Mtac: A Monad for Typed Tactic Programming in Coq

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A simple problem in Coq

$$\frac{x\ y\ z{:}A \qquad s{:}\mathsf{list}\ A}{x \in \mathsf{append}\ s\ (z :: x :: y :: [])}$$

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Proof.

A simple problem in Coq

$$\text{inR} : a \in r \to a \in \text{append } l\ r$$

$$\frac{x\ y\ z{:}A \qquad s{:}\text{list } A}{x \in \text{append } s\ (z :: x :: y :: [])}$$

Proof.

apply: inR.

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Proof.

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A simple problem in Coq

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A simple problem in Coq

$$\text{in\_head} : a \in (a :: l)$$

$$\frac{x\ y\ z{:}A \qquad s{:}\text{list}\ A}{x \in (x :: y :: [])}$$

Proof.

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apply: in_tail.

apply: in_head.

A simple problem in Coq

$$\frac{x\ y\ z{:}A \qquad s{:}\mathsf{list}\ A}{x \in (x :: y :: [])}$$

□

Proof.

apply: inR.

apply: in_tail.

apply: in_head.

Qed.

A simple problem in Coq

This script is:
Boring.

$$\frac{x\ y\ z{:}A \qquad s{:}\mathsf{list}\ A}{x \in (x :: y :: [])}$$

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Proof.
apply: inR.
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A simple problem in Coq

This script is:

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Fragile.

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This script is:

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$$\frac{x\ y\ z{:}A \qquad s{:}\mathsf{list}\ A}{x \in (y :: x :: [])}$$

$\square$

Proof.

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Qed.

search in Ltac

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(1)   Ltac search x s :=
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search in Ltac

Logic of Coq includes a funct. language

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search in Ltac

Logic of Coq includes a pure funct. language

search requires impure features

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search in Ltac

#2: Unbounded recursion

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search in Ltac

#3: Control flow (backtracking)

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search in Ltac

Untyped!

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No spec: search :???

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search in Ltac

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Debugging hell!

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Use Monads

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} Mtac

search in Mtac

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Specification (type) should be:

$$\text{search} : \forall (x : A)\ (s : \text{list}\ A).\ \text{M}\ (x \in s).$$

search in Mtac

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(1)  Definition search (x : A) :=
(2)      mfix f (s : list A) : M (x ∈ s) :=
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search in Mtac

#2: Unbounded recursion

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search in Mtac

#3: Control flow (exceptions)

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search in Mtac

Typed!

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search in Mtac

Typed!

Spec: $\forall x\ s.\ \mathsf{M}\ (x \in s)$

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Type checker

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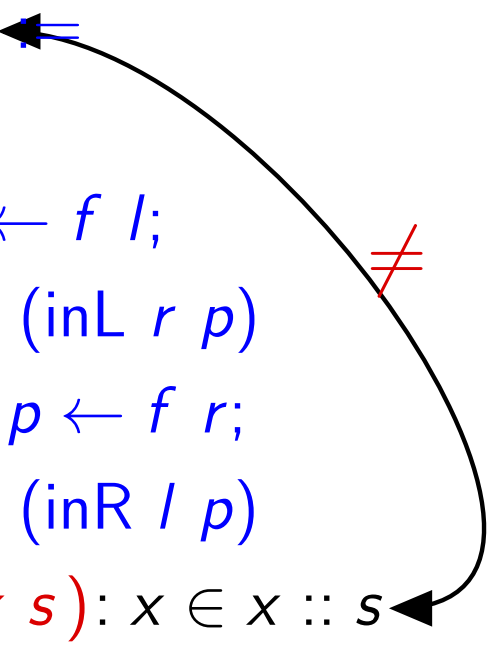

search in Mtac

Typed!

Type inference

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search in Mtac

Typed! Interactive programming

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search in Mtac

Typed!

Interactive programming

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(5)                                 ret _
(6)                            with _ ⇒ p ← f r;
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A simple problem in Coq, with Mtac

$$\frac{x\ y\ z : A \qquad s : \mathsf{list}\ A}{x \in \mathsf{append}\ s\ (z :: x :: y :: [])}$$

Proof.

A simple problem in Coq, with Mtac

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```
Proof.
  apply: run (search _ _).
Qed.
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A simple problem in Coq, with Mtac

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run is a first-class citizen

A simple problem in Coq, with Mtac

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run is a first-class citizen

With lemma $L : a \in s \to P\ a$

A simple problem in Coq, with Mtac

$$\frac{x\ y\ z : A \qquad s : \text{list } A}{x \in \text{append } s\ (z :: y :: x :: [])}$$

Proof.
  apply: **run** (search _ _).
Qed.

run is a first-class citizen

With lemma $L : a \in s \to P\ a$
to solve $P\ x$ we can apply L (**run** (search x _)).

Use Monads

1. Encapsulate tactical effects in a monad.

   Give tactical effects a Coq type M *A*.

2. Execute (**run**) tactics at type inference.

} Mtac

Mtac: tactic language

Key idea #1:

- Put tactical effects in a monad M.

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Add typechecker.

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~~Add typechecker.~~ Coq is Mtac's typechecker!

Mtac: tactic language

Key idea #1:

- Put tactical effects in a monad M.

~~Add typechecker.~~ Coq is Mtac's typechecker!

Inductive M : Type → Type :=

| ret : $A \to$ M A

| bind : M $A \to (A \to$ M $B) \to$ M B

| mtry : M $A \to$ (Exception → M $A) \to$ M A

| raise : Exception → M A

| mfix : $((\forall x : A.$ M $(B\ x)) \to (\forall x : A.$ M $(B\ x))) \to \forall x : A.$ M $(B\ x)$

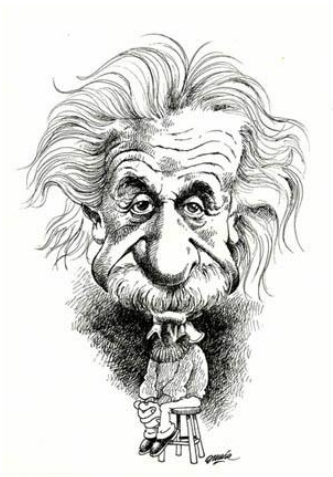
...

Mtac: tactic execution

Key idea #2:

- Execute (**run**) tactics at type inference.

Proof dev



Type inference

Kernel

Mtac: tactic execution

Key idea #2:

- Execute (**run**) tactics at type inference.

$(\lambda x.\ x)\ 1$

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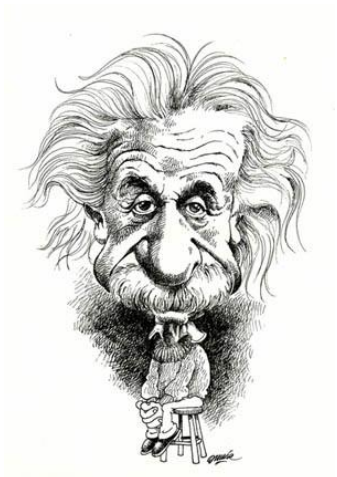

Type inference

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Mtac: tactic execution

Key idea #2:

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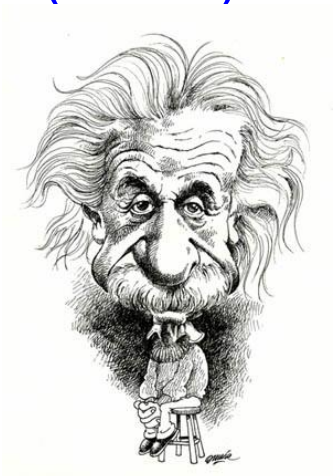
$(\lambda x.\ x)\ 1 \longrightarrow (\lambda x.\ x)\ 1$

Proof dev



Type inference

Kernel

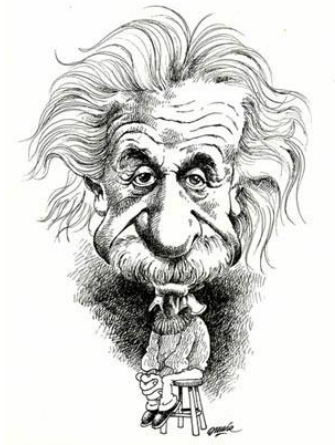
Mtac: tactic execution

Key idea #2:

- Execute (**run**) tactics at type inference.

$(\lambda x.\ x)\ 1 \longrightarrow (\lambda x : \mathsf{nat}.\ x)\ 1$

Proof dev



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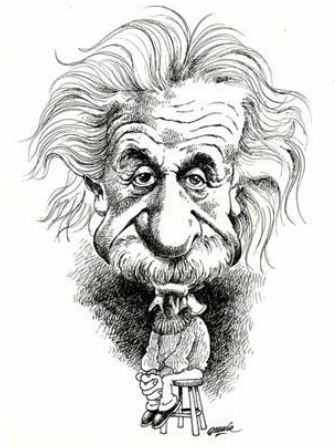

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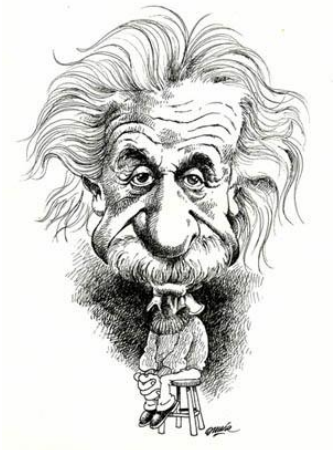


Type inference

Kernel

Mtac: tactic execution

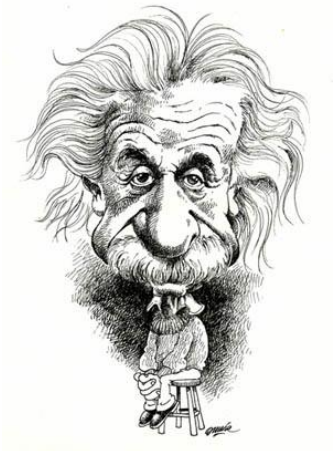
Key idea #2:

- Execute (**run**) tactics at type inference.

$$\Gamma \vdash e \hookrightarrow e' : A$$

$(\lambda x.\ x)\ 1 \longrightarrow (\lambda x : \mathsf{nat}.\ x)\ 1 \longrightarrow (\lambda x : \mathsf{nat}.\ x)\ 1$

Proof dev



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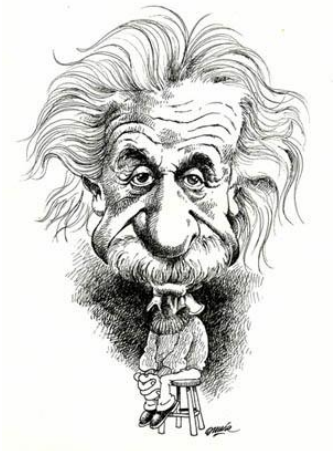

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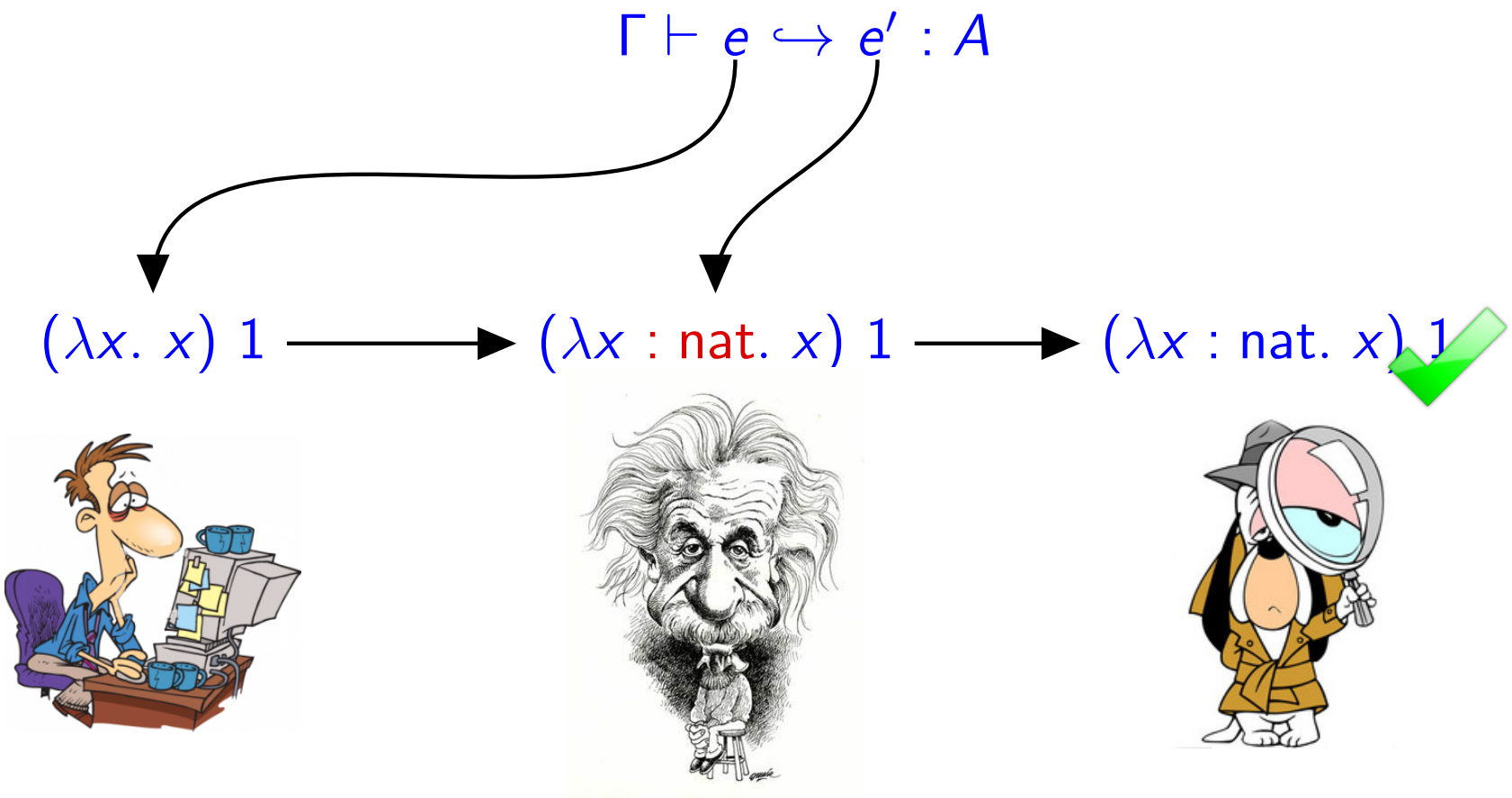


Mtac: tactic execution

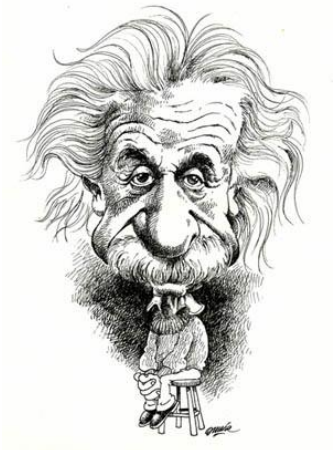
Key idea #2:

- Execute (**run**) tactics at type inference.

$$\frac{\Gamma \vdash t \hookrightarrow t' : \mathsf{M}\ A \qquad \Gamma \vdash t' \rightsquigarrow^* \mathbf{ret}\ e}{\Gamma \vdash \mathbf{run}\ t \hookrightarrow e : A}$$

$(\lambda x.\ x)\ 1 \longrightarrow (\lambda x : \mathsf{nat}.\ x)\ 1 \longrightarrow (\lambda x : \mathsf{nat}.\ x)\ 1$

Proof dev



Type inference

Kernel

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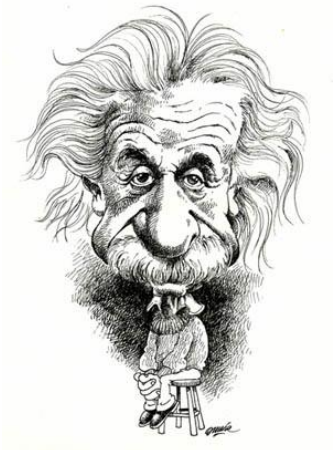

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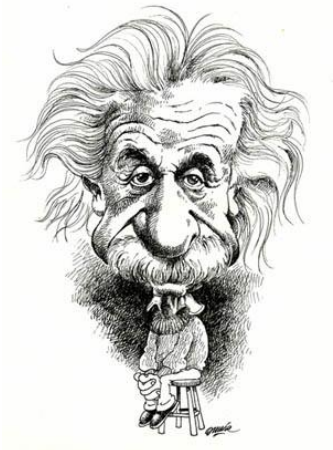

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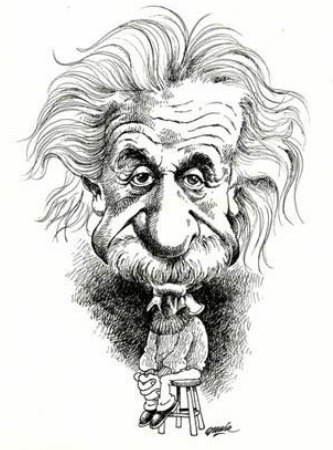

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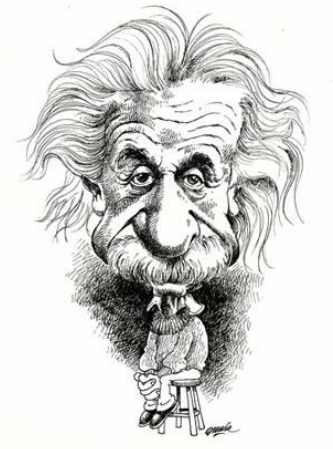

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Kernel

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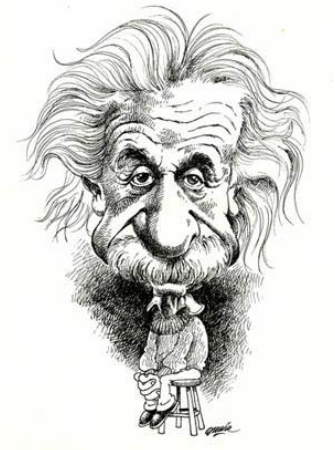

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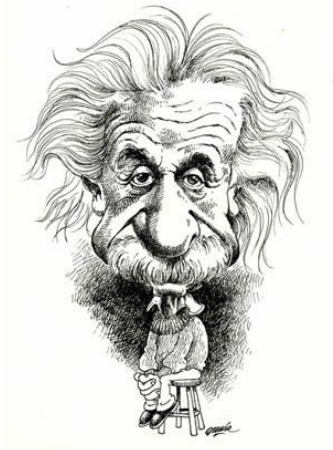

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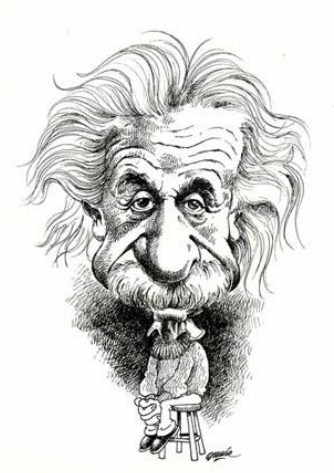

Type inference

Kernel unmodified

Mtac: tactic execution

Key idea #2:

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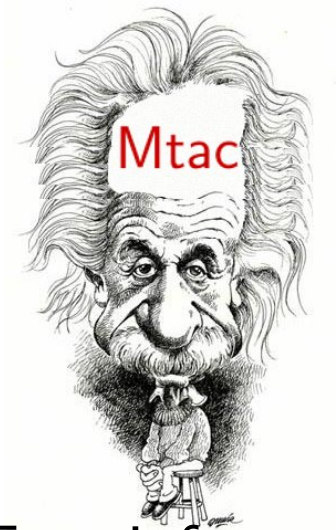


Type inference

Kernel unmodified

Related work

(Some) other typed tactic languages:

- VeriML (Stampoulis & Shao '10)
  Beluga (Pientka '09)
  Delphin (Poswolsky & Schürmann '08)

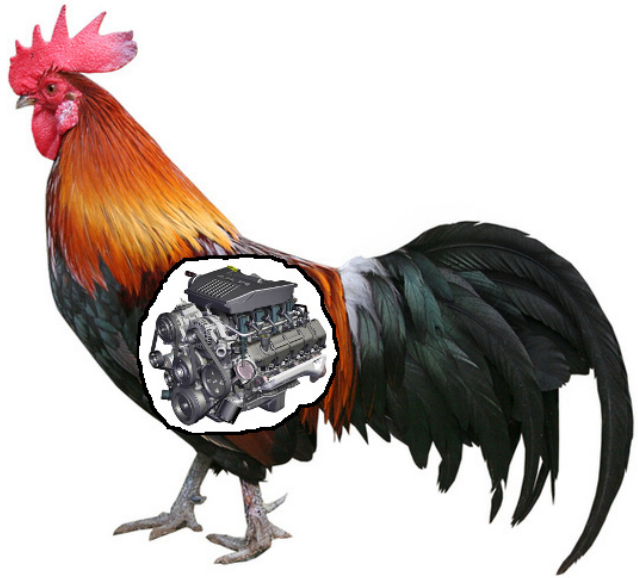
  Stronger typechecker, difficult to incorporate into Coq.

- Lemma Overloading (Gonthier, Ziliani, *et al.*, '11)

  Logic programming style, convoluted semantics.

In the paper

- Binder manipulation.
- Examples.
- Formalization.
- Implementation details.
- Related work.



http://plv.mpi-sws.org/mtac/