Fair Quantitative Games

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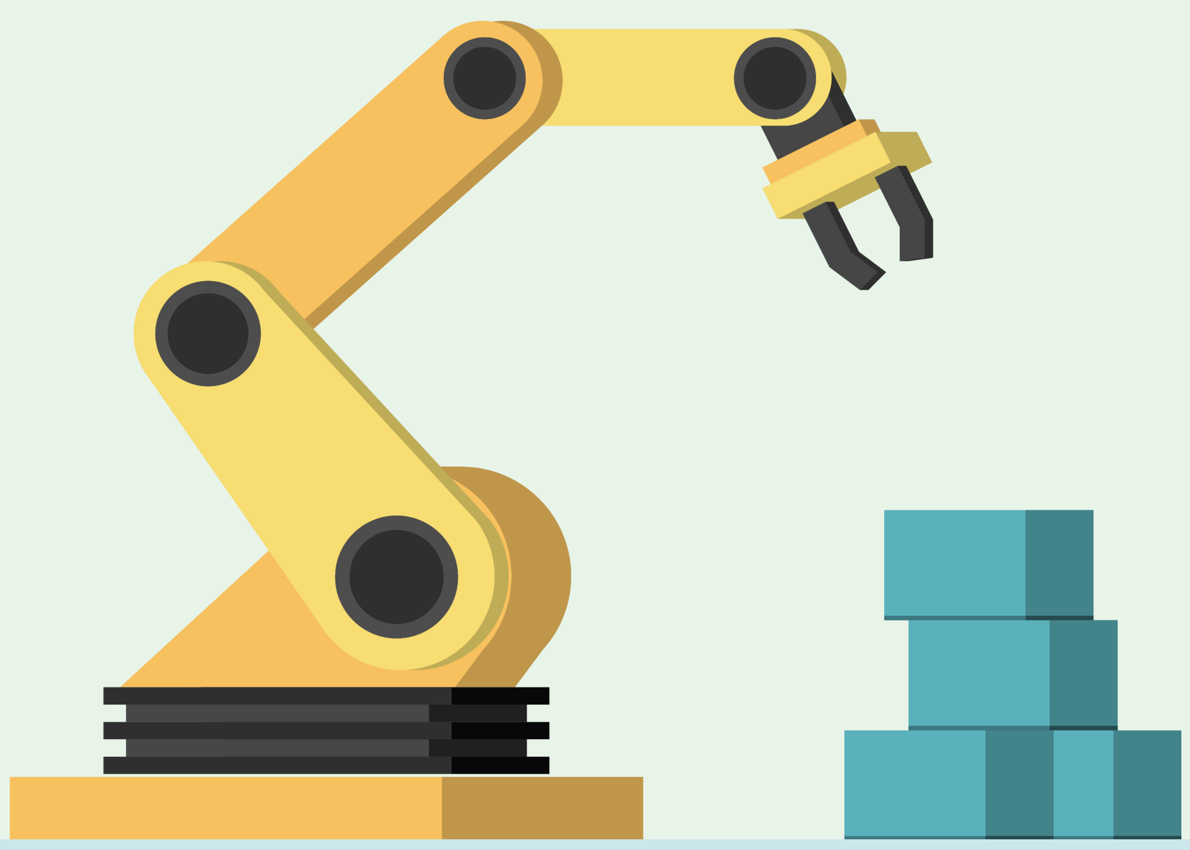
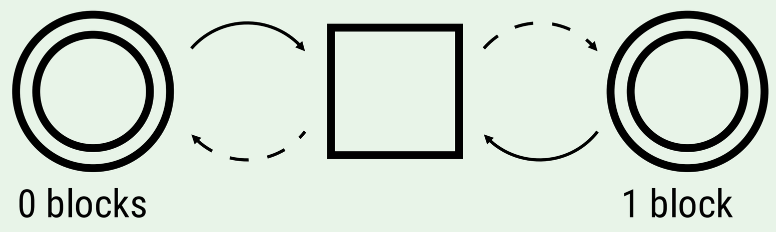
○ System (Player 1) □ Environment (Player 2)

Fairness assumptions eliminate unrealistic scenarios.

Fairness: Whenever the source node of a dashed edge is taken infinitely often, the dashed edge is also taken infinitely often.



Fairness in Synthesis

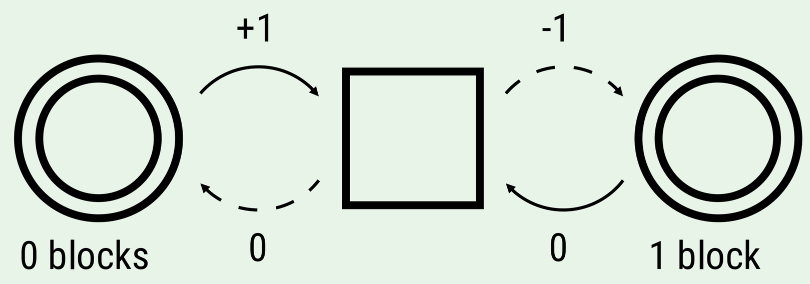


Can we guarantee the robot will put a block at the mark, and remove it, infinitely often?

Qualitative
Safety, Parity, Rabin…

Fairness is easy on qualitative games

Can be solved in **same time complexity** as the original game [1, 2]



Can we guarantee the robot will put a block at the mark, and remove it, infinitely often, without the battery running out, *for some initial value of the battery*?

Quantitative
Energy, Mean-payoff

What about fairness on quantitative games?

Fairness on system can be solved in **super-exponential time** using current approaches, whereas there is no known approach for fairness on environment.

Contributions

	Determined?	Complexity (Pseudopolynomial)	Reduction
1-fair MP	Yes	$O(n^3mW)$	To MP on 6n nodes and max absolute weight
2-fair MP	Yes	$O(n^3mW)$	To MP on 6n nodes and max absolute weight
1-fair Energy	Yes	$O(n^4mW)$	To Energy on 8n nodes and max absolute weight
2-fair Energy	No	$O(n^3mW)$	Player 1 winning region reduces to that of an energy on the same graph, Player 2 winning region reduces to that of 2-fair MP game on the same graph

Fairness in Quantitative Games

A play ρ is **fair** iff for every node $v \in inf(\rho)$ that has fair *(dashed)* outgoing edges $E^f(v) \neq \emptyset$, $E^f(v) \subseteq inf(\rho)$.

1-Fair Games: Player 1 nodes have fair outgoing edges.

2-Fair Games: Player 2 nodes have fair outgoing edges.

1-Fair Mean Payoff

Does there exist a strategy σ such that, long run average payoff of every σ-play is **non-negative AND the play is fair?**

1-Fair Energy

Does there exist an initial credit **c** and a strategy σ such that, total energy level along every σ-play stays **non-negative AND the play is fair?**

2-Fair Mean Payoff

Does there exist a strategy σ such that, long run average payoff of every σ-play is **non-negative OR the play is NOT fair?**

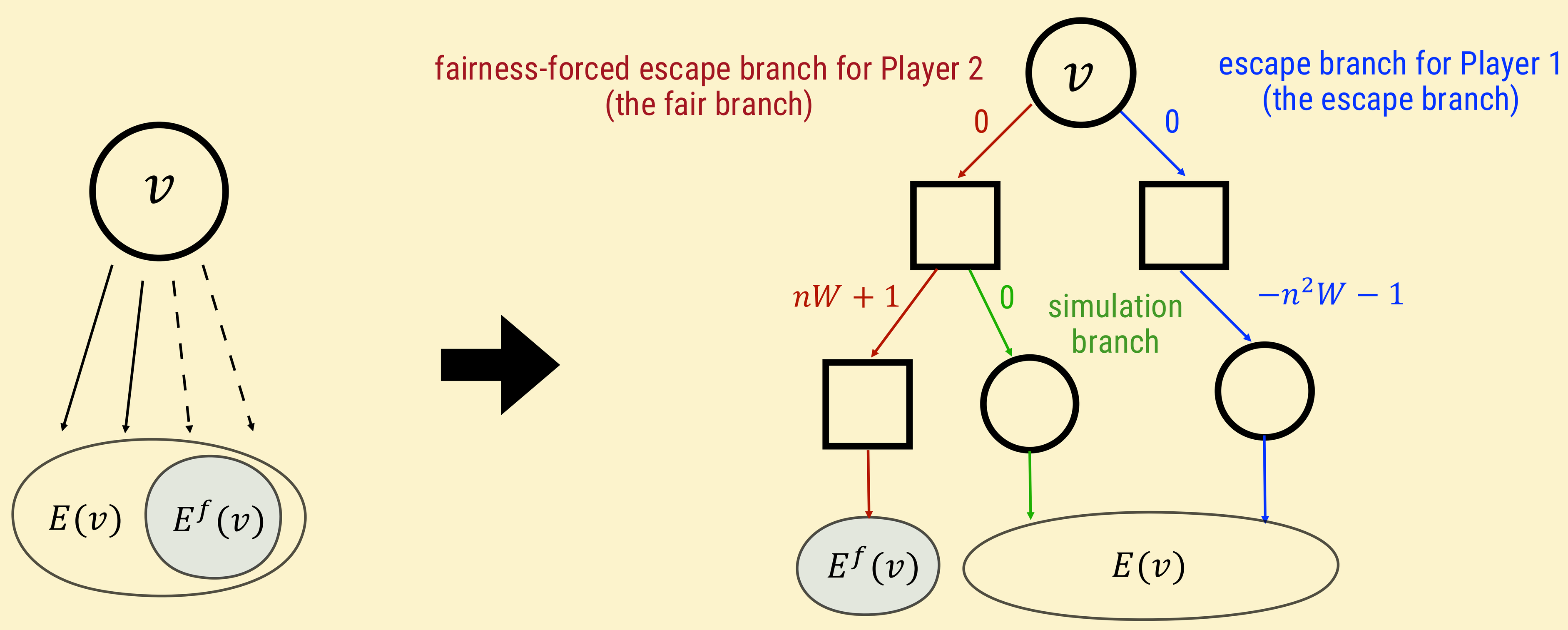
2-Fair Energy

Does there exist an initial credit **c** and a strategy σ such that, total energy level along a play stays **non-negative OR the play is NOT fair?**

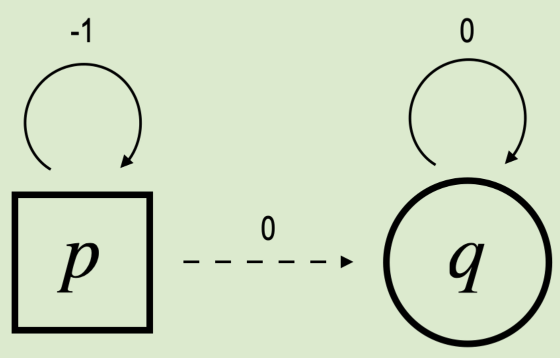
Gadgets for Reducing Fair Games to Standard games

For each fair node v in the 1-fair MP game

replace v with the following v-gadget



2-fair energy games are not determined



A node is won by Player 1 if there exists a Player 1 strategy σ and a credit c s.t. every $\sigma-$play is won by Player 1 for credit c.

A node is won by Player 2 if there exists a Player 2 strategy π s.t. every $\pi-$play's total payoff goes below $-c$ for every credit c.

[1] Banerjee, T., Majumdar, R., Mallik, K., Schmuck, A., Soudjani, S.: Fast symbolic algorithms for omega-regular games under strong transition fairness. TACAS'22
[2] Hausmann, D., Piterman, N., Saglam, I., Schmuck, A.: Fair ømega-regular games. FoSSaCS'24

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