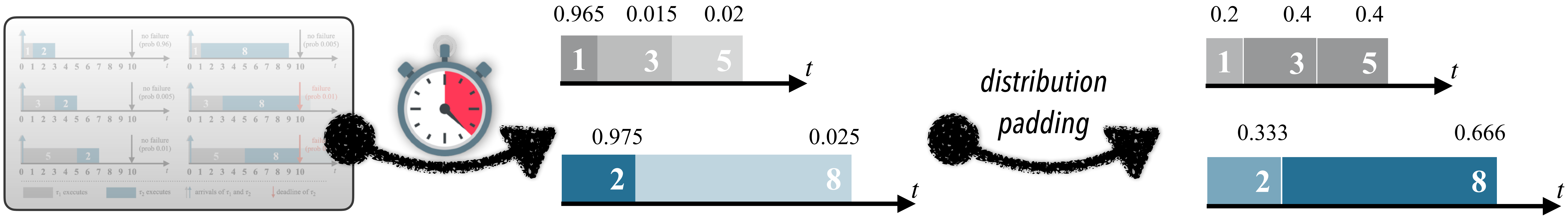
Output: **0.000875 < 0.02 (ground-truth DFP)**

Conclusion: Ignoring task dependence (correlation) risks unsound DFP estimation.

pWCET: PESSIMISM "BAKED IN"

A distribution designed to "hide" dependence while being analytically convenient.

Input: Probabilistic Worst-Case Execution Time (pWCET)

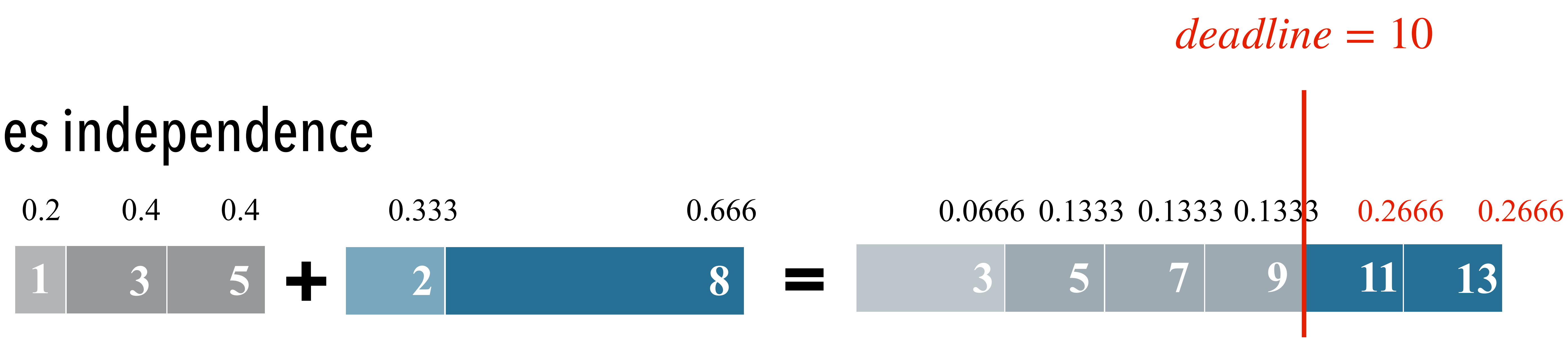


Ground-truth behavior

per-task ground-truth execution-time distributions

pWCET distributions

Analysis: assumes independence

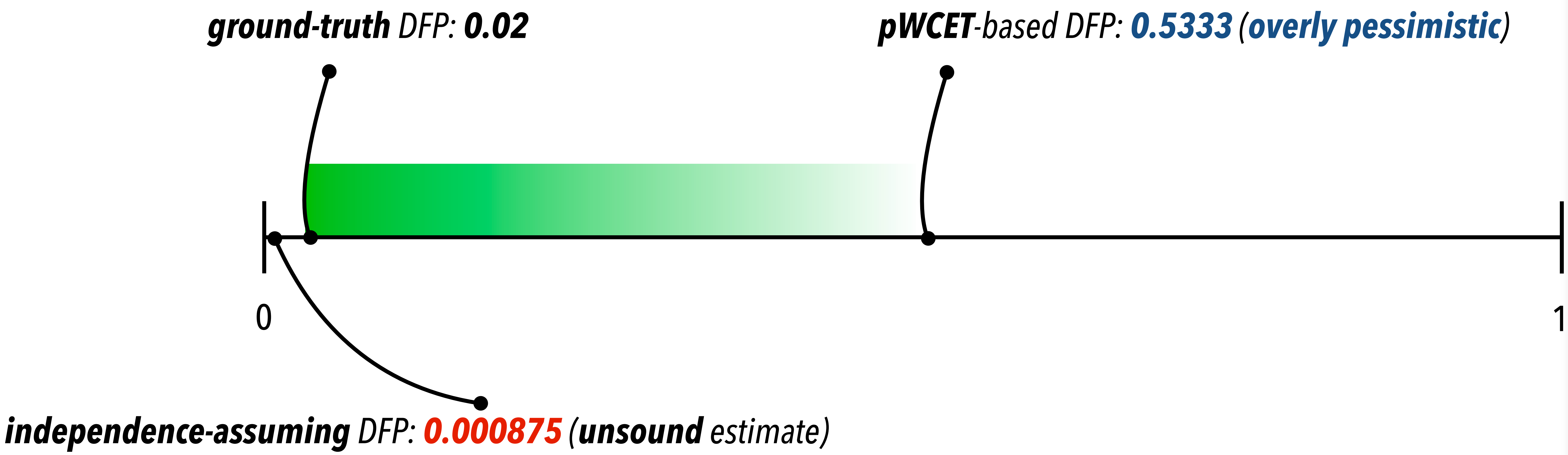


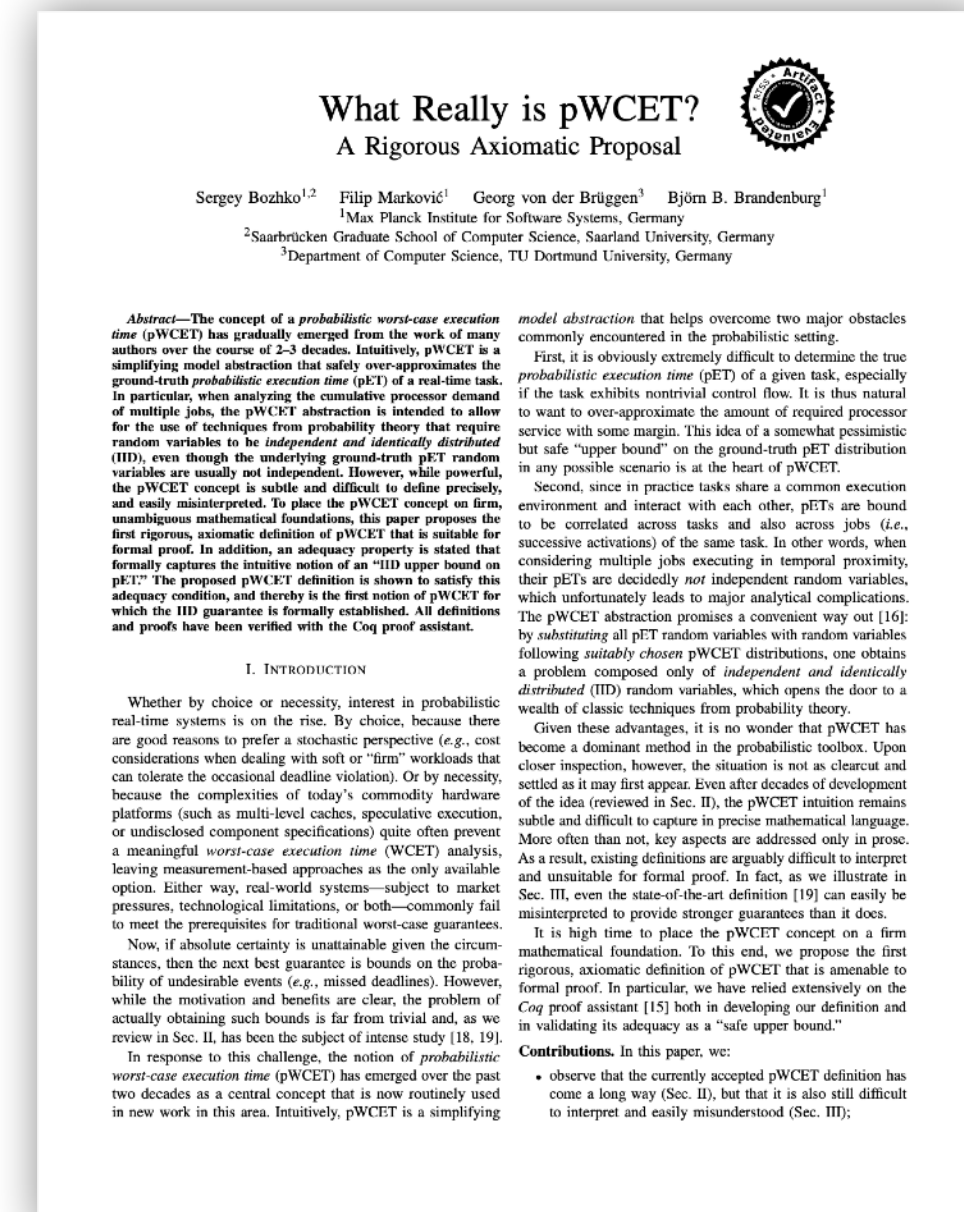
Output: 0.533333 > 0.02

Conclusion: pWCET-based analysis can be inherently pessimistic.

SUMMARY

*Ignoring correlation can be **unsound**, while pWCET-based approaches can be **overly pessimistic**.*

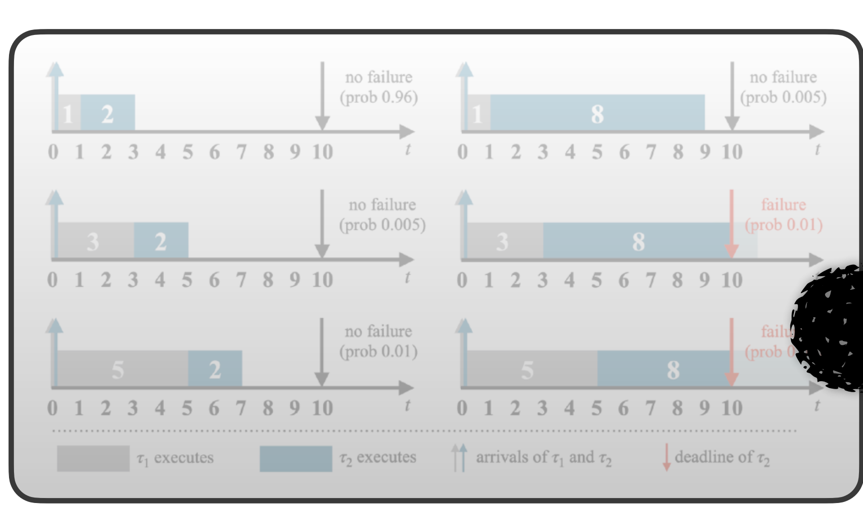



What Really is pWCET?
A Rigorous Axiomatic Proposal

Sergey Bozhko[1,2] Filip Marković[1] Georg von der Brüggen[3] Björn B. Brandenburg[1]
[1]Max Planck Institute for Software Systems, Germany
[2]Saarbrücken Graduate School of Computer Science, Saarland University, Germany
[3]Department of Computer Science, TU Dortmund University, Germany

I. Introduction

Also, **pWCET** derivation is not trivial



Ground-truth behavior

per-task ground-truth execution distributions

pWCET distributions

THIS PAPER:

A CORRELATION-TOLERANT ANALYSIS

(Not using pWCET!)

CANTELLI'S INEQUALITY

*Given a random variable with a known expected value and standard deviation and some threshold t, it **bounds the exceedance probability**.*

Input: expected value and variance of some random variable X.

But how does this translate to our **RT problem**?

$$\forall t > \mathbb{E}(X),$$
$$\mathbb{P}[X > t] \leq \frac{(\sigma(X))^2}{(\sigma(X))^2 + (t - \mathbb{E}(X))^2}$$

Variance of the response-time distribution of a task

Response-time distribution of a task

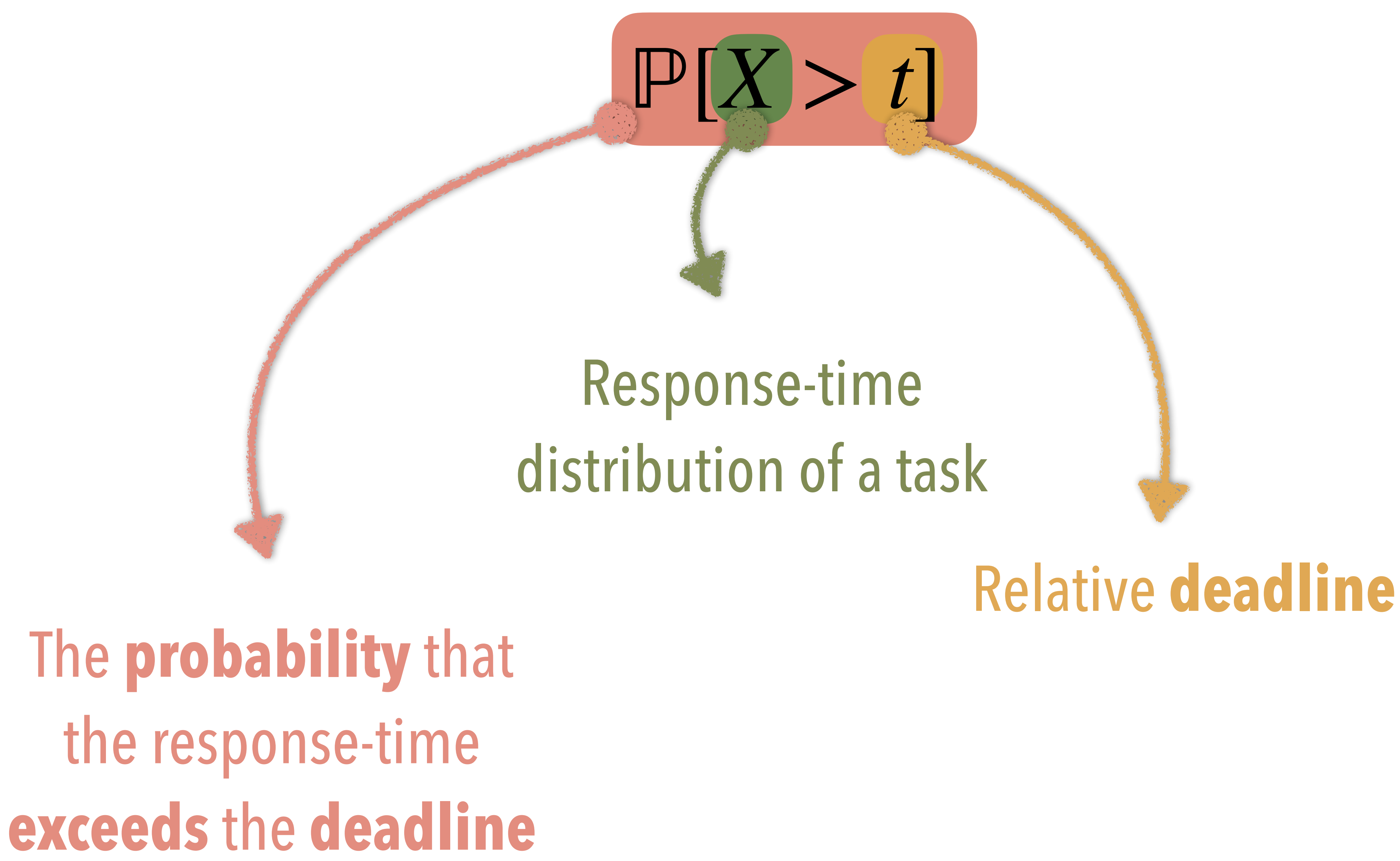
Relative **deadline**

The **probability** that the response-time **exceeds** the **deadline**

Expected value of the response-time distribution of a task

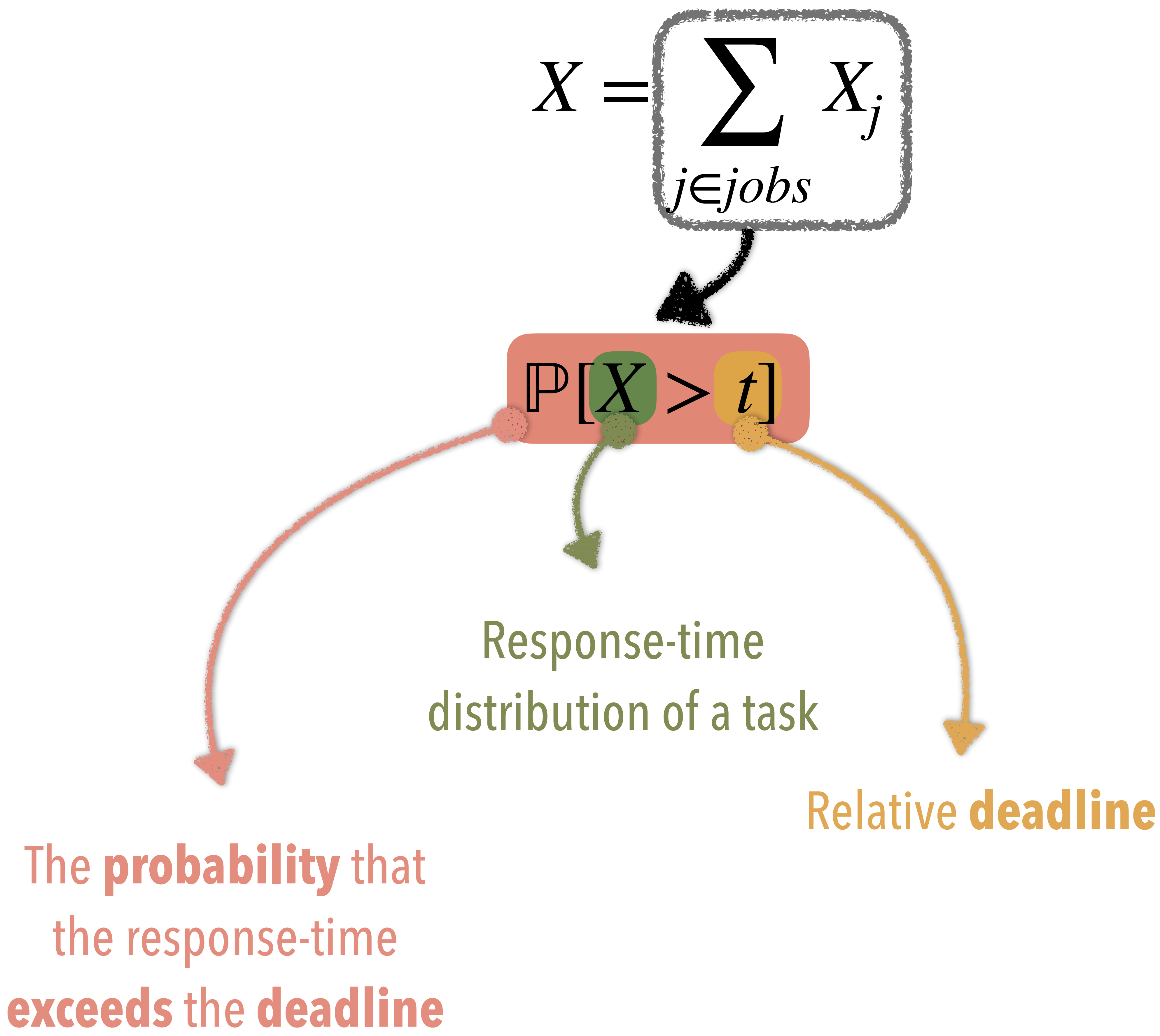
A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

We can apply Cantelli's Inequality to a sum of possibly correlated random variables.



A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

We can apply Cantelli's Inequality to a sum of possibly correlated random variables.



A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

We apply the same substitution to the rest of the inequality.

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right]$$

Distribution of an **interfering** job

Relative **deadline**

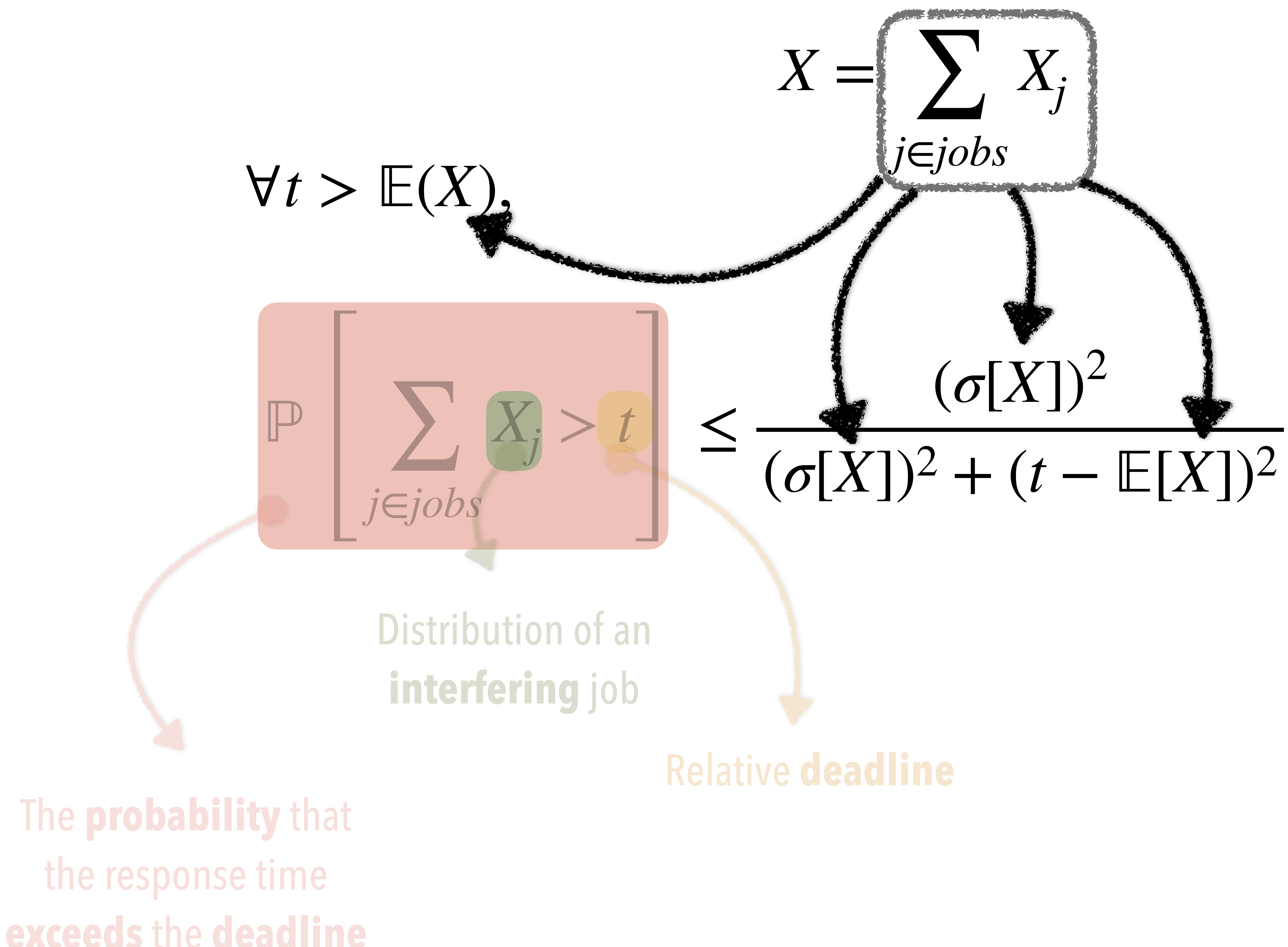
The **probability** that the response time **exceeds** the **deadline**

A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

We apply the same substitution to the rest of the inequality.

$$X = \sum_{j \in jobs} X_j$$

$$\forall t > \mathbb{E}(X), \quad \mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{(\sigma[X])^2}{(\sigma[X])^2 + (t - \mathbb{E}[X])^2}$$



A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

We apply the same substitution to the rest of the inequality. ✓

$$\forall t > \mathbb{E}\left[\sum_{j \in jobs} X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \mathbb{E}\left[\sum_{j \in jobs} X_j\right]\right)^2}$$

Distribution of an **interfering** job

Relative **deadline**

The **probability** that the response time **exceeds** the **deadline**

A SUM OF POSSIBLY CORRELATED RANDOM VARIABLES

We apply the same substitution to the rest of the inequality. ✓

$$\forall t > \mathbb{E}\left[\sum_{j \in jobs} X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \mathbb{E}\left[\sum_{j \in jobs} X_j\right]\right)^2}$$

Distribution of an **interfering** job

Relative **deadline**

The **probability** that the response time **exceeds** the **deadline**

EXPECTATION OF THE SUM

What is the expected value of the sum of possibly correlated random variables?

Expected value of the response time distribution

$$\forall t > \mathbb{E}\left[\sum_{j \in jobs} X_j\right], \quad \mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \mathbb{E}\left[\sum_{j \in jobs} X_j\right]\right)^2}$$

Distribution of an **interfering** job

Relative **deadline**

The **probability** that the response time **exceeds** the **deadline**

EXPECTATION OF THE SUM

What is the expected value of the sum of possibly correlated random variables?

$$\forall t > \mathbb{E}\left[\sum_{j \in jobs} X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \mathbb{E}\left[\sum_{j \in jobs} X_j\right]\right)^2}$$

$$\mathbb{E}\left[\sum_{j \in jobs} X_j\right] = \sum_{j \in jobs} \mathbb{E}\left[X_j\right]$$

Expected value of the response time distribution

Distribution of an **interfering** job

Relative **deadline**

The **probability** that the response time **exceeds** the **deadline**

Expected value of **an interfering** job

EXPECTATION OF THE SUM

What is the expected value of the sum of possibly correlated random variables?

Equal to *the sum of per-RV expected values.*

$$\forall t > \mathbb{E}\left[\sum_{j \in jobs} X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \mathbb{E}\left[\sum_{j \in jobs} X_j\right]\right)^2}$$

$$= \sum_{j \in jobs} \mathbb{E}\left[X_j\right]$$

linearity of expectation

Expected value of the response time distribution

Expected value of **an interfering** job

The **probability** that the response time **exceeds** the **deadline**

Distribution of an **interfering** job

Relative **deadline**

EXPECTATION OF THE SUM

What is the expected value of the sum of possibly correlated random variables?

Equal to *the sum of per-RV expected values.*

$$\forall t > \mathbb{E}\left[\sum_{j \in jobs} X_j\right], \quad \mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \mathbb{E}\left[\sum_{j \in jobs} X_j\right]\right)^2}$$

$$\mathbb{E}\left[\sum_{j \in jobs} X_j\right] = \sum_{j \in jobs} \mathbb{E}\left[X_j\right]$$

linearity of expectation

Expected value of the response time distribution

Distribution of an **interfering** job

Relative **deadline**

The **probability** that the response time **exceeds** the **deadline**

Expected value of **an interfering** job

EXPECTATION OF THE SUM

What is the expected value of the sum of possibly correlated random variables?

Equal to *the sum of per-RV expected values.* ✓

$$\forall t > \sum_{j \in jobs} \mathbb{E}\left[X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \sum_{j \in jobs} \mathbb{E}\left[X_j\right]\right)^2}$$

Distribution of an **interfering** job

Relative **deadline**

The **probability** that the response time **exceeds** the **deadline**

Expected value of **an interfering** job

VARIANCE OF THE SUM

What is the value of the variance of the sum of possibly correlated random variables?

$$\forall t > \sum_{j \in jobs} \mathbb{E}\left[X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \sum_{j \in jobs} \mathbb{E}\left[X_j\right]\right)^2}$$

Variance of the response-time distribution

Distribution of an **interfering** job

Relative **deadline**

The **probability** that the response time **exceeds** the **deadline**

Expected value of **an interfering** job

VARIANCE OF THE SUM

What is the value of the variance of the sum of possibly correlated random variables?

$$\forall t > \sum_{j \in jobs} \mathbb{E}\left[X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \sum_{j \in jobs} \mathbb{E}\left[X_j\right]\right)^2}$$

Cauchy-Schwarz Inequality

$$\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 \leq \left(\sum_{j \in jobs} \sigma\left[X_j\right]\right)^2$$

Standard deviation of **an interfering** job

Variance of the response-time distribution

The **probability** that the response time **exceeds** the **deadline**

Distribution of an **interfering** job

Relative **deadline**

Expected value of **an interfering** job

VARIANCE OF THE SUM

What is the value of the variance of the sum of possibly correlated random variables?

***Less** than or equal to the sum of per-RV standard deviations.*

$$\forall t > \sum_{j \in jobs} \mathbb{E}\left[X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \le \frac{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2}{\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 + \left(t - \sum_{j \in jobs} \mathbb{E}\left[X_j\right]\right)^2}$$

$$\left(\sigma\left[\sum_{j \in jobs} X_j\right]\right)^2 \le \left(\sum_{j \in jobs} \sigma\left[X_j\right]\right)^2$$

Cauchy-Schwarz Inequality

Standard deviation of **an interfering** job

Variance of the response-time distribution

The **probability** that the response time **exceeds** the **deadline**

Distribution of an **interfering** job

Relative **deadline**

Expected value of **an interfering** job

VARIANCE OF THE SUM

What is the value of the variance of the sum of possibly correlated random variables?

***Less** than the sum of per-RV standard deviations.*

$$\forall t > \sum_{j \in jobs} \mathbb{E}\left[X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sum_{j \in jobs} \sigma\left[X_j\right]\right)^2}{\left(\sum_{j \in jobs} \sigma\left[X_j\right]\right)^2 + \left(t - \sum_{j \in jobs} \mathbb{E}\left[X_j\right]\right)^2}$$

Standard deviation of **an interfering** job

Distribution of an **interfering** job

Relative **deadline**

The **probability** that the response time **exceeds** the **deadline**

Expected value of **an interfering** job

CORRELATION-TOLERANT INEQUALITY

Now, we can use the expected values and standard deviations of the ***individual****,* ***possibly correlated*** *random variables.*

$$\forall t > \sum_{j \in jobs} \mathbb{E}\left[X_j\right],$$

$$\mathbb{P}\left[\sum_{j \in jobs} X_j > t\right] \leq \frac{\left(\sum_{j \in jobs} \sigma\left[X_j\right]\right)^2}{\left(\sum_{j \in jobs} \sigma\left[X_j\right]\right)^2 + \left(t - \sum_{j \in jobs} \mathbb{E}\left[X_j\right]\right)^2}$$

Standard deviation of **an interfering** job

Distribution of an **interfering** job

Relative **deadline**

The **probability** that the response time **exceeds** the **deadline**

Expected value of **an interfering** job

THERE IS MUCH MORE IN THE PAPER

Concentration Inequality

- based on Cantelli's Ineq,
- Verified in Coq.

The Coq Proof Assistant

coq.inria.fr

Ground-Truth System Model

- jobs aborted upon deadline miss
- fully-preemptive scheduling
- fixed priority, uniprocessor

Correlation-Tolerant Analysis (CTA)

- connects the other two pillars
- soundly-upper bounds DFP
- closed-form solution

LET US APPLY CTA TO THE EXAMPLE

Input: expected value and standard deviation upper bounds on task execution time distributions.

$\mathbb{E}\left(\begin{smallmatrix} 0.965 & 0.015 & 0.02 \\ 1 & 3 & 5 \end{smallmatrix}\right) = 1.11 \leq 1.12$ $\sigma\left(\begin{smallmatrix} 0.965 & 0.015 & 0.02 \\ 1 & 3 & 5 \end{smallmatrix}\right) \approx 0.606 \leq 0.61$

$\mathbb{E}\left(\begin{smallmatrix} 0.975 & 0.025 \\ 2 & 8 \end{smallmatrix}\right) = 2.15 \leq 2.16$ $\sigma\left(\begin{smallmatrix} 0.975 & 0.025 \\ 2 & 8 \end{smallmatrix}\right) \approx 0.937 \leq 0.94$ $deadline = 10$

LET US APPLY CTA TO THE EXAMPLE

Input: expected value and standard deviation upper bounds on task execution time distributions.

$\mathbb{E}\left(\begin{smallmatrix} 0.965 & 0.015 & 0.02 \\ 1 & 3 & 5 \end{smallmatrix}\right) = 1.11 \leq 1.12$ $\quad$ $\sigma\left(\begin{smallmatrix} 0.965 & 0.015 & 0.02 \\ 1 & 3 & 5 \end{smallmatrix}\right) \approx 0.606 \leq 0.61$

$\mathbb{E}\left(\begin{smallmatrix} 0.975 & 0.025 \\ 2 & 8 \end{smallmatrix}\right) = 2.15 \leq 2.16$ $\quad$ $\sigma\left(\begin{smallmatrix} 0.975 & 0.025 \\ 2 & 8 \end{smallmatrix}\right) \approx 0.937 \leq 0.94$ $\quad$ $deadline = 10$

Analysis:

$$\mathbb{P}[X > t] \leq \frac{(\sigma(X))^2}{(\sigma(X))^2 + (t - \mathbb{E}(X))^2}$$

LET US APPLY CTA TO THE EXAMPLE

Input: expected value and standard deviation upper bounds on task execution time distributions.

$$\mathbb{E}\left(\begin{smallmatrix}0.965 & 0.015 & 0.02\\ 1 & 3 & 5\end{smallmatrix}\right) = 1.11 \leq 1.12 \qquad \sigma\left(\begin{smallmatrix}0.965 & 0.015 & 0.02\\ 1 & 3 & 5\end{smallmatrix}\right) \approx 0.606 \leq 0.61$$

$$\mathbb{E}\left(\begin{smallmatrix}0.975 & 0.025\\ 2 & 8\end{smallmatrix}\right) = 2.15 \leq 2.16 \qquad \sigma\left(\begin{smallmatrix}0.975 & 0.025\\ 2 & 8\end{smallmatrix}\right) \approx 0.937 \leq 0.94 \qquad \textit{deadline} = 10$$

CTA:

$$\mathbb{P}\left[\begin{smallmatrix}0.965 & 0.015 & 0.02\\ 1 & 3 & 5\end{smallmatrix} + \begin{smallmatrix}0.975 & 0.025\\ 2 & 8\end{smallmatrix} > 10\right] \leq \frac{(0.61+0.94)^2}{(0.61+0.94)^2 + (10-(1.12+2.16))^2} \approx 0.05$$

LET US APPLY CTA TO THE EXAMPLE

The CTA-derived DFP over-approximates the ground-truth DFP, being more accurate than pWCET-DFP.

Input: expected value and standard deviation upper bounds on task execution time distributions.

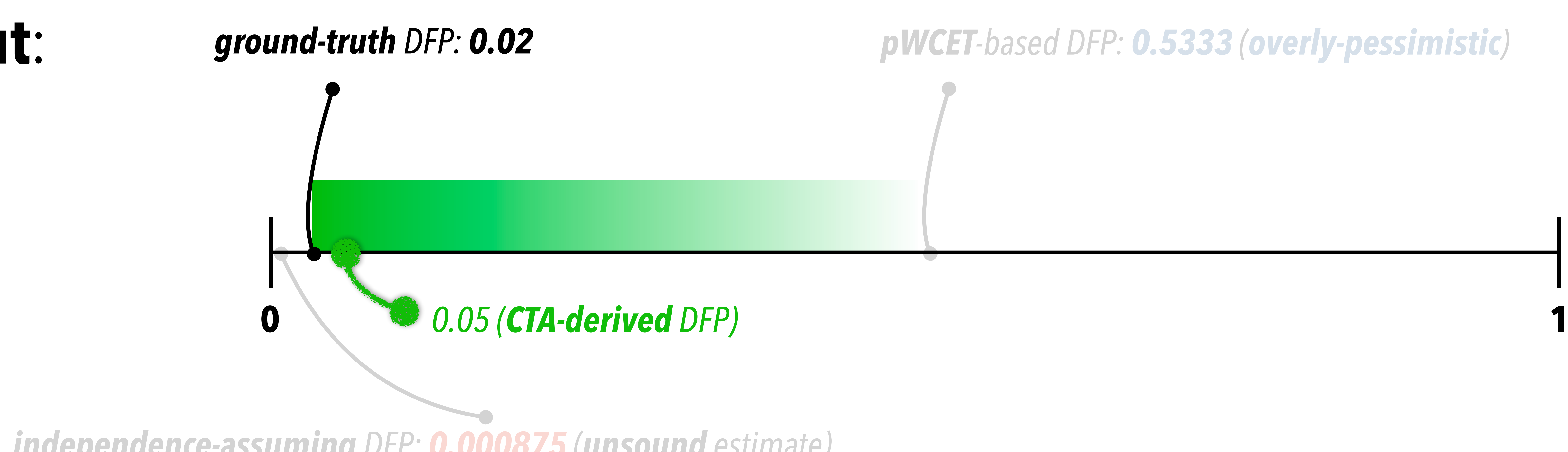
$\mathbb{E}([1^{0.965}, 3^{0.015}, 5^{0.02}]) = 1.11 \leq 1.12$ $\quad\sigma([1^{0.965}, 3^{0.015}, 5^{0.02}]) \approx 0.606 \leq 0.61$

$\mathbb{E}([2^{0.975}, 8^{0.025}]) = 2.15 \leq 2.16$ $\quad\sigma([2^{0.975}, 8^{0.025}]) \approx 0.937 \leq 0.94$ $\quad deadline = 10$

CTA:

$$\mathbb{P}\left[[1^{0.965}, 3^{0.015}, 5^{0.02}] + [2^{0.975}, 8^{0.025}] > 10\right] \leq \frac{(0.61+0.94)^2}{(0.61+0.94)^2 + (10-(1.12+2.16))^2} \approx 0.05$$

Output:



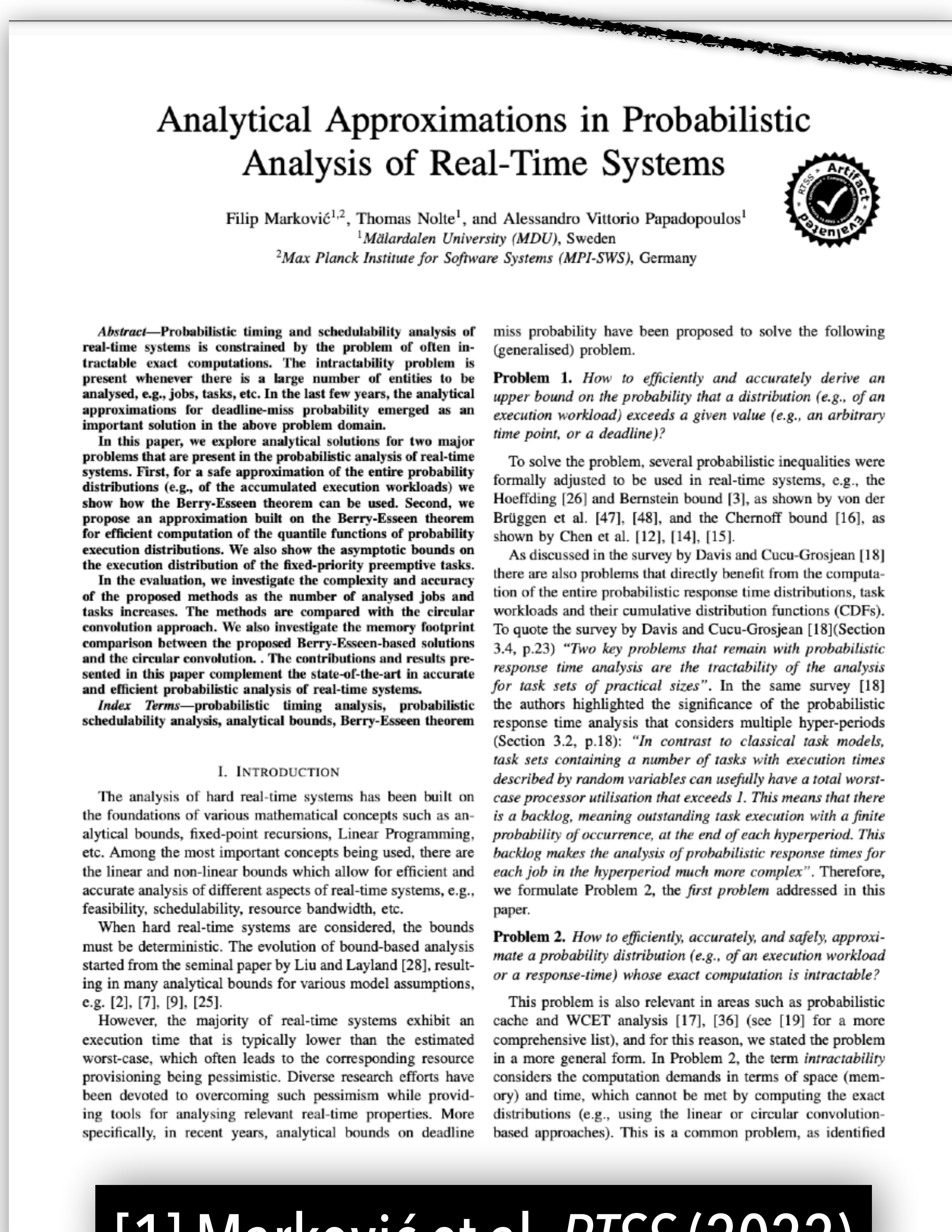
EVALUATION

EVALUATION

How does CTA compare to the pWCET-based analyses in general?

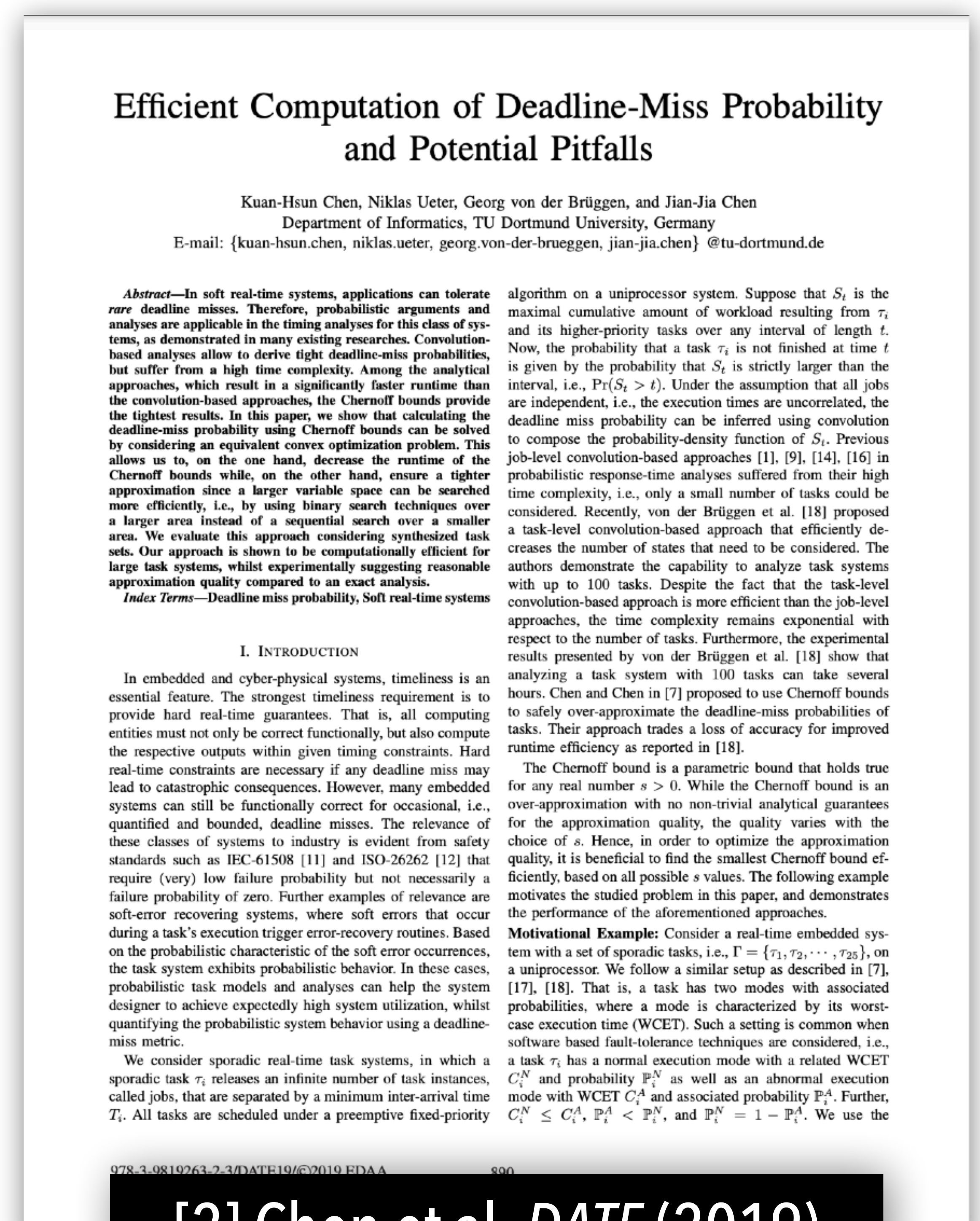
We compared **CTA** to the following baselines:

- **Berry–Essen**: DFP lower bound computed with the Berry-Esseen theorem.
- **Chernoff**: DFP upper bound computed with Chernoff bound


Analytical Approximations in Probabilistic Analysis of Real-Time Systems

Filip Marković[1,2], Thomas Nolte[1], and Alessandro Vittorio Papadopoulos[1]
[1]Mälardalen University (MDU), Sweden
[2]Max Planck Institute for Software Systems (MPI-SWS), Germany

[1] Marković et al. *RTSS* (2022)


Efficient Computation of Deadline-Miss Probability and Potential Pitfalls

Kuan-Hsun Chen, Niklas Ueter, Georg von der Brüggen, and Jian-Jia Chen
Department of Informatics, TU Dortmund University, Germany
E-mail: {kuan-hsun.chen, niklas.ueter, georg.von-der-brueggen, jian-jia.chen} @tu-dortmund.de

[2] Chen et al. *DATE* (2019)

EVALUATION SETUP

Synthetic task sets were randomly generated to highlight differences between pWCET and CTA analysis.

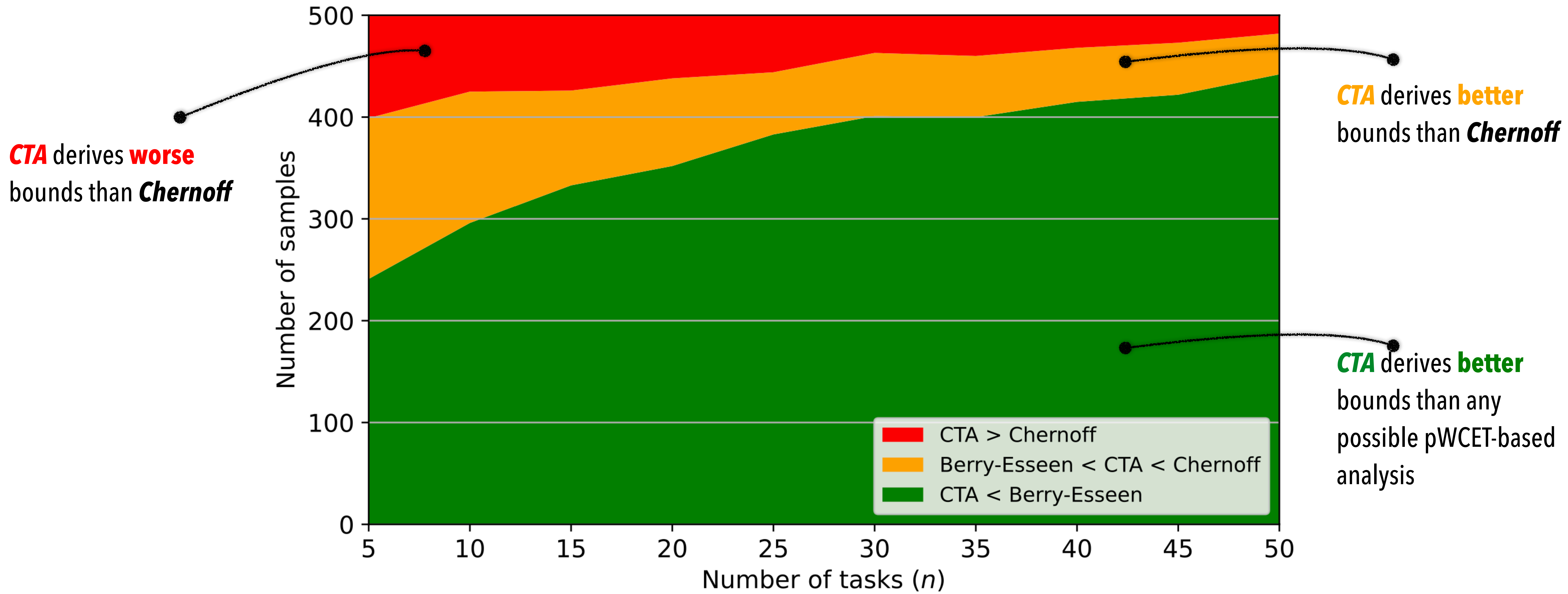
Four experiments were conducted to investigate:

1. Influence of the **task set size** on DFP,
2. The influence of the **expected utilization** according to pWCET distributions,
3. The influence of the **expected utilization** according to CTA inputs,
4. The influence of the **maximum standard deviation** on CTA.

In this talk, we focus on (1)

EVALUATION, EXPERIMENT 1

Investigating the influence of the task-set size



As the number of tasks in a set **increases**, the CTA method's **advantage** over pWCET-based baselines **grows**.

This is because pWCET can be overly pessimistic in the presence of correlations.

The level of pessimism increases at a faster rate than the expectation used by CTA with new interfering tasks.

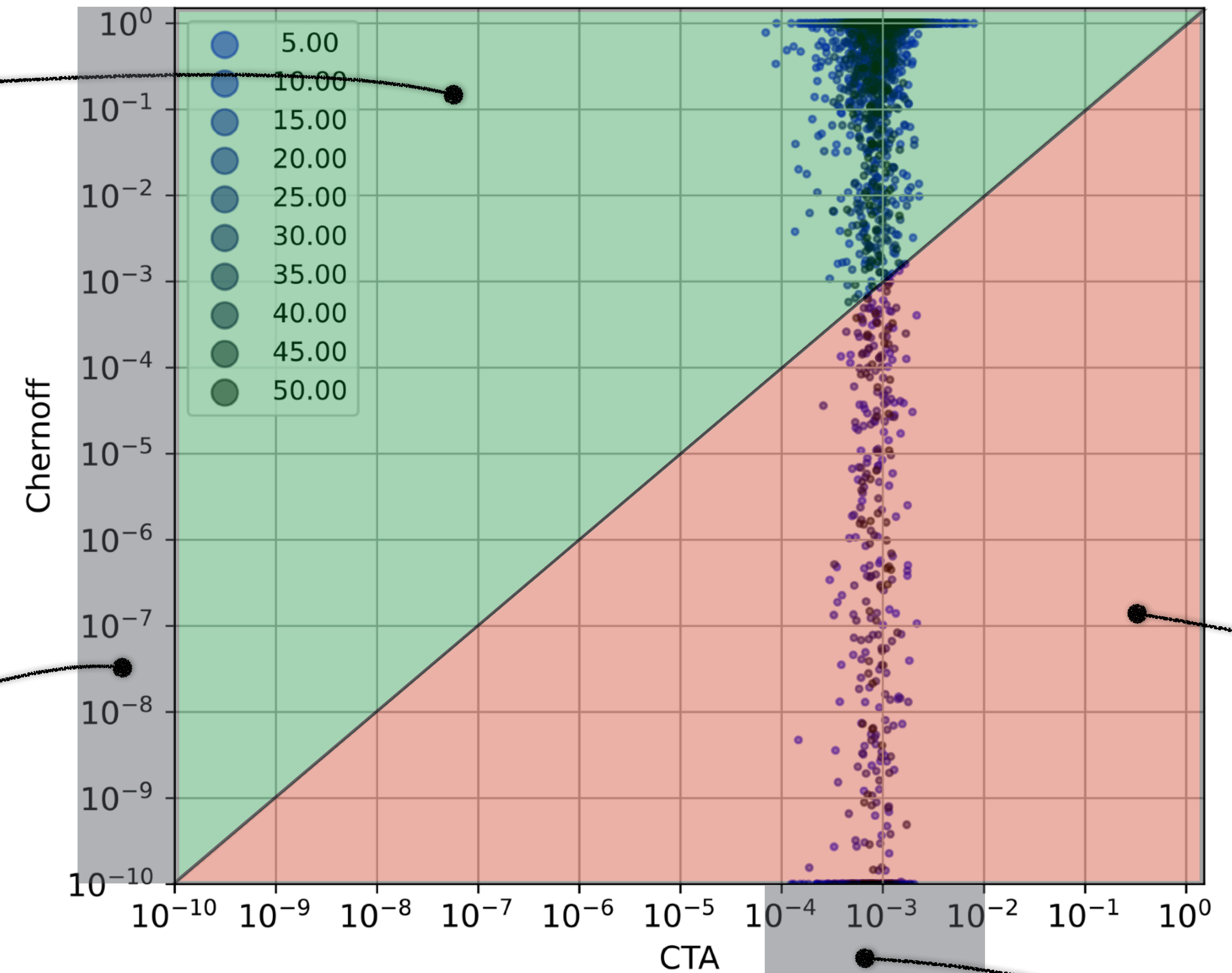
EVALUATION, EXPERIMENT 1

Investigating the influence of the task-set size

CTA derives **better** bounds than ***Chernoff***

$\approx 88\,\%$

Range of DFP solutions for ***Chernoff***



CTA derives **worse** bounds than ***Chernoff***

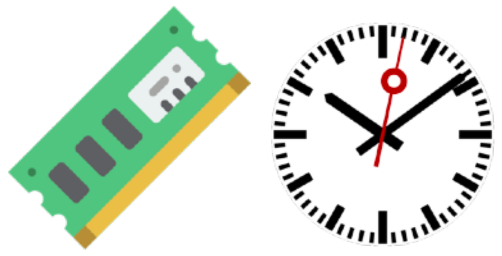
$\approx 12\,\%$

Range of DFP solutions for ***CTA***

CTA typically offers **lower bounds**,
but its reliance on simple summary statistics **can limit the range** of obtainable DFP bounds.

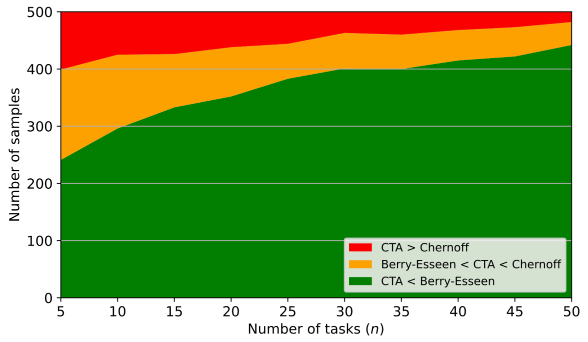
SUMMARY

SUMMARY

Efficient: minimize 

space and **time** complexity

- CTA relies on a **closed-form** expression; its run-time and space complexity are **negligible**
- CTA **tolerates dependence** by construction
- CTA **does not** require pWCET nor any similar independence-implying construct

Accurate: minimize over-approximation 

- The results are promising, but pWCET can still be useful under certain conditions.

A novel analysis with a lot of potential.