

# A ROS 2 Response-Time Analysis Exploiting Starvation Freedom and Execution-Time Variance

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**Abstract**—Robots are commonly subject to real-time constraints. To ensure that such constraints are met, recent work has analyzed the response times of processing chains under ROS 2, a popular robotics framework. However, prior work supports only scalar worst-case execution time bounds and does not exploit that the ROS 2 scheduling mechanism is starvation-free.

This paper proposes a novel response-time analysis for ROS 2 processing chains that accounts for both the high execution-time variance typically encountered in robotics workloads and the starvation freedom of the default ROS 2 callback scheduler. Experimental results from both synthetic callback graphs and a real ROS 2 workload empirically show the proposed analysis to be much more accurate (often by a factor of  $2\times$  or more).

## I. INTRODUCTION

Among the frameworks commonly used to program robots, ROS [1] is undoubtedly one of the most widespread: according to the ROS community metrics released in July 2020 [2], it is used by thousands of users, with a 33% increase in users over 2019. Reasons for its popularity include the flexible communication infrastructure, a low barrier to entry, and foremost a vibrant ecosystem, which allows developers to quickly integrate extensively tested components instead of re-implementing common functionality from scratch.

Timing correctness is a central challenge in such systems. In recent years, the real-time community has started to address this issue by developing response-time analyses for ROS 2 systems [3]–[5]. Compared to traditional real-time systems, ROS<sup>1</sup> systems impose two additional challenges. First, ROS systems do not consist of independent tasks, but of a network of interacting callbacks. The analysis has to cope with dependencies among callbacks and must bound the response times of entire *chains* of callbacks. Second, callbacks are multiplexed onto shared *executor threads*, which execute callbacks using a custom scheduling policy that differs significantly from common real-time schedulers. The analysis thus not only has to consider the scheduling of executor threads by the OS, but also callback sequencing *within* executors.

In this paper, we improve upon prior analyses of ROS along three dimensions: a refined model of processor demand, a less pessimistic callback activation model, and an improved response-time analysis that exploits starvation freedom in the ROS executor’s callback scheduling algorithm.

The improved processor-demand model addresses the problem that prior analyses support only scalar *worst-case execu-*

*tion times* (WCETs), meaning that the processor demand of any activation of a callback is characterized with a single, often pessimistic bound. In contrast, ROS callbacks often exhibit highly variable execution times [6], which are poorly represented by a single scalar. Such execution times are better described as *execution-time curves*. First introduced by Quinton *et al.* as  $ET^+$  functions [7], execution-time curves are a general way to describe the cumulative processor demand of multiple consecutive instances. In this paper, we model the processor demand of callbacks with execution-time curves. Taking advantage of the richer description, however, poses significant challenges in the analysis (Sec. IV-D).

In an orthogonal direction, we improve the callback activation model to better account for activations among callbacks assigned to the same executor. As a result, we obtain more accurate bounds on intra-executor activations (Sec. V), without imposing additional constraints on the callback graph.

Finally, we develop a new analysis to exploit a key property of the scheduling mechanism in ROS executors, namely that it provides *starvation freedom*: executors execute callbacks in a round-robin fashion, preventing any individual callback from hogging the executor. It has been previously recognized that this makes it difficult, or even impossible, to properly prioritize callbacks [3, 5]. However, this design can also *reduce* worst-case response times by limiting interference from competing callbacks, in particular if competing callbacks are activated in bursts, but prior analyses fail to capture this positive aspect. In this paper, we propose a new response-time analysis that exploits the benefits of the round-robin property (Sec. IV).

We evaluated the proposed analysis on both synthetic callback graphs and a widely used ROS package. The empirical results show that each of the three improvements leads to tighter response-time bounds, and that jointly they yield much more accurate bounds overall. It also shows that the new round-robin analysis and the traditional busy-window approach are most effective under different circumstances, so that a combination of both yields the best response-time bounds (Sec. VI).

## II. BACKGROUND

The ROS framework is composed of a stack of layers, as illustrated in Fig. 1. ROS users and developers interact with the language-specific layers at the top, which provide a language-specific API to define a ROS application. In this work, we

<sup>1</sup>For brevity we omit the version and refer to ROS 2 simply as ROS.

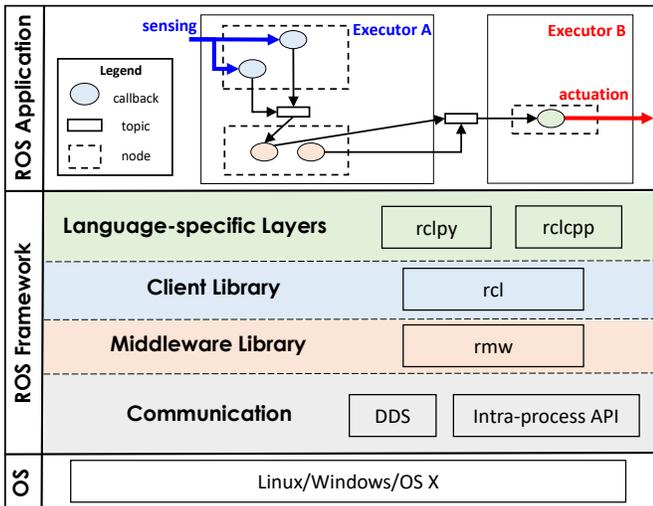


Fig. 1: Layered structure of ROS.

focus on the C++ interface, which we consider the presently most suitable choice for time-sensitive applications.

From a logical point of view, ROS applications are composed of callbacks and topics. Communication follows the *publish-subscribe* paradigm: callbacks publish messages to topics, where each message triggers the callbacks subscribing to the topic. Alternatively, callbacks may also be triggered by timers. The resulting graph of interacting callbacks and topics (illustrated in the upper part of Fig. 1) implements the complex and dynamic behavior of the robot.

ROS additionally provides an asynchronous remote procedure call (RPC) facility via *services* and the associated *service handler* and *client* callbacks. From the perspective of the ROS scheduler, these callbacks behave identically to regular subscription callbacks. We thus do not distinguish between topics and services in this paper; our analysis applies to both equally.

Below the language-specific layer, the *rcl* library implements the core functionalities of ROS. This guarantees consistent behavior across different programming languages and provides a common system model. The *rcl* layer sits on top of the *rmw* layer, which provides a common API to the underlying communication middleware (usually DDS [8]).

The ROS scheduling policy and execution model is implemented by *executors*, which choose the order in which pending callbacks are executed. While ROS supports in principle arbitrary user-defined executors, it provides two built-in implementations, a single- and a multi-threaded one, which are commonly used. This paper focuses exclusively on the single-threaded executor provided by ROS, the default solution.

The executor selects the next callback to run by considering each class of callbacks in order: first timers, then subscriptions, service handlers, and finally clients. Within each class, callbacks are considered in registration order, i.e., the order in which they were registered with the runtime system at process startup. Overall, the combination of callback type and registration time uniquely determines the priority of each callback. Once

selected, the chosen callback must run to completion before the executor can select the next callback to run.

One peculiarity of the ROS scheduler is that the list of ready instances is not updated before each scheduling decision. For most classes of callbacks, each callback’s readiness status is polled only irregularly from the underlying layers and cached in the *rclcpp* layer. We refer to such callbacks as *polled* callbacks. In contrast, the readiness status of a class of *privileged* callbacks is updated before each scheduling decision.

Whether a callback is polled or privileged is determined by the ROS implementation. In versions up to ROS 2 “*Dashing*,” timers are privileged but all other classes are polled [3]. In later versions, timers are polled callbacks as well [9].

The executor runs callbacks in the order described above. Polled callbacks are removed from the cache on completion. The executor polls the list of ready callbacks to refresh the cache only when all callbacks have been removed, i.e., when the cache has become empty, and then starts anew with the processing of cached callbacks. We refer to points in time when the executor’s pending callback cache is refreshed as *polling points*, and to the interval between two successive polling points as a *processing window*. An important property of the cache is that it stores at most one ready instance of each callback at a time. The executor thus runs at most one instance of each polled callback per processing window.

It is worth stressing how the ROS callback scheduler differs from common schedulers in the literature. First, it prioritizes callbacks by kind: timers are inherently prioritized over subscriptions, which are prioritized over service handlers, which in turn are prioritized over clients. Second, it does not consider all pending callbacks as eligible for execution. Instead, polled callbacks only become eligible at the first polling point after their activation. Third, the executor executes at most one instance of each polled callback per processing window, irrespective of the number of queued activations.

### III. SYSTEM MODEL

Following prior work [3], we model the system as a set of callbacks  $\mathcal{C}$ , each of which activates a potentially infinite sequence of instances. Each callback  $c_i \in \mathcal{C}$  is statically assigned to one of  $k$  single-threaded executors  $E_1, \dots, E_k$ ; for notational convenience, we let  $e_i$  denote callback  $c_i$ ’s assigned executor. We assume a steady-state system (i.e., callbacks neither leave nor join the system at runtime) and a discrete-time model wherein all time parameters are integer multiples of a basic time unit  $\epsilon \triangleq 1$  (e.g., a processor cycle).

Some of the inputs of a ROS system usually come from outside the callback system, for example, devices. This is commonly implemented through external driver threads that convert external stimuli into ROS publications. For consistency, we model such threads as executors with a single pseudo-callback. This callback, which we refer to as an *event source*, represents the computational demand and the communication behavior of the driver thread.

We divide the set of all callbacks by type and let  $\mathcal{C}^{\text{tmr}}$ ,  $\mathcal{C}^{\text{evt}}$ ,  $\mathcal{C}^{\text{sub}}$ ,  $\mathcal{C}^{\text{srv}}$ , and  $\mathcal{C}^{\text{clt}}$  denote the set of all timers, event sources,

subscribers, services, and clients, respectively. The last three categories are collectively referred to as the set of *message-driven callbacks*  $\mathcal{C}^{\text{msg}} = \mathcal{C}^{\text{sub}} \cup \mathcal{C}^{\text{srv}} \cup \mathcal{C}^{\text{clt}}$ . We further let  $\mathcal{C}_k$  denote the subset of all callbacks assigned to executor  $E_k$ , so that  $\mathcal{C}_k^{\text{tmr}}$  denotes the subset of all timers assigned to executor  $E_k$ , etc. Finally,  $lp_k(c_i)$  and  $hp_k(c_i)$  denote all callbacks in  $\mathcal{C}_k$  with lower or higher priority than  $c_i$ , respectively.

We distinguish between the set of polled callbacks  $\mathcal{C}^{\text{pol}}$  and the set of privileged callbacks  $\mathcal{C}^{\text{prv}}$ , with  $\mathcal{C}^{\text{pol}} \cup \mathcal{C}^{\text{prv}} = \mathcal{C}$ . In ROS versions up to “*Dashing*,” only message-driven callbacks are polled:  $\mathcal{C}^{\text{pol}} \triangleq \mathcal{C}^{\text{msg}}$ . In later versions, timers are polled, too:  $\mathcal{C}^{\text{pol}} \triangleq \mathcal{C}^{\text{msg}} \cup \mathcal{C}^{\text{tmr}}$ . Event sources are always privileged.

Each callback  $c_i$  is described by an *activation curve*  $\eta_i(\Delta)$ , which upper-bounds the number of activations within any time window of length  $\Delta$  [10]–[12] ( $\eta_i(\Delta) = 0$  if  $\Delta \leq 0$ ). For timer callbacks and event sources, the activation curve is given as part of the problem specification. For message-driven callbacks, the activation curve is derived by the analysis (Sec. V).

**Instance lifecycle.** Each callback activates an instance whenever a type-dependent event occurs: an external stimulus for event sources, a new period for timers, or an incoming message for message-driven callbacks. We refer to the  $k$ -th instance of a callback  $c_i$  as  $c_i^k$ . Each callback instance passes through several phases, which are illustrated in Fig. 2.

Conceptually, an instance of a message-driven callback  $c_i^k$  is *triggered* when a message is published on the topic to which  $c_i$  subscribes. However, the triggering message may incur a *propagation delay* while it traverses the ROS stack and potentially the network. Consequently, the instance  $c_i^k$  *activates* only some time later when the triggering message arrives at  $c_i$ ’s assigned executor  $e_i$ . We let  $\delta_{i,j}$  denote an upper bound on the propagation delay between any two callbacks  $c_i$  and  $c_j$ , and further assume that there is negligible propagation delay within the same executor, i.e.,  $e_i = e_j \Rightarrow \delta_{i,j} = 0$ .

Once activated, an instance  $c_i^k$  is said to be *pending* until it completes. If  $c_i$  is a polled callback, then  $c_i^k$  resides in a middleware buffer after activation until it is *sampled* by its executor at a future polling point. Once sampled,  $c_i^k$  becomes eligible for execution in the subsequent processing window. After all higher-priority sampled instances have executed, the executor then *selects*  $c_i^k$  and runs it until  $c_i^k$  *completes*. Due to the polling-point semantics, there is at most one sampled instance of a polled callback at a time.

The lifecycle of a polled, message-driven callback instance is illustrated in the top part of Fig. 2, where instance  $c_2^1$  is triggered by a message published to  $c_2$ ’s topic. Once the message arrives at  $E_1$ , it activates  $c_2^1$ , which is sampled by  $E_1$  after some delay at the next polling point.

A privileged callback’s lifecycle is somewhat simpler since it does not involve a separate sampling step. There is hence no difference between the time of activation and the time of sampling. In contrast to polled callbacks, there can be multiple simultaneously sampled instances of a privileged callback.

The bottom part of Fig. 2 illustrates the lifecycle of a privileged timer callback  $c_4$ . Executor  $E_2$  immediately samples

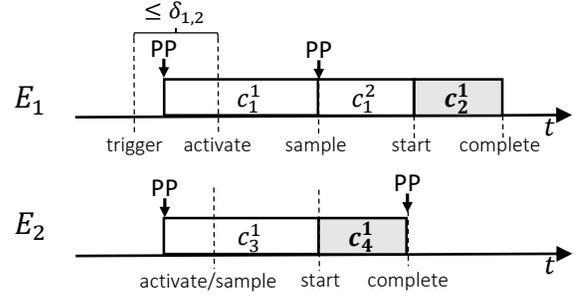


Fig. 2: Lifecycle of a message-driven callback  $c_2$  and a privileged timer  $c_4$

$c_4^1$  upon activation, and, once the current instance  $c_3^1$  completes, the executor selects and starts  $c_4^1$  without further delay.

**Execution-time model.** In most of the real-time systems literature (including Casini *et al.* [3]), execution-time requirements are modeled as scalar worst-case execution times (WCETs). While this is both safe and convenient from an analysis perspective, it can be overly pessimistic for many ROS applications. Since callbacks typically execute different code paths depending on the contents of the messages, they can exhibit varying execution-time patterns over time [6].

We therefore model the execution-time needs of each callback  $c_i$  as a *cumulative execution-time curve*  $ET_i(n)$  [7], which bounds the maximum cumulative execution time of any  $n$  consecutive instances of  $c_i$ . This model is strictly more expressive: it can represent the scalar WCET model (in which case  $ET_i(n)$  is linear), but can also incorporate additional information about series of activations.

**Processing chains.** The trigger relationships among callbacks form a directed acyclic graph (DAG)  $\mathcal{D} = \{\mathcal{C}, \mathcal{E}\}$ . An edge  $(c_i, c_j) \in \mathcal{E}$  encodes that an instance of  $c_i$  may trigger one instance of  $c_j$  at some point during its execution. We define the set of predecessors and successors associated with each callback  $c_i$  as  $pred(c_i) = \{c_j \in \mathcal{C} : \exists (c_j, c_i) \in \mathcal{E}\}$  and  $succ(c_i) = \{c_j \in \mathcal{C} : \exists (c_i, c_j) \in \mathcal{E}\}$ , respectively. A callback without predecessors is said to be a *source callback*.

In ROS, a conceptually single piece of functionality is often realized jointly by multiple consecutive callbacks. We refer to such a path through the callback graph as a *callback chain*. Given such a chain  $\gamma_i = (c_s, \dots, c_e)$ , a *chain instance*  $\gamma_i^k = (c_s^k, \dots, c_e^k)$  is a sequence of instances of the comprising callbacks where each instance triggers the next instance in the sequence. The chain instance activates when its first element activates and completes when its last element completes.

**Executors.** Callbacks are subject to hierarchical scheduling at two levels. The first level of scheduling is the operating system scheduler, which schedules all executor threads as well as any unrelated threads. To support a wide range of systems, we do not assume any particular process scheduling algorithm. Instead, we assume only the existence of a *supply-bound function*  $sbf_k(\Delta)$  that lower-bounds the amount of service provided to an executor  $E_k$  within any time window of length  $\Delta$ . Such

supply-bound functions are known for many commonly-used OS schedulers [13]–[16].

The second level of scheduling is performed by each executor when it selects the next instance to run. As described in Sec. II, this scheduler has the following properties:

**SQ** *Sequentiality*: Different instances of the same callback are executed in activation order.

**NP** *Non-Preemptiveness*: The executor selects a new instance only when the previous instance has completed.

**WC** *Work-Conservation*: The executor never idles if an instance is pending.

**PP** *Polling Points*: A polling point occurs when the executor needs to select the next instance to execute but no sampled instance is available.

**SM** *Sampling*: At a polling point, the executor samples up to one instance of each polled callback in activation order. Instances of privileged callbacks are sampled immediately upon activation.

**PR** *Selective prioritization*: The executor runs sampled instances (and only sampled instances) in priority order.

While the algorithm shares the first three properties with any standard fixed-priority non-preemptive scheduler, the last three properties represent the unique behavior of the ROS executor.

#### IV. ROUND-ROBIN ANALYSIS

We now present a response-time analysis for processing chains based on the round-robin behavior of the ROS callback scheduler. We begin with some definitions. The *worst-case response time* of a callback  $c_i$  or a chain  $\gamma_i$  is the largest possible difference between activation and completion of any instance of  $c_i$  (respectively,  $\gamma_i$ ). We let  $R(c_i)$  (respectively,  $R(\gamma_i)$ ) denote the response-time bound of  $c_i$  (respectively,  $\gamma_i$ ).

A callback instance  $c_i^x$  is pending during an interval  $[t_1, t_2)$  if it is pending at any instant  $t \in [t_1, t_2)$ .  $c_i^x$  suffers *interference* from another instance  $c_j^y$  at time  $t$  if  $c_j^y$  occupies the shared executor at  $t$  and  $c_i^x$  is incomplete at  $t$ . Interference is *direct* if  $c_i^x$  is pending at time  $t$  and *indirect* if  $c_i^x$  is not yet pending. As a special case, we call interference by prior instances of  $c_i$  *self-interference*. A chain instance suffers interference if any of its callback instances suffers interference.

##### A. Motivation

Property **SM** of the ROS callback scheduler leads to round-robin-like behavior: no more than one instance per polled callback runs in each processing window, independently of its priority or the number of pending instances.

This scheduling approach ensures a notion of fairness among polled callbacks. Consider two polled callbacks,  $c_1$  and  $c_2$ . Assume  $c_1$  is triggered periodically and has two pending instances, whereas  $c_2$  is triggered in infrequent bursts and has ten pending instances. Due to Property **SM**, only two instances of  $c_2$  interfere with  $c_1$ 's two pending instances. A traditional busy-window analysis (e.g., [3]) would pessimistically account for ten interfering instances instead. The analysis proposed in this section improves upon prior work by bounding the number of processing windows needed to complete a chain instance.

##### B. Interference Bounds

In the following, we exploit Property **SM** to establish bounds on total interference. As a preliminary, Lemma 1 bounds the number of pending callback instances in arbitrary time intervals.

**Lemma 1.** *Let  $c_i$  be any callback. In any interval of length  $\Delta$ , at most  $\eta_i(\Delta + R(c_i) - \epsilon)$  instances of  $c_i$  are pending.*

*Proof.* Consider an arbitrary interval  $[t, t + \Delta)$ . If  $\Delta = 0$ , then the bound holds trivially, so assume  $\Delta > 0$ . Clearly, callback instances activated at or after time  $t + \Delta$  are not pending before  $t + \Delta$ . By definition of the response-time bound  $R(c_i)$ , instances of  $c_i$  activated at or before  $t - R(c_i)$  are complete by time  $t$ . Thus, only instances activated in  $(t - R(c_i), t + \Delta)$  are pending during  $[t, t + \Delta)$ . The lemma follows since the length of  $(t - R(c_i), t + \Delta)$  is  $\Delta + R(c_i) - \epsilon$ . ■

Next, we introduce a bound that exploits Property **SM** to bound the number of callback instances that directly interfere with a callback  $c_i$ . The bound depends only on  $c_i$ 's activation curve and not on the activation curve of the interfering callbacks. In the following lemmas, let  $N$  denote a given upper bound on the number of polling points in an arbitrary interval  $[t_1, t_2)$ . We later show in Lemma 7 how to obtain such a bound  $N$ . For brevity, we let  $\llbracket p \rrbracket_1$  denote the *indicator function* that evaluates to 1 if the predicate  $p$  is true and to 0 otherwise.

**Lemma 2.** *Let  $c_i \in C_k^{\text{pol}}$  and  $c_j \in C_k^{\text{pol}} \setminus \{c_i\}$ . Let  $N$  be a bound on the number of polling points in  $E_k$  during an interval  $[t_1, t_2)$ . If an instance of  $c_i$  completes at time  $t_2$ , then at most  $N + \llbracket c_j \in hp_k(c_i) \rrbracket_1$  instances of  $c_j$  run during  $[t_1, t_2)$ .*

*Proof.* Consider separately the last polling point before time  $t_2$  (called *last polling point* hereafter), and the up to  $N - 1$  polling points in  $[t_1, t_2)$  that precede the last polling point (called *internal polling points* hereafter). Each of these polling points samples at most one instance of  $c_j$  (Property **SM**). In the case of internal polling points, all such instances run before the next polling point and thus before  $t_2$ . In the case of the last polling point, the instance of  $c_i$  that completes at  $t_2$  is among the sampled instances. During the processing window, the executor  $E_k$  runs only callbacks of higher priority than  $c_i$  before  $c_i$  (Property **SM**). Thus, instances of callbacks in  $lp_k(c_i)$  run after  $t_2$ , and instances in  $hp_k(c_i) \cup c_i$  run before  $t_2$ . There may be further instances running in  $[t_1, t_2)$  that have been sampled before  $t_1$ . These instances are all sampled at the same polling point, namely the last polling point preceding  $t_1$  (Property **PP**). There is thus at most one such instance per callback (Property **SM**). Overall,  $c_j$  executes in  $[t_1, t_2)$  up to  $N - 1$  instances sampled at internal polling points, at most 1 instance sampled before  $t_1$ , and, if  $c_j \in hp_k(c_i)$ , at most 1 instance sampled at the last polling point, for a total of at most  $N + \llbracket c_j \in hp_k(c_i) \rrbracket_1$  instances. ■

Lemmas 1 and 2 bound the number of callbacks that run during an interval in different ways. The minimum of both bounds is a safe bound, too, as the following corollary notes.

**Corollary 1.** Let  $c_i \in \mathcal{C}_k^{\text{pol}}$  and  $c_j \in \mathcal{C}_k^{\text{pol}} \setminus \{c_i\}$ . Let  $N$  be an upper bound on the number of polling points in  $E_k$  during an interval  $[t_1, t_2]$ . If an instance of  $c_i$  completes at time  $t_2$ , then for any  $0 \leq \Delta \leq t_2 - t_1$  at most

$$\min(\eta_j(\Delta + R(c_j) - \epsilon), N + \llbracket c_j \in hp_k(c_i) \rrbracket_1)$$

instances of  $c_j$  run during  $[t_1, t_1 + \Delta]$ .

Thanks to Corollary 1, we can bound the total amount of direct interference a callback  $c_i$  suffers from other callbacks.

**Definition 1.** For any polled callback  $c_i$  assigned to  $E_k$ , the direct interference bound function  $I_i(\Delta, N)$  is given by

$$\begin{aligned} I_i(\Delta, N) \triangleq & \sum_{c_j \in \mathcal{C}_k^{\text{prv}}} ET_j(\eta_j(\Delta + R(c_j) - \epsilon)) \\ & + \sum_{c_j \in \mathcal{C}_k^{\text{pol}} \setminus \{c_i\}} ET_j(v_j) \end{aligned}$$

with  $v_j = \min(\eta_j(\Delta + R(c_j) - \epsilon), N + \llbracket c_j \in hp_k(c_i) \rrbracket_1)$ .

Lemma 3 shows  $I_i(\Delta, N)$  to be sound.

**Lemma 3.** Let  $c_i \in \mathcal{C}_k^{\text{pol}}$ . Let  $N$  be an upper bound on the number of polling points in  $E_k$  during an interval  $[t_1, t_2]$ . If an instance of  $c_i$  completes at time  $t_2$ , then for any  $0 \leq \Delta \leq t_2 - t_1$ , instances of callbacks in  $\mathcal{C}_k \setminus c_i$  consume at most  $I_i(\Delta, N)$  units of processor service during  $[t_1, t_1 + \Delta]$ .

*Proof.* First note that, w.r.t. each callback, instances running during  $[t_1, t_1 + \Delta]$  form a consecutive sequence of instances. Thus, if  $n$  instances of an interfering callback  $c_j$  run during  $[t_1, t_1 + \Delta]$ , then their total demand is bounded by  $ET_j(n)$ . In case of privileged callbacks only pending instances can run. Therefore, Lemma 1 yields a bound on the number of callbacks that may be executed in  $[t_1, t_1 + \Delta]$ . In case of polled callbacks, Corollary 1 shows that  $v_j$  bounds the number of instances that may run in  $[t_1, t_1 + \Delta]$ . Since each callback in  $\mathcal{C}_k$  is either privileged or polled,  $I_i(\Delta, N)$  bounds the total demand across all callbacks in  $\mathcal{C}_k \setminus \{c_i\}$ . ■

Next, we bound the direct self-interference caused by earlier instances of the callback under analysis.

**Definition 2.** For any  $c_i \in \mathcal{C}^{\text{pol}}$ , the self-interfering instances bound is given by  $si_i(\Delta) \triangleq \max(0, \eta_i(\Delta + R(c_i) - \epsilon) - 1)$ .

Lemma 4 shows Definition 2 to be sound.

**Lemma 4.** Let  $c_i$  be an arbitrary callback and  $c_i^x$  one of  $c_i$ 's instances. During any interval  $[t_1, t_1 + \Delta]$  for any  $\Delta \geq 0$ , at most  $si_i(\Delta)$  instances directly self-interfere with  $c_i^x$ .

*Proof.* If  $\Delta = 0$ , then  $[t_1, t_1 + \Delta]$  is empty and does not contain any self-interfering instance, and hence  $si_i(\Delta) \geq 0$  is an upper bound. If  $\Delta > 0$ , then  $\Delta + R(c_i) - \epsilon \geq 0$ . By Lemma 1, at most  $\eta_i(\Delta + R(c_i) - \epsilon) = si_i(\Delta) + 1$  instances of  $c_i$  are pending during any interval of length  $\Delta$ . To suffer direct self-interference,  $c_i^x$  must be pending at some point in  $[t_1, t_1 + \Delta]$ . By definition of the response-time bound  $R(c_i)$ , instances released at or prior to  $t_1 - R(c_i)$  complete by time  $t_1$ ;

$c_i^x$  is hence activated after time  $t_1 - R(c_i)$ . Thus, one of the  $si_i(\Delta) + 1$  pending instances is  $c_i^x$  itself, which implies that at most  $si_i(\Delta)$  instances cause direct self-interference. ■

### C. Response-Time Bound

With Definitions 1 and 2 in place, we now bound the response time of any given subchain  $\gamma = (c_s, \dots, c_e)$  assigned to executor  $E_k$  ending in a polled callback  $c_e$ . Let  $\gamma^a = (c_s^x, \dots, c_e^y)$  be an arbitrary instance of this subchain. Let  $A$  denote  $\gamma^a$ 's activation time and  $F$  its completion time, so that its response time is given by  $F - A$ .

As a first step towards a response-time bound, we note that throughout  $[A, F)$  some callback of  $\gamma^a$  is always pending.

**Lemma 5.** At any time in  $[A, F)$ , at least one of the callback instances comprising  $\gamma^a = (c_s^x, \dots, c_e^y)$  is pending.

*Proof.* Since all callbacks in  $\gamma^a$  are assigned to  $E_k$  and the intra-executor propagation delay is zero, each callback instance  $c_i^q \in \gamma^a \setminus c_e^y$  is still running when its successor  $c_i^w \in \gamma^a$  is activated, with  $c_j \in succ(c_i)$ . Thus, at least one of the callbacks in  $\gamma^a$  is pending throughout  $[A, F)$ . ■

Since at least one callback instance in  $\gamma^a$  is pending at every polling point during  $[A, F)$ , each polling point samples at least one instance of a callback in  $\gamma$ , which implies an upper bound on the number of polling points.

**Definition 3.** For any callback  $c_i$ , its polling-point bound  $pp(c_i)$  is defined as  $pp(c_i) \triangleq \eta_i(R(c_i))$  if  $c_i \in \mathcal{C}^{\text{pol}}$ , and simply as  $pp(c_i) \triangleq 0$  otherwise. For a subchain  $\gamma$ , the aggregate bound  $pp(\gamma)$  is defined as  $pp(\gamma) \triangleq \sum_{c_i \in \gamma} pp(c_i)$ .

Lemmas 6 and 7 prove the correctness of these bounds.

**Lemma 6.** Let  $c_i^x$  be an arbitrary callback instance. Let  $t_a$  denote  $c_i^x$ 's activation time and  $t_f$  denote its completion time. There are at most  $pp(c_i)$  polling points in  $[t_a, t_f)$ .

*Proof.* If  $c_i$  is not a polled callback,  $c_i^x$  is sampled immediately upon activation. Since a polling point occurs only if there are no sampled instances (Property **PP**), there can be no polling point in  $[t_a, t_f)$ . If  $c_i$  is a polled callback each polling point in  $[t_a, t_f)$  samples one instance of  $c_i$  since at least one instance of  $c_i$ , namely  $c_i^x$ , is pending during the entire interval  $[t_a, t_f)$ . The number of polling points in  $[t_a, t_f)$  is therefore bounded by the number of instances of  $c_i$  that are sampled in  $[t_a, t_f)$ . The last of these instances is  $c_i^x$  (Property **PP**), which is pending at  $t_a$ . Due to Property **SQ**, any instance of  $c_i$  that is sampled at or after  $t_a$ , but before  $c_i^x$  is sampled, must be activated before  $c_i^x$ , which implies that any such instance is also pending at time  $t_a$ . The number of polling points in  $[t_a, t_f)$  is thus bounded by the number of instances of  $c_i$  pending at time  $t_a$ . By Lemma 1 with  $\Delta = \epsilon$ , at most  $\eta_i(R(c_i))$  instances of  $c_i$  are pending at time  $t_a$  (i.e., during  $[t_a, t_a + \epsilon)$ ). ■

**Lemma 7.** There are at most  $pp(\gamma)$  polling points in  $[A, F)$ .

*Proof.* By Lemma 5, at every time in  $[A, F)$ , at least one callback of  $\gamma^a$  is pending. Every polling point in  $[A, F)$  lies

thus between activation and completion of at least one of the callbacks of  $\gamma^a$  (Property **SM**). By Lemma 6,  $pp(c_i)$  bounds the number of polling points between the activation and the completion of each callback  $c_i^y$  of  $\gamma^a$ . The sum of the individual polling-point bounds of the callbacks comprising  $\gamma$  hence yields an upper bound on the total number of polling points between the activation and completion of  $\gamma^a$ . ■

Since  $\gamma^a$ 's last callback instance  $c_e^y$  completes at time  $F$ , this bound fulfills the condition on  $N$  and the associated interval  $[t_1, t_2)$  in Lemmas 2 and 3 and Corollary 1.

In the last preparatory step, we observe a simple structural property of self-interference.

**Lemma 8.** *Let  $c_i^y, \dots, c_i^{y+n}$  be  $n + 1$  consecutive instances of a callback  $c_i$ . If the last instance  $c_i^{y+n}$  runs for  $\omega_i^{y+n}$  time units, then the first  $n$  instances demand at most  $\min(ET_i(n + 1) - \omega_i^{y+n}, ET_i(n))$  time units of processor service.*

*Proof.* As  $c_i^y, \dots, c_i^{y+n}$  is a sequence of  $n + 1$  consecutive callback instances, their overall execution time is bounded by  $ET_i(n + 1)$ . The last element  $c_i^{y+n}$ , by assumption, runs for  $\omega_i^{y+n}$  time units. The first  $n$  elements thus run for at most  $ET_i(n + 1) - \omega_i^{y+n}$  time units. Similarly,  $c_i^y, \dots, c_i^{y+n-1}$  is a sequence of  $n$  consecutive callback instances; their total execution time hence is bounded by  $ET_i(n)$ . ■

The next two lemmas finally bound the response time  $F - A$  by bounding first the *start time*  $S$  of the last callback instance, and then its completion time  $F$ . The start time is a useful stepping stone because, from this point on, other callbacks can no longer interfere with  $\gamma^a$  (Property **NP**).

Using the established interference bounds, Lemma 9 finds the latest point in time where  $c_e^y$ , the last callback instance of  $\gamma^a$ , must consume its first unit of supply, which bounds  $S$ .

**Lemma 9.** *Let  $\gamma^a$  be an arbitrary instance of subchain  $\gamma$ , let  $A$  denote  $\gamma^a$ 's activation time, let  $c_e^y$  denote the last callback instance in  $\gamma^a$ , suppose that  $c_e^y$  requires  $\omega_e^y$  time units of processor service, and let  $N \triangleq pp(\gamma)$ . If  $S^*$  is the least positive solution (if any) of the inequality*

$$sbf_k(S^*) \geq \epsilon + I_e(S^*, N) + \min(ET_e(s_{i_e}(S^*) + 1) - \omega_e^y, ET_e(s_{i_e}(S^*))),$$

*then  $c_e^y$  starts running in  $[A, A + S^*)$ .*

*Proof.* By Lemmas 3 and 7,  $\epsilon + I_e(S^*, pp(\gamma))$  strictly exceeds the total direct interference due to all callbacks in  $\mathcal{C}_k \setminus \{c_e\}$ . By Lemma 4, there are at most  $s_{i_e}(S^*)$  directly self-interfering instances of  $c_e$ . Since the self-interfering instances are consecutive, the total interference due to these instances is bounded by Lemma 8 (with  $n = s_{i_e}(S^*)$ ). We now show that  $c_e^y$  completes in  $[A, A + S^*)$ . Since  $A$  is the activation time of  $\gamma^a$ , by Lemma 5, there is always a pending callback instance of  $\gamma^a$  until  $c_e^y$  completes, which implies that the executor does not idle (Property **WC**). Since  $S^*$  satisfies the stated inequality, the amount of service supplied by the executor exceeds the total demand by callback instances directly interfering with  $c_e^y$

(either due to other callbacks or self-interference) by at least  $\epsilon$  units of service. It follows that the only instance that can run while  $c_e^y$  is pending without interfering with it is  $c_e^y$  itself. Therefore,  $c_e^y$  starts running in  $[A, A + S^*)$ . ■

From  $S^*$ , we obtain a bound on the response time of  $\gamma^a$ .

**Theorem 1.** *Let  $\gamma^a$  be an arbitrary instance of subchain  $\gamma$ , let  $A$  denote  $\gamma^a$ 's activation time, let  $c_e^y$  denote the last callback instance in  $\gamma^a$ , and suppose that  $c_e^y$  requires  $\omega_e^y$  time units of processor service. Let  $S^*$  be defined as in Lemma 9. If  $R^*$  is the least positive solution (if any) of the inequality*

$$sbf_k(R^*) \geq sbf_k(S^*) - \epsilon + \omega_e^y,$$

*then  $R^*$  is a response-time bound for  $\gamma$  (i.e.,  $F - A \leq R^*$ ).*

*Proof.* Due to Property **NP**, a callback instance cannot be interfered with once it starts running. Recall from Lemma 9 that  $c_e^y$  starts running in  $[A, S^*)$ , and that it suffers at most  $sbf_k(S^*) - \epsilon$  time units of direct interference before it starts running. By assumption,  $c_e^y$  runs for  $\omega_e^y$  time units. Therefore,  $c_e^y$  necessarily completes once the executor has provided  $sbf_k(S^*) - \epsilon + \omega_e^y$  units of supply. Since  $R^*$  satisfies the stated inequality, the executor provides at least  $sbf_k(S^*) - \epsilon + \omega_e^y$  units of supply in  $[A, A + R^*)$ . Consequently,  $c_e^y$  completes in  $[A, A + R^*)$  and  $A + R^* - A = R^*$  is a response-time bound for  $\gamma^a$ . Furthermore, since  $\gamma^a$  is an arbitrary instance of  $\gamma$  upon which we have placed no restrictions,  $R^*$  bounds the response time of *any* instance of  $\gamma$ . ■

#### D. Eliminating $\omega_e^y$

Both  $S^*$  and  $R^*$  in Theorem 1 depend on  $\omega_e^y$ , the exact runtime of the last component of the chain under analysis, which is unknown at analysis time. The bound thus cannot be directly applied in a response-time analysis. While  $\omega_e^y$  can be trivially bounded by 0 from below and by  $ET_e(1)$  from above, such an estimate would be needlessly pessimistic. In the following, we refine Lemma 9 and Theorem 1 to be independent of  $\omega_e^y$  (hereafter simply referred to as  $\omega$ ).

Since the argument does not depend on details of the interference bounds (and will be reused in Sec. V), we consider a more general version of the problem. Let  $f, g : \mathbb{N} \rightarrow \mathbb{N}$  be any two monotonically increasing functions with  $f(t) > 0$  and  $g(t) \geq 0$  for all  $t$ . For a given  $\omega \in \mathbb{N}$  s.th.  $0 \leq \omega \leq ET_e(1)$ , let  $s(\omega)$  denote the least positive  $S^* \in \mathbb{N}$  that satisfies

$$sbf_k(S^*) \geq f(S^*) + \min(ET_e(g(S^*)), ET_e(g(S^*) + 1) - \omega) \quad (1)$$

and let  $r(\omega)$  denote the least positive  $R^* \in \mathbb{N}$  that satisfies

$$sbf_k(R^*) \geq sbf_k(s(\omega)) - \epsilon + \omega. \quad (2)$$

We refer to Ineqs. (1) and (2) as the *defining inequality* of  $s(\omega)$  and  $r(\omega)$ , respectively.

In the following, we derive a bound on  $\max_{\omega \geq 0} r(\omega)$  for arbitrary  $f$  and  $g$  that is independent of  $\omega$ . This bound is then applied to Lemma 9 and Theorem 1, which are a special case of the above system of inequalities (with  $f(x) \triangleq \epsilon + I_e(x, N)$

and  $g(x) \triangleq si_e(x)$ ). Our argument makes use of the following trivial property of supply-bound functions.

**Property 1.** An executor cannot provide more than  $\epsilon$  units of supply in an interval of length  $\epsilon$ :  $\forall x. sbf_k(x) \leq sbf_k(x - \epsilon) + \epsilon$ .

To get started, we establish that  $s(\omega)$  not only satisfies its defining inequality (Ineq. (1)), but in fact yields an equality. For brevity, let  $z(S^*)$  denote the right-hand side of Ineq. (1).

**Lemma 10.** *If  $0 \leq \omega \leq ET_e(1)$  and Ineq. (1) has a positive solution, then  $sbf_k(s(\omega)) = z(s(\omega))$ .*

*Proof.* By contradiction: suppose  $sbf_k(s(\omega)) > z(s(\omega))$ . By Property 1,  $sbf_k(s(\omega) - \epsilon) \geq sbf_k(s(\omega)) - \epsilon > z(s(\omega)) - \epsilon$ . Since time is discrete,  $sbf_k(s(\omega) - \epsilon) > z(s(\omega)) - \epsilon$  implies  $sbf_k(s(\omega) - \epsilon) \geq z(s(\omega))$ .  $s(\omega) - \epsilon$  is thus a solution of Ineq. (1), too. Since  $s(\omega)$  is by definition the least positive solution of Ineq. (1), it follows that  $s(\omega) - \epsilon = 0$ , which implies  $z(s(\omega)) = 0$  since  $sbf_k(s(\omega) - \epsilon) = sbf_k(0) = 0$ . However, as  $f(s(\omega)) > 0$ , this implies  $\min(ET_e(g(s(\omega))), ET_e(g(s(\omega)) + 1) - \omega) < 0$ , which is impossible since  $\forall x. ET_e(x) \geq 0$  and  $\omega \leq ET_e(1) \leq ET_e(g(s(\omega)) + 1)$ . ■

The next three lemmas characterize  $s(\omega)$  by identifying a value  $\omega^m$  such that  $s$  is constant up to  $\omega^m$  and monotonically decreasing afterwards. Based on  $\omega^m$ , we then identify the maximum of  $r(\omega)$ . We begin by establishing monotonicity.

**Lemma 11.**  *$s(\omega)$  is monotonically decreasing.*

*Proof.* Since  $\omega \in \mathbb{N}$ , it suffices to establish  $s(\omega + \epsilon) \leq s(\omega)$  for any  $\omega \geq 0$ . To this end, we show that  $S^* = s(\omega)$  is a solution to the defining inequality of  $s(\omega + \epsilon)$ , of which  $s(\omega + \epsilon)$  is by definition the least solution.

$$\begin{aligned} & f(S^*) + \min(ET_e(g(S^*)), ET_e(g(S^*) + 1) - (\omega + \epsilon)) \\ & \leq f(S^*) + \min(ET_e(g(S^*)), ET_e(g(S^*) + 1) - \omega) \\ & = f(s(\omega)) + \min(ET_e(g(s(\omega))), ET_e(g(s(\omega)) + 1) - \omega) \\ & \leq sbf_k(s(\omega)) = sbf_k(S^*) \quad \{\text{Def. } s(\omega)\} \quad \blacksquare \end{aligned}$$

Lemma 11 implies that  $s$  is maximized at  $\omega = 0$ . We next observe that there exists a ‘‘tipping point’’ such that the minimum term in Ineq. (1) resolves to its first argument below the tipping point and to its second argument otherwise.

**Lemma 12.** *There is an  $\omega^m$  such that*

$$\omega \geq \omega^m \Leftrightarrow ET_e(g(s(\omega))) \geq ET_e(g(s(\omega)) + 1) - \omega.$$

*Proof.* For brevity, we refer to the right-hand side of the stated equivalence as the  $\omega^m$ -criterion. There is always at least one value that fulfills the  $\omega^m$ -criterion, namely  $ET_e(1)$ , since  $ET_e$  is sub-additive ( $ET_e(n) \geq ET_e(n+1) - ET_e(1)$  for any  $n$ ).

Therefore, there also exists a *least* value of  $\omega$  for which the  $\omega^m$ -criterion holds. We now show that this least value satisfies the stated equivalence: that is, the value of  $\omega^m$  is given by the least  $\omega$  for which the  $\omega^m$ -criterion holds.

$\Leftarrow$ : follows immediately, since by definition  $\omega^m$  is the least value that satisfies the  $\omega^m$ -criterion.

$\Rightarrow$ : We show that  $\omega$  *not* fulfilling the  $\omega^m$ -criterion implies  $\omega < \omega^m$ . Let  $\omega^*$  be a value of  $\omega$  for which  $ET_e(g(s(\omega^*))) < ET_e(g(s(\omega^*)) + 1) - \omega^*$ . (If no such  $\omega^*$  exists, then  $\omega^m = 0$  and the claim holds trivially.) Then  $s(\omega^*)$  fulfills  $s(\omega^m)$ 's defining inequality: by definition of  $\omega^*$  and  $s(\omega^*)$ , we have  $sbf_k(s(\omega^*)) \geq f(s(\omega^*)) + ET_e(g(s(\omega^*)))$ , and since  $\forall x. ET_e(g(s(\omega^*))) \geq \min(ET_e(g(s(\omega^*)), x)$ , hence also  $sbf_k(s(\omega^*)) \geq f(s(\omega^*)) + \min(ET_e(g(s(\omega^*))), ET_e(g(s(\omega^*)) + 1) - \omega^m)$ , which is  $s(\omega^m)$ 's defining inequality. Since  $s(\omega^*)$  is positive and  $s(\omega^m)$  is the least positive solution of  $\omega^m$ 's defining inequality, this implies  $s(\omega^*) \geq s(\omega^m)$ , which implies  $\omega^* \leq \omega^m$  since  $s$  is monotonically decreasing (Lemma 11). Further,  $\omega^* \neq \omega^m$  since  $\omega^m$  fulfills the  $\omega^m$ -criterion while  $\omega^*$  does not. Therefore,  $\omega^* < \omega$ . ■

As a result,  $s(\omega)$  is a constant function for any  $\omega < \omega^m$ , which implies that all values in  $[0, \omega^m)$  maximize  $s$ .

**Lemma 13.** *If  $0 \leq \omega < \omega^m$ , then  $s(\omega) = s(0)$ .*

*Proof.* If  $\omega < \omega^m$ , then  $ET_e(g(s(\omega)) + 1) - \omega > ET_e(g(s(\omega)))$  by Lemma 12. Therefore,  $s(\omega)$  is the least positive solution of the inequality  $sbf_k(S^*) \geq f(S^*) + ET_e(g(S^*))$ , which obviously does not depend on  $\omega$ . Since by assumption  $\omega^m > 0$  (otherwise  $\omega < \omega^m$  does not exist since  $0 \leq \omega$ ), we have  $s(\omega) = s(0)$  for  $0 \leq \omega < \omega^m$ . ■

Based on Lemmas 10 to 13, we now identify the possible maxima of the  $r$  function for  $0 \leq \omega \leq ET_e(1)$ .

**Lemma 14.**  $\max_{0 \leq \omega \leq ET_e(1)} r(\omega) \in \{\omega^m, \omega^m - \epsilon\}$

*Proof.* We distinguish two cases:  $\omega < \omega^m$  and  $\omega \geq \omega^m$ .

*Case 1:* If  $\omega < \omega^m$ , then  $s(\omega) = s(0)$  (Lemma 13) and thus  $sbf_k(r(\omega)) \geq sbf_k(s(0)) - \epsilon + \omega$ . Since  $sbf_k$  is monotonically increasing,  $r(\omega)$  is hence also monotonically increasing and is thus maximized if  $\omega$  is maximized, i.e., at  $\omega = \omega^m - \epsilon$ .

*Case 2:* If  $\omega \geq \omega^m$ , then by Lemma 12 the minimum term in Ineq. (1) is equal to  $ET_e(g(s(\omega)) + 1) - \omega$ . By Lemma 10, Ineq. (1) is in fact an equality, which allows us to replace  $sbf_k(s(\omega))$  with the right-hand side of Ineq. (1). Therefore,  $r(\omega)$  is the least positive value satisfying:

$$\begin{aligned} sbf_k(r(\omega)) & \geq sbf_k(s(\omega)) - \epsilon + \omega \\ & = f(s(\omega)) + ET_e(g(s(\omega)) + 1) - \omega - \epsilon + \omega \\ & = f(s(\omega)) + ET_e(g(s(\omega)) + 1) - \epsilon. \end{aligned}$$

$r(\omega)$  is thus monotonically increasing in  $s(\omega)$  (as  $sbf_k$ ,  $f$ ,  $g$ , and  $ET_e$  are all monotonically increasing). Since  $s(\omega)$  is monotonically *decreasing* (Lemma 11),  $s(\omega)$  is maximized if  $\omega$  is minimized, i.e., at  $\omega = \omega^m$ . ■

Based on the maxima of  $r(\omega)$ , the next lemma provides two simplified inequalities that no longer depend on  $\omega$ .

**Lemma 15.** *If  $S^*$  is the least positive solution of*

$$sbf_k(S^*) \geq f(S^*) + ET_e(g(S^*)), \quad (3)$$

$\Omega \triangleq ET_e(g(S^*) + 1) - ET_e(g(S^*))$ , and  $R^*$  is the least positive solution of

$$sbf_k(R^*) \geq sbf_k(S^*) - \epsilon + \Omega, \quad (4)$$

then  $R^* \geq \max_{0 \leq \omega \leq ET_e(1)} r(\omega)$ .

*Proof.* We first show that  $S^*$  upper-bounds  $s(\omega^m)$  and, if  $\omega^m > 0$ ,  $s(\omega^m - \epsilon)$ . If  $\omega^m = 0$ , then by Lemma 12  $ET_e(g(s(\omega^m))) \geq ET_e(g(s(\omega^m)) + 1)$ . Since  $ET_e$  is monotonically increasing, this implies  $ET_e(g(s(\omega^m))) = ET_e(g(s(\omega^m)) + 1)$ . As a result, Ineq. (3) is equivalent to the defining inequality of  $s$  (Ineq. (1)) for  $\omega = \omega^m$  and  $S^*$  is therefore equal to  $s(\omega^m)$ . If  $\omega^m > 0$ , Ineq. (3) is equivalent to the defining inequality of  $s$  (Ineq. (1)) for  $\omega = \omega^m - \epsilon$  (Lemma 12), and hence  $S^* = s(\omega^m - \epsilon)$ . As  $s$  is monotonically decreasing (Lemma 11), this implies  $S^* \geq s(\omega^m)$ .

We now show that  $\Omega \geq \omega^m$ . If  $\omega^m = 0$ , then  $\Omega$  is trivially an upper bound, so assume  $\omega^m > 0$  and thus  $S^* = s(\omega^m - \epsilon)$ . By the definition of  $\omega^m$ , it then holds that:

$$\begin{aligned} ET_e(g(s(\omega^m - \epsilon))) &< ET_e(g(s(\omega^m - \epsilon)) + 1) - (\omega^m - \epsilon) \\ \Leftrightarrow ET_e(g(S^*)) &< ET_e(g(S^*) + 1) - (\omega^m - \epsilon) \\ \Leftrightarrow (\omega^m - \epsilon) &< ET_e(g(S^*) + 1) - ET_e(g(S^*)) \\ \Leftrightarrow \omega^m &< ET_e(g(S^*) + 1) - ET_e(g(S^*)) + \epsilon \\ \Leftrightarrow \omega^m &< \Omega + \epsilon \Leftrightarrow \omega^m \leq \Omega \end{aligned}$$

Finally, we establish that  $R^*$  upper-bounds both  $r(\omega^m)$  and, if  $\omega^m > 0$ ,  $r(\omega^m - \epsilon)$  by showing that  $R^*$  fulfills  $r$ 's defining inequality (Ineq. (2)) for  $\omega^* \in \{\omega^m, \omega^m - \epsilon\}$ .

$$\begin{aligned} sbf_k(R^*) &\geq sbf_k(S^*) - \epsilon + \Omega \\ \Rightarrow sbf_k(R^*) &\geq sbf_k(S^*) - \epsilon + \omega^* \quad \{\Omega \geq \omega^* \geq \omega^* - \epsilon\} \\ \Rightarrow sbf_k(R^*) &\geq sbf_k(s(\omega^*)) - \epsilon + \omega^* \quad \{S^* \geq s(\omega^*)\} \end{aligned}$$

Thus, by Lemma 14,  $R^* \geq r(\omega)$  for  $0 \leq \omega \leq ET_e(1)$ . ■

Having solved the general case, we now apply Lemma 15 to the response-time analysis in Lemma 9 and Theorem 1.

**Theorem 2.** Let  $\gamma^a$  be an arbitrary subchain instance. Let  $A$  be  $\gamma^a$ 's activation time, let  $F$  be  $\gamma^a$ 's completion time, and let  $c_e^y$  denote the last callback instance in  $\gamma^a$ . Let  $S^*$  be the least positive solution (if any) of the following inequality.

$$sbf_k(S^*) \geq \epsilon + I_e(S^*, pp(\gamma)) + ET_e(si_e(S^*)) \quad (5)$$

Let  $\Omega \triangleq ET_e(si_e(S^*) + 1) - ET_e(si_e(S^*))$ , and let  $R^*$  be the least positive solution (if any) of the following inequality.

$$sbf_k(R^*) \geq sbf_k(S^*) - \epsilon + \Omega \quad (6)$$

Then  $R^*$  is a response-time bound for  $\gamma$ :  $F - A \leq R^*$ .

*Proof.* Follows from Lemma 15 for  $f(x) \triangleq \epsilon + I_e(x, N)$  and  $g(x) \triangleq si_e(x)$ , Lemma 9, and Theorem 1. ■

Since Theorem 2 does not depend on  $\omega_e^y$ , it is suitable for an *a priori* response-time analysis. Specifically, an implementation can find the least solution  $S^*$  through fixed-point iteration and

then compute a response-time bound. If a solution  $S^*$  cannot be found, then Theorem 2 is not applicable.

Furthermore, since the computation of  $\gamma^a$ 's response-time depends on other response-time bounds in a cyclical fashion (e.g., Definitions 1 and 2), another outer fixed-point iteration is necessary [11] until the response-time bounds for all callbacks have reached a global fixed point (or until some predefined threshold is exceeded). This process always terminates since the response-time estimates never decrease during the search.

## V. BUSY-WINDOW ANALYSIS

We now integrate the busy-window principle with the preceding analysis. A key benefit of extending the analysis horizon in this manner is that it allows for the derivation of less pessimistic callback activation curves.

First, some preliminary definitions: We say that a callback instance is *carried in* at time  $t$  if it is pending at both  $t$  and  $t - 1$ . An instant  $t$  is a *quiet time* of executor  $E_k$  if no instances assigned to  $E_k$  are carried-in. An interval  $[t_1, t_2)$  is a *busy window* w.r.t. a callback instance  $c_i^x$  if both  $t_1$  and  $t_2$  are quiet times of  $c_i^x$ 's executor, no quiet time of  $c_i^x$ 's executor occurs between  $t_1$  and  $t_2$ , and  $c_i^x$  is activated in  $[t_1, t_2)$ .

Since the propagation delay within executors is zero, all components of a subchain instance  $\gamma^x$  share the same busy window. We thus define the busy window of a chain instance  $\gamma^x$  as the busy window of its components.

We again refer to the subchain instance under analysis as  $\gamma^a$ , which is activated at time  $A$  and completes at time  $F$ . For simplicity (and w.l.o.g.), we assume that the time axis is normalized such that  $\gamma^a$ 's busy window starts at time zero.

### A. Activation Curves

With basic definitions in place, we now introduce the improved callback activation curves. To begin with, first recall the general case from prior work [3], wherein the activation curve  $\eta_j(\Delta)$  for any non-source callback  $c_j$  is defined in terms of the activation curves of  $c_j$ 's predecessors:

$$\eta_j(\Delta) \triangleq \sum_{c_i \in \text{pred}(c_j)} \eta_i(\Delta + R(c_i) - \epsilon + \delta_{i,j}). \quad (7)$$

The bound follows since instances activated in  $[0, \Delta)$  must be triggered in  $[-\delta_{i,j}, \Delta)$ . An instance must be pending to trigger a successor callback, and by Lemma 1 at most  $\eta_i(\Delta + \delta_{i,j} + R(c_i) - \epsilon)$  instances of  $c_i$  are pending in  $[-\delta_{i,j}, \Delta)$ .

Under the busy-window assumption, the analysis can further exploit that no instance served by  $\gamma_a$ 's executor is carried in at time 0. If two callbacks  $c_i$  and  $c_j$  are assigned to  $\gamma_a$ 's executor, then instances of  $c_i$  activated before time 0 cannot activate instances of  $c_j$  at or after time 0. Definition 4 and Lemma 16 provide an activation curve variant that exploits this property within  $\gamma_a$ 's executor and falls back to  $\eta_i(\Delta)$  otherwise.

**Definition 4.** For a message-driven callback  $c_j$  assigned to  $\gamma_a$ 's executor, the *busy-window activation curve*  $\eta_j^b(\Delta)$  is given by

$$\eta_j^b(\Delta) \triangleq \sum_{c_i \in \text{pred}(c_j)} \begin{cases} \eta_i^b(\Delta) & e_i = e_j, \\ \eta_i(\Delta + \delta_{i,j} + R(c_i) - \epsilon) & \text{otherwise.} \end{cases}$$

If  $c_j$  is not message-driven, then  $\eta_j^b(\Delta) \triangleq \eta_j(\Delta)$  instead.

**Lemma 16.** Let  $\Delta > 0$ , and  $c_j \in \mathcal{C}_k$ . If time 0 is a quiet time of  $c_j$ 's executor, then at most  $\eta_j^b(\Delta)$  instances of  $c_j$  are activated during  $[0, \Delta)$ .

*Proof.* By induction over the callback graph.

*Induction base:* For source callbacks,  $\eta_j^b(\Delta) = \eta_j(\Delta)$ , which bounds the number of activations of  $c_j$  in general.

*Induction step:* Consider any edge  $(c_i, c_j)$ . If  $c_i$  and  $c_j$  do not share an executor, then the number of activations along  $(c_i, c_j)$  is bounded by  $\eta_j(\Delta + \delta_{i,j} + R(c_i) - \epsilon)$ , which bounds the number of activations along an arbitrary edge (Eq. (7)). If  $c_i$  and  $c_j$  share an executor, then propagation delay is zero and no instance of  $c_i$  is carried in at time 0. Thus, all instances of  $c_i$  pending in  $[0, \Delta)$  are also activated in  $[0, \Delta)$ . Only instances pending in  $[0, \Delta)$  can trigger an instance of  $c_j$  during  $[0, \Delta)$ . By the induction hypothesis, at most  $\eta_i^b(\Delta)$  instances of  $c_i$  are activated in  $[0, \Delta)$ . Since each instance triggers at most one instance of  $c_j$ ,  $\eta_i^b(\Delta)$  bounds the activations along the edge  $(c_i, c_j)$ . The above argument holds for each predecessor of  $c_j$ , independently of any other predecessors. We can thus repeat the argument for all predecessors of  $c_j$  and bound the total activations of  $c_j$  by the sum of all per-predecessor bounds. ■

## B. Response-Time Bound

The improved activation curve replaces  $\eta_i$  in the interference bound to reduce pessimism. In addition to  $\Delta$ , the length of the interval to consider, and  $N$ , a bound on the number of polling points, the function takes  $c_i^x$ 's activation time  $t_a$  as a third parameter. It then combines two ways to bound the number of instances of a polled callback  $c_j$  during  $[0, \Delta)$ . The first way is to bound all activations in  $[0, \Delta)$  as  $\eta_j^b(\Delta)$ , which exploits that the considered interval starts at a quiet time but does not exploit Property **SM**. The second way is to bound all activations in  $[0, t_a)$  as  $\eta_j^b(t_a)$ , and to then bound the interference in  $[t_a, \Delta)$  (i.e., after  $c_i^x$ 's activation) under consideration of Property **SM**.

**Definition 5.** For any polled callback  $c_i$  assigned to  $E_k$ , the *busy-window interference bound* is given by

$$I_i^b(\Delta, N, t_a) \triangleq \sum_{c_j \in \mathcal{C}_k^{\text{pvt}}} ET_j(\eta_j^b(\Delta)) + \sum_{c_j \in \mathcal{C}_k^{\text{pol}} \setminus \{c_i\}} \min \begin{cases} ET_j(\eta_j^b(\Delta)) \\ ET_j(v_j) \end{cases}$$

with  $v_j = \eta_j^b(t_a) + N + \llbracket c_j \in hp_k(c_i) \rrbracket_1$ .

Lemma 17 proves the bound to be sound.

**Lemma 17.** Let  $c_i \in \mathcal{C}_k^{\text{pol}}$ . Let  $t_a$  and  $t_f$  denote the activation time and completion time of an instance of  $c_i$ . Let  $N$  upper-bound the number of polling points in  $E_k$  during the interval  $[t_a, t_f)$ . If time 0 is a quiet time of  $E_k$  then for any  $0 \leq \Delta \leq t_f$ , instances of callbacks in  $\mathcal{C}_k \setminus \{c_i\}$  consume at most  $I_i^b(\Delta, N, t_a)$  units of processor service during  $[0, \Delta)$ .

*Proof.* Since  $I_i^b$  sums over all callbacks in  $\mathcal{C}_k$ , it suffices to show that both  $ET_j(\eta_j^b(\Delta))$  and, in the case of polled callbacks,  $ET_j(v_j)$  bound how many instances of each interfering callback  $c_j$  run during  $[0, \Delta)$ . By Lemma 16,  $\eta_j^b(\Delta)$  bounds how many instances of  $c_j$  are activated in the interval  $[0, \Delta)$  (since by assumption time 0 is a quiet time of  $E_k$ ).

It remains to be shown that  $v_j$  is a valid upper bound, too. Lemma 16 again shows that  $\eta_j^b(t_a)$  bounds how many instances of  $c_j$  run during  $[0, t_a)$ . By Lemma 2 and Corollary 1,  $N + \llbracket c_j \in hp_k(c_i) \rrbracket_1$  upper-bounds the number of instances running in  $[t_a, t_f)$ . Their sum therefore bounds the number of instances running in  $[0, t_a) \cup [t_a, t_f) = [0, t_f) \supseteq [0, \Delta)$ . ■

Analogously to Definition 2, we also define a busy-window-aware self-interference bound  $si_i^b(t_a) \triangleq \eta_i^b(t_a + \epsilon) - 1$ , using  $si_i^b(t_a)$  in place of  $si_i(\Delta)$  to leverage the busy-window activation curve  $\eta_i^b(\Delta)$ . Unlike in Definition 2, no  $\max(0, \dots)$  term is required since  $t_a + \epsilon > 0$  and therefore  $\eta_i^b(t_a + \epsilon) \geq 1$ .

Put together, the two new interference bounds yield a result similar to Theorem 2.

**Theorem 3.** Let  $\gamma^a$  be an arbitrary instance of subchain  $\gamma$ . Let  $A$  denote  $\gamma^a$ 's activation time,  $F$  its completion time,  $c_e^y$  the last callback instance in  $\gamma^a$ , and  $t_a$  the activation time of  $c_e^y$ . If time 0 is a quiet time of  $\gamma$ 's executor,  $S^*$  is the least positive solution (if any) of

$$sbfk(S^*) \geq \epsilon + I_e^b(S^*, pp(\gamma), t_a) + ET_e(si_e^b(t_a)),$$

$\Omega \triangleq ET_e(si_e^b(t_a) + 1) - ET_e(si_e^b(t_a))$ , and  $F^*$  is the least positive solution (if any) of the inequality

$$sbfk(F^*) \geq sbfk(S^*) - \epsilon + \Omega,$$

then  $F \leq F^*$  and  $F^* - A$  is a response-time bound for  $\gamma^2$ .

*Proof.* Recall there are at most  $pp(\gamma)$  polling points in  $[A, F)$  (Lemma 7), hence also in  $[t_a, F) \subseteq [A, F)$  ( $t_a \geq A$ , since  $c_e^y$  is part of  $\gamma^a$ ). By Lemma 17,  $\epsilon + I_e^b(S^*, pp(\gamma), t_a)$  hence strictly exceeds the total interference due to all callbacks in  $\mathcal{C}_k \setminus \{c_e\}$  during  $[0, S^*)$ . Since no instance of  $c_e$  is carried in at time 0,  $si_e^b(t_a)$  bounds the number of instances of  $c_e$  except  $c_e^y$  that are pending (and hence self-interfering) in  $[0, t_a)$ . By Lemma 8, the total self-interference is then given by  $\min(ET_e(si_e^b(t_a) + 1) - \omega_e^y, ET_e(si_e^b(t_a)))$ , where  $\omega_e^y$  is  $c_e^y$ 's execution cost. Analogously to the proofs of Lemma 9 and Theorems 1 and 2, the claim then follows via Lemma 15 with  $f(x) \triangleq \epsilon + I_e^b(x, pp(\gamma), t_a)$  and  $g(x) \triangleq si_e^b(t_a)$ . ■

<sup>2</sup>Compared to the conference version of this paper that appeared at RTSS'21, Theorem 3 and Lemmas 18 and 19 have been revised to correct an accidental confusion of  $t_a$  and  $A$ . We reran the experiments reported on in Sec. VI and did not observe significant differences as a result of this correction.

Theorem 3 yields a response-time bound, namely  $F^* - A$ , which however depends on two unknown offsets,  $A$  and  $t_a$ . In the case where  $\gamma$  consists of a single callback,  $A$  is equal to  $t_a$  and does not need to be considered separately. Otherwise, it suffices to consider  $A = 0$ ;  $A$  does not influence the computation of  $F^*$ , and  $A = 0$  therefore maximizes the term  $F^* - A$ . We next derive a sparse, finite set of offsets  $t_a$  that suffice to be considered.

### C. Search Space for the Activation Offset $t_a$

To start, Lemma 18 derives an upper bound on  $t_a$ .

**Lemma 18.** *Let  $c_i$  be an arbitrary callback and let  $t_a^*$  denote the least positive solution (if any) of*

$$sbf_k(t_a^*) \geq \epsilon + I_i^b(t_a^*, pp(c_i), t_a^*) + ET_i(\eta_i^b(t_a^*)).$$

*Then any instance of  $c_i$  is activated strictly less than  $t_a^*$  time units after the beginning of its busy window.<sup>3</sup>*

*Proof.* By contradiction: suppose an instance  $c_i^a$  is activated at time  $t_a \geq t_a^*$ . W.l.o.g. let time 0 denote the start of  $c_i^a$ 's busy window. By Lemma 17,  $I_i^b(t_a^*, pp(c_i), t_a)$  bounds the total interference that callbacks other than  $c_i$  impose upon  $c_i^a$  during the time window  $[0, t_a^*)$ .  $ET_i(\eta_i^b(t_a^*))$  bounds the demand that  $c_i$  imposes upon  $c_i^a$  during  $[0, t_a^*)$ . By the inequality in the lemma statement, the supply available to callbacks served by  $E_k$  during  $[0, t_a^*)$  then necessarily exceeds the possible total demand during that time. Since  $c_i^a$  is not pending before  $t_a \geq t_a^*$ , this implies that the executor must idle at some point during  $[0, t_a^*)$ , i.e., there is a quiet time in  $(0, t_a^*)$ . Since  $c_i^a$ 's busy window starts at time 0, this implies that  $c_i^a$ 's busy window ends before  $c_i^a$ 's activation, which is a contradiction. Therefore, any instance  $c_i^a$  activates at most  $t_a^* - \epsilon$  time units after the beginning of its busy window. ■

To obtain a sparse search space, the next lemma identifies that only “steps” in the  $\eta_j^b$  bounds for the callbacks in  $E_k$  need to be considered to find a response-time bound.

**Lemma 19.** *Let  $c_i^x$  be an arbitrary instance of  $c_i$  and let  $t_a$  denote  $c_i^x$ 's activation time. If  $t_a > 0$ ,*

$$\forall c_j \in \mathcal{C}_k^{\text{pol}} \setminus \{c_i\} . \eta_j^b(t_a) = \eta_j^b(t_a - \epsilon), \text{ and}$$

$$\eta_i^b(t_a) = \eta_i^b(t_a + \epsilon),$$

*then the response-time bound (given by Theorem 3) for  $\gamma^a = \langle c_i^x \rangle$  is lower than that for an instance activated at time  $t_a - \epsilon$ .*

*Proof.* Let  $c_i^w$  be an instance of  $c_i$  activated at time  $t_a - \epsilon$ . We show that  $I_j^b(\Delta, N, t_a) = I_j^b(\Delta, N, t_a - \epsilon)$  and  $si_i^b(t_a) = si_i^b(t_a - \epsilon)$ . Hence  $F^*$  as defined in Theorem 3 (with  $\gamma$  being the chain consisting of only  $c_i$ ) is the same for  $c_i^w$  and  $c_i^x$ . The lemma follows since  $F^* - t_a < F^* - (t_a - \epsilon)$ .

In the definition of  $I_i^b(\Delta, N, t_a)$  only the term  $ET_j(v_j)$  depends on the  $t_a$  parameter. This term appears for any  $c_j \in \mathcal{C}_k^{\text{pol}} \setminus \{c_i\}$ . Since for each such  $c_j$  by assumption  $\eta_j^b(t_a) =$

<sup>3</sup>Addendum: this document has been revised to correct an accidental misuse of notation in Lemma 18 (confusion of  $\eta_i^b(t_a^*)$  and  $si_i^b(t_a)$ ).

$\eta_j^b(t_a - \epsilon)$ , it follows that  $v_j$  is equal for both activation times,  $t_a$  and  $t_a - \epsilon$ . Therefore,  $I_j^b(\Delta, N, t_a) = I_j^b(\Delta, N, t_a - \epsilon)$ .

In the definition of  $si_i^b(t_a)$ , only the term  $\eta_i^b(t_a + \epsilon)$  depends on  $t_a$ . Since  $\eta_i^b(t_a + \epsilon) = \eta_i^b(t_a)$  it follows that  $si_i^b(t_a) = si_i^b(t_a - \epsilon)$  ■

The search space for activation offsets of a callback  $c_i$  is thus defined as

$$\mathcal{A} \triangleq \{0\} \cup \{t_a < t_a^* \mid \eta_i^b(t_a) \neq \eta_i^b(t_a + \epsilon) \vee \exists c_j \in \mathcal{C}_k^{\text{pol}} \setminus \{c_i\} : \eta_j^b(t_a) \neq \eta_j^b(t_a - \epsilon)\}.$$

The analysis needs to consider only  $\mathcal{A}$  as possible values of  $t_a$ . The response-time bound for the subchain under analysis is then given by the maximum result obtained via Theorem 3 for each  $t_a \in \mathcal{A}$ . If  $t_a^*$ , or either of the fixed-point solutions  $F^*$  and  $S^*$  for any  $t_a$ , cannot be found, then Theorem 3 is not applicable.

### D. Combined Analysis

Theorems 2 and 3 are independent analyses that should be used jointly: as neither dominates the other, it is generally advisable to apply both analyses to each subchain and to use the lesser of the two bounds on a per-subchain basis.

In the derivations of Theorems 2 and 3, no assumptions have been placed on the number of callbacks in the chain under analysis  $\gamma$ . Both analyses can therefore also be used to bound the response time of an individual polled callback by interpreting the callback under analysis as a single-element chain.

## VI. EVALUATION

We evaluated the proposed analyses on two case studies: a synthetic callback graph, designed to assess each analysis's advantages and disadvantages, and a real-world callback graph, to evaluate the analyses under realistic conditions.

The analysis by Casini *et al.* [3] serves as a baseline. We compare it to the round-robin analysis in Theorem 2 (*RR-only*), the busy-window analysis in Theorem 3 (*BW-only*), and the combined analysis that computes the minimum of both approaches (*this-paper*).

To distinguish effects due to analysis improvements from effects due to the refined execution-time model, we also compare against variants of our analysis using only linear execution-time curves (i.e.,  $ET(n) \triangleq n \cdot ET(1)$  for  $n \geq 1$ ). The resulting execution-time curves are equivalent to assuming a scalar WCET of  $ET(1)$  for each callback instance. Consequently, this analysis variant removes the precision advantage of execution-time curves while keeping analysis improvements in place. In the figures, these variants are marked with the suffix “(wcet).”

**Synthetic workload.** The first case study, depicted in Fig. 3, consists of a single executor containing only polled callbacks. One callback,  $c_0$ , is triggered in bursts of up to  $b$  activations at once. The bursts are separated by at least 10 ms. A second set of callbacks,  $c_1$  to  $c_l$ , forms an intra-executor chain of length  $l = 6$ . Callback  $c_1$  is activated by  $f$  callbacks (the *fan-in*) located in the same executor, each with an activation curve that mandates a 10  $\mu$ s distance between any two activations, but

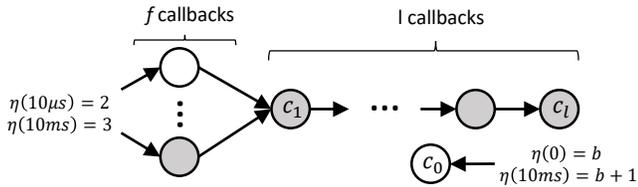


Fig. 3: Synthetic setup. The chain under analysis is shaded gray.

10 ms between any three activations. To simplify comparisons with the baseline, which supports only scalar WCETs, the setup is limited to scalar WCETs. The callback  $c_0$  has a WCET of  $10\ \mu\text{s}$ , each callback in the chain has a WCET of  $50\ \mu\text{s}$ , and the fan-in callbacks have a WCET of  $1\ \mu\text{s}$  each. The executor is provisioned with a periodic supply of  $700\ \mu\text{s}$  every one ms.

The experiment varies two parameters:  $b$ , the burst length of  $c_0$ , and  $f$ , the width of the fan-in. As the parameters vary, we observe the response-time bound of the chain marked in Fig. 3.

Fig. 4a shows the response-time bounds reported by the three analyses as  $b$  changes (for a fixed fan-in width  $f = 1$ ). Since *this-paper* is simply the minimum of *RR-only* and *BW-only*, it is not shown separately. The plot shows a steadily increasing response-time bound for both the baseline and *BW-only*. In contrast, the *RR-only* bound stays flat after  $b = 14$ . At this point, all processing windows involved in the chain are saturated with instances of  $c_0$ . Activating more instances of  $c_0$  therefore does not increase the interference suffered by the chain. The results show the importance of accounting for starvation freedom, as both the baseline and the *BW-only* analysis significantly overestimate the interference from  $c_0$ .

Fig. 4b shows a second experiment in which we varied  $f$ , the width of the fan-in, for a fixed burst length of  $b = 10$ . The plot shows that all variants achieve similar bounds at  $f = 1$ . As the fan-in width increases, the *BW-only* response-time bound also increases in reaction to the increased activation rate of the chain. The *RR-only* analysis starts out similarly, but exhibits a more rapid increase starting at  $f = 5$ . The *baseline* analysis performs much worse, with particularly large jumps at  $f = 2$  and  $f = 5$ . Overall, the *BW-only* bound improves upon the baseline by a factor of at least  $2\times$  for  $f > 1$ .

The baseline’s lower analysis precision is caused by its lack of support for intra-executor fan-in. Since  $c_1$  has multiple predecessors for  $f > 1$ , the baseline analysis needs to separately bound the response time of the first fan-in callback and the following  $l$ -element chain, oblivious to the fact that they necessarily share a busy window. The *BW-only* analysis, in contrast, analyzes the entire chain as a whole and correctly accounts for busy-window constraints in all predecessors.

Overall, the two experiments show that both analysis approaches, *RR-only* (Theorem 2) and *BW-only* (Theorem 3), improve upon the baseline. Since they excel in different situations, combining both approaches is advisable.

**Real workload.** In the second case study, we evaluated the analyses on a callback graph extracted from a real ROS 2 system using the Navigation 2 package [17]. The model stems

from prior work [6], where it was obtained by measuring all model-relevant parameters (such as execution-time curves and activation frequencies) at runtime on a *Turtlebot 3* robot performing a simple navigation task. The robot ran ROS 2 version “Dashing,” so timers were privileged.

Fig. 5 shows the results for the 19 (out of 54) callbacks in the system for which the response-time bounds are not trivial (e.g., due to lack of interference or extremely sparse activations). Since the absolute values of the bounds vary heavily between the callbacks, we instead show each bound as a ratio normalized by the *this-paper* bound. All ratios exceed 1, which shows that the analysis proposed in this paper produces lower response-time bounds than the baseline and all three depicted variants of the proposed analysis. For some callbacks (e.g., *7469/scan*), the baseline yields response-time bounds that are almost 80 times larger than the bounds produced by our analysis. The baseline’s bound is still about 25 times larger than the bounds given by versions of the proposed analyses using linear execution-time bounds (i.e., with the “(wcet)” suffix), which shows that the bulk of the improvement in this case stems from analysis improvements, specifically the round-robin analysis. Similar gains are visible for, e.g., *7533/scan* and *7556/tf-static*.

The *BW-only* analysis also achieves significant improvements over the baseline in some of the callbacks (e.g., *7469/scan* and *7533/tf-static*). For about half the callbacks, the *BW-only* analysis does not improve upon the baseline, though. The reason is that the considered benchmark contains only few intra-executor edges, which limits the impact of the improved activation-curve propagation highlighted in Fig. 4b.

The results further show that the busy-window analyses (*baseline* and *BW-only*) are not dominated by the round-robin analysis: for instance, the *RR-only* bounds are significantly worse for the *7469/tf* callback. Again, this shows that it is best to combine both analyses instead of using either in isolation.

Finally, the experiment clearly shows the benefits of using a processor demand model that is more expressive than scalar WCETs. Consider callback *7469/tf* again: the response-time bounds of the WCET-based version of *this-paper* differ from the execution-time curve version by a factor of  $60\times$ . Assuming that each callback instance runs for its full WCET obviously leads to tremendous pessimism in the analysis of this callback.

Overall, the experiments confirm that both the round-robin analysis (Sec. IV) and the busy-window analysis (Sec. V) are needed, as both perform better than the other in some cases. The proposed analysis performs significantly better than the baseline, reducing response-time bounds by almost up to  $80\times$  in the real-world case study.

## VII. RELATED WORK

Casini *et al.* proposed the first response-time analysis of ROS 2 processing chains [3]. Tang *et al.* [4] subsequently provided a more precise analysis for the special case of independent linear processing chains, at the cost of more limited applicability (i.e., each callback can belong to only one chain). Consequently, Tang *et al.*’s analysis [4] is not applicable to

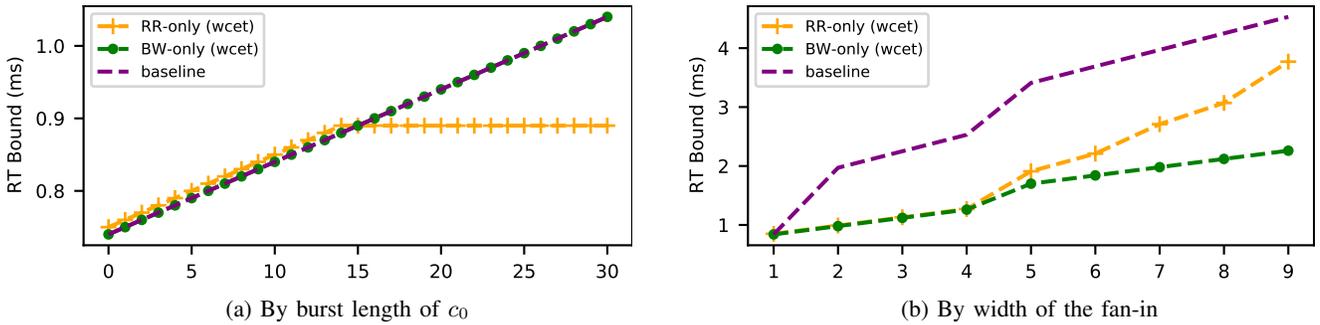


Fig. 4: Response-time bound of the chain (synthetic workload).

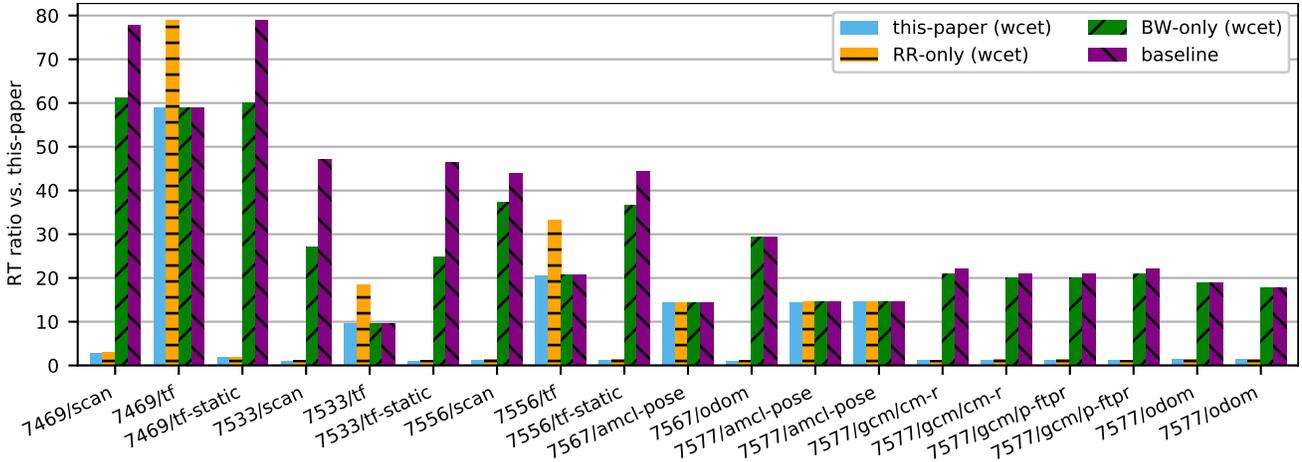


Fig. 5: Response-time bound of callbacks compared to the proposed analysis (lower is better).

the workloads considered in Sec. VI, which contain branching processing chains.

In more distantly related work, Tang *et al.* [18] recently developed an analysis for a ROS-inspired scheduler that also exhibits a round-robin-like property. As their analysis considers only systems of independent tasks, it does not transfer easily to the kind of general ROS systems considered herein. Furthermore, prior timing analyses for other graph-based frameworks like OpenMP [19, 20] and Tensorflow [21] share similar goals and some of the challenges studied herein.

Besides providing analytical guarantees, researchers have explored real-time requirements in ROS 2 in a number of other ways, including empirical studies on latency effects [22]–[26]. In another line of work, Choi *et al.* [5] and Staschulat *et al.* [27] have explored alternative ROS executor designs with improved time-predictability. In contrast, our focus is mainline ROS as it is deployed by most ROS users today. Also targeting unmodified mainline ROS deployments, Blass *et al.* [6] proposed an automated, introspection-based method for provisioning ROS executors under reservation-based scheduling.

This paper adopts the execution-time curve model [7] to represent processor demand. While this is a natural choice in our context due to their similarity with activation curves, the literature provides various alternative processor demand models of varying complexity and expressiveness [28]–[30].

## VIII. CONCLUSION

We have proposed a more precise response-time analysis for ROS systems. The analysis improves upon prior work through three key techniques: modeling processor demand as execution-time curves instead of scalar WCETs, accounting for the effects of quiet times and busy windows in the activation curve derivation, and exploiting the round-robin behavior of the ROS callback scheduler. The resulting analysis was demonstrated in both a synthetic and a real-world case study to yield significantly tighter response-time bounds compared to prior work on the ROS default scheduler.

In future work, it would be interesting to revisit how recent alternative executor designs [5, 27] compare to the default executor once the round-robin property is taken into account.

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